Educate.ie
MATHEMATICS SOLUTIONS
Leaving Certificate
Ordinary Level

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Sample Paper 1: Paper 1

Question 1 (25 marks)

Question 1 (a)
The value of the house has depreciated by 25%, which means that it is now 75% of its original value.
€450 000 × 0.75 = €337 500

Question 1 (b)
The contents have a value of 95% of the original value after 1 year, and 95% of this value after 2 years, and so on.
€150 000 × 0.95^5 = €116 067

Question 1 (c) (i)
Value in 2007:
€450 000 + €150 000 = €600 000

Question 1 (c) (ii)
Value in 2012:
€337 500 + €116 067 = €453 567

Question 1 (d)
Decrease in value of contents and building = €600 000 – €453 567 = €146 433

$$\% \text{ decrease} = \frac{146433}{600000} \times 100\% = 24.4\%$$

Question 1 (e)
€$780 \times 0.244 =$ €190.32 [The premium should decrease by 24.4%.]
$\therefore$ €$780 -$ €$190.32 =$ €589.68

Question 2 (25 marks)

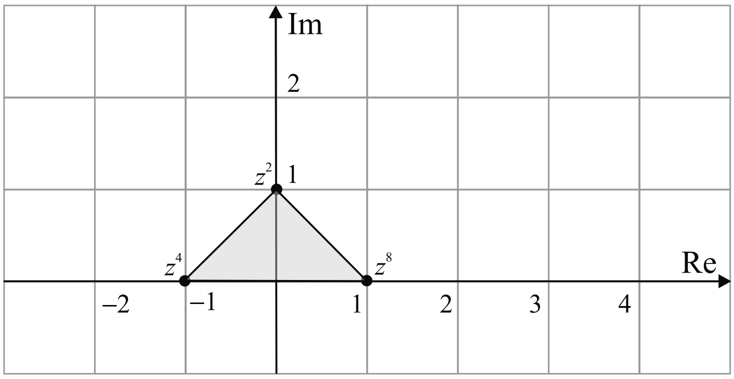
Question 2 (a)

$$z=\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}i$$

$$z^2=\left(\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}i\right)\left(\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}i\right)=\frac{1}{2}+\frac{1}{2}i+\frac{1}{2}i+\frac{1}{2}i^2=\frac{1}{2}+i-\frac{1}{2}=i$$

$$z^4=z^2\times z^2=(i)(i)=i^2=-1$$

$$z^8=z^4\times z^4=(-1)(-1)=1$$



$$\text{Area}=\tfrac{1}{2}\times\text{Base}\times\text{Height}=\tfrac{1}{2}(2)(1)=1$$

Question 2 (b)

$$z^{32}=(z^8)^4=1^4=1$$

$$z^{33}=z^{32}\times z^1=1\times\left(\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}i\right)=\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}i$$

Question 3 (25 marks)

Question 3 (a)

$$f(x)=ax+b$$

$$f(0)=1\Rightarrow a(0)+b=1$$

$$b=1$$

$$f(-2)=-5\Rightarrow -2a+1=-5$$

$$6=2a$$

$$\therefore a=3$$

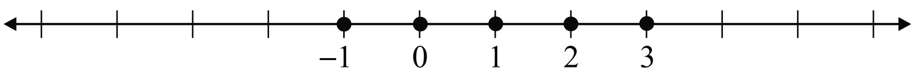
$$f(x)=3x+1$$

Question 3 (b)

$$-2\le 3x+1\le 10$$

$$-3\le 3x\le 9$$

$$-1\le x\le 3$$



Question 3 (c)

$$\frac{4^{\frac{3}{2}}}{\sqrt{2}} = \frac{(2^2)^{\frac{3}{2}}}{2^{\frac{1}{2}}} = \frac{2^3}{2^{\frac{1}{2}}} = 2^{3-\frac{1}{2}} = 2^{\frac{5}{2}}$$

$$4^{f(x)} = \frac{4^{\frac{3}{2}}}{\sqrt{2}}$$

$$4^{3x+1} = 2^{\frac{5}{2}}$$

$$(2^2)^{3x+1} = 2^{\frac{5}{2}}$$

$$2^{6x+2} = 2^{\frac{5}{2}}$$

$$\therefore 6x+2 = \tfrac{5}{2}$$

$$6x = \tfrac{5}{2} - 2 = \tfrac{1}{2}$$

$$\therefore x = \tfrac{1}{12}$$

Question 4 (25 marks)

Question 4 (a)

$$\frac{3(x+3)}{4} - \frac{2(x-3)}{3} = \frac{x+1}{2}$$ [Multiply by 12.]

$$9(x+3) - 8(x-3) = 6(x+1)$$

$$9x + 27 - 8x + 24 = 6x + 6$$

$$27 + 24 - 6 = 6x - 9x + 8x$$

$$45 = 5x$$

$$\therefore x = 9$$

Question 4 (b)

$$4x^2 - 8x - 21 = 0$$

$$(2x+3)(2x-7) = 0$$

$$\therefore x = -\tfrac{3}{2},\ \tfrac{7}{2}$$

$$4(k+\tfrac{1}{2})^2 - 8(k+\tfrac{1}{2}) - 21 = 0$$

$$\therefore (k+\tfrac{1}{2}) = -\tfrac{3}{2} \Rightarrow k = -\tfrac{3}{2} - \tfrac{1}{2} = -2$$

$$\therefore (k+\tfrac{1}{2}) = \tfrac{7}{2} \Rightarrow k = \tfrac{7}{2} - \tfrac{1}{2} = 3$$

Question 4 (c)

$$2\sqrt{x+1} = \sqrt{6}$$ [Square both sides.]

$$(2\sqrt{x+1})^2 = (\sqrt{6})^2$$

$$4(x+1) = 6$$

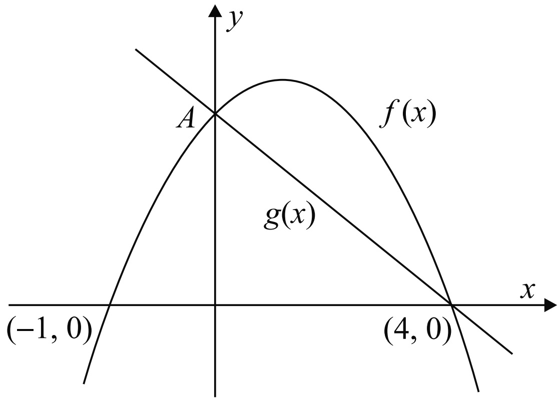
$$4x + 4 = 6$$

$$4x = 2$$

$$\therefore x = \tfrac{1}{2}$$

Question 5 (25 marks)

Question 5 (a)



$g(x) = -2x + d$

$(4, 0) \in g(x) \Rightarrow -2(4) + d = 0$

$-8 + d = 0$

$\therefore d = 8$

Question 5 (b)

$g(x) = -2x + 8$

$x = 0 : g(x) = -2(0) + 8 = 8$

$\therefore A(0, 8)$

Question 5 (c)

$f(x) = ax^2 + bx + c$

$(0, 8) \in f(x) \Rightarrow a(0)^2 + b(0) + c = 8$

$\therefore c = 8$

$f(x) = ax^2 + bx + 8$

$(-1, 0) \in f(x) \Rightarrow a(-1)^2 + b(-1) + 8 = 0$

$\therefore a - b = -8....(\mathbf{1})$

$(4, 0) \in f(x) \Rightarrow a(4)^2 + b(4) + 8 = 0$

$16a + 4b + 8 = 0$

$\therefore 4a + b = -2....(\mathbf{2})$

$(\mathbf{1}) + (\mathbf{2}) : 5a = -10 \Rightarrow a = -2$

$-2 - b = -8....(\mathbf{1}) \Rightarrow b = 6$

$\therefore f(x) = -2x^2 + 6x + 8$

Question 5 (d)

$f(x) = -2x^2 + 6x + 8$

$f'(x) = -4x + 6$

$f'(x) = 0 \Rightarrow -4x + 6 = 0$

$\therefore x = \frac{3}{2}$

$f(\frac{3}{2}) = -2(\frac{3}{2})^2 + 6(\frac{3}{2}) + 8 = \frac{25}{2}$

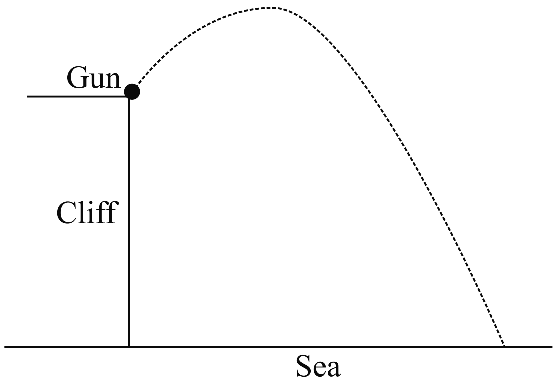
Local maximum: $(\frac{3}{2}, \frac{25}{2})$

QUESTION 6 (25 MARKS)

Question 6 (a)

$h = 80 + 30t - 5t^2$

$t = 0 : h = 80 + 30(0) - 5(0)^2 = 80 \text{ m}$



Question 6 (b)

$h = 80 + 30t - 5t^2$

$t = 4 : h = 80 + 30(4) - 5(4)^2$

$= 80 + 120 - 80$

$= 120 \text{ m}$

Question 6 (c)

$h = 80 + 30t - 5t^2$

$h = 0 : 0 = 80 + 30t - 5t^2$

$5t^2 - 30t - 80 = 0$

$t^2 - 6t - 16 = 0$

$(t + 2)(t - 8) = 0$

$\therefore t = 8 \text{ s}$

Question 6 (d)

$h = 80 + 30t - 5t^2$

$\frac{dh}{dt} = 30 - 10t$

$\frac{dh}{dt} = 0 \Rightarrow 30 - 10t = 0$

$30 = 10t$

$\therefore t = 3 \text{ s}$

$h = 80 + 30(3) - 5(3)^2$

$= 80 + 90 - 45$

$= 125 \text{ m}$

Question 7 (50 marks)

Question 7 (a)

$$T_n = \frac{g^n - (1-g)^n}{\sqrt{5}}$$

$$T_1 = \frac{1.618034^1 - (1-1.618034)^1}{\sqrt{5}} \approx 1$$

$$T_2 = \frac{1.618034^2 - (1-1.618034)^2}{\sqrt{5}} \approx 1$$

$$T_3 = \frac{1.618034^3 - (1-1.618034)^3}{\sqrt{5}} \approx 2$$

$$T_4 = \frac{1.618034^4 - (1-1.618034)^4}{\sqrt{5}} \approx 3$$

$$T_5 = \frac{1.618034^5 - (1-1.618034)^5}{\sqrt{5}} \approx 5$$

First 5 terms of the Fibonacci Sequence:
1, 1, 2, 3, 5

Question 7 (b)

1 + 1 = 2
2 + 1 = 3
3 + 2 = 5
5 + 3 = 8
8 + 5 = 13
13 + 8 = 21

Next 3 terms of the Fibonacci Sequence: 8, 13, 21

Question 7 (c)

1 + 1 + 2 + 3 + 5 + 8 = 4(5)
20 = 20

Question 7 (d)

4, –2, 2, 0, 2, 2, 4, 6

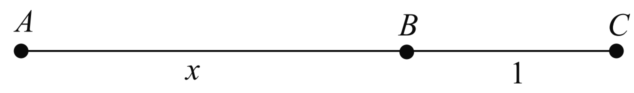
Question 7 (e)

$x, y, x + y, x + 2y, 2x + 3y, 3x + 5y$

$x + y + x + y + x + 2y + 2x + 3y + 3x + 5y = 4(2x + 3y)$
$8x + 12y = 8x + 12y$

Question 7 (f) (i)

$$\frac{|AB| + |BC|}{|AB|} = \frac{|AB|}{|BC|} \Rightarrow \frac{x+1}{x} = \frac{x}{1}$$



Question 7 (f) (ii)

$$\frac{x+1}{x} = x$$

$$x + 1 = x^2$$

$$x^2 - x - 1 = 0 \qquad \boxed{x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}}$$

$$a = 1,\ b = -1,\ c = -1$$

$$\therefore x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(-1)}}{2(1)} = \frac{1 \pm \sqrt{1+4}}{2} = \frac{1 \pm \sqrt{5}}{2} = 1.618034$$

Question 8 (50 marks)

Question 8 (a) (i)

19			
13	6		
3	10	–4	
–3	6	4	–8

Question 8 (a) (ii)

10			
$x + 2y + 3$	$y + 12$		
$x + y$	$y + 3$	9	
x	y	3	6

$10 = x + 2y + 3 + y + 12$

$10 = x + 3y + 15$

$-5 = x + 3y$

$x + 3y = -5 \; (\times -2)$

$2x - y = 4$

$-2x - 6y = 10$

$2x - y = 4$

$-7y = 14 \Rightarrow y = -2$

$x + 3(-2) = -5$

$x - 6 = -5$

$\therefore x = 1$

Question 8 (a) (iii)

kx		
x^2	$6x - x^2$	
$x(x - 2)$	$2x$	$4x - x^2$

$kx = x^2 + 6x - x^2$

$kx = 6x$

$\therefore k = 6$

Question 8 (b) (i)

The container has 40% acid and 60% water.

Volume of acid = $2 \times 0.4 = 0.8$ l

Volume of water = $2 \times 0.6 = 1.2$ l

Question 8 (b) (ii)

The container has 35% acid and 65% water.
Volume of acid = $0.35x$ litres
Volume of water = $0.65x$ litres

Question 8 (c) (i)

$x + y = 12$

Question 8 (c) (ii)

Bottle 1: Volume of acid = $0.5x$
Bottle 2: Volume of acid = $0.2y$
Mixture: Volume of acid $= 12 \times 0.3 = 3.6$

$0.5x + 0.2y = 3.6$

Question 8 (c) (iii)

$x + y = 12 \ (\times -0.2)$
$0.5x + 0.2y = 3.6$

$$\begin{array}{l} -0.2x - 0.2y = -2.4 \\ \ \ 0.5x + 0.2y = 3.6 \\ \hline \ \ 0.3x \qquad\quad = 1.2 \Rightarrow x = 4 \ 1 \end{array}$$

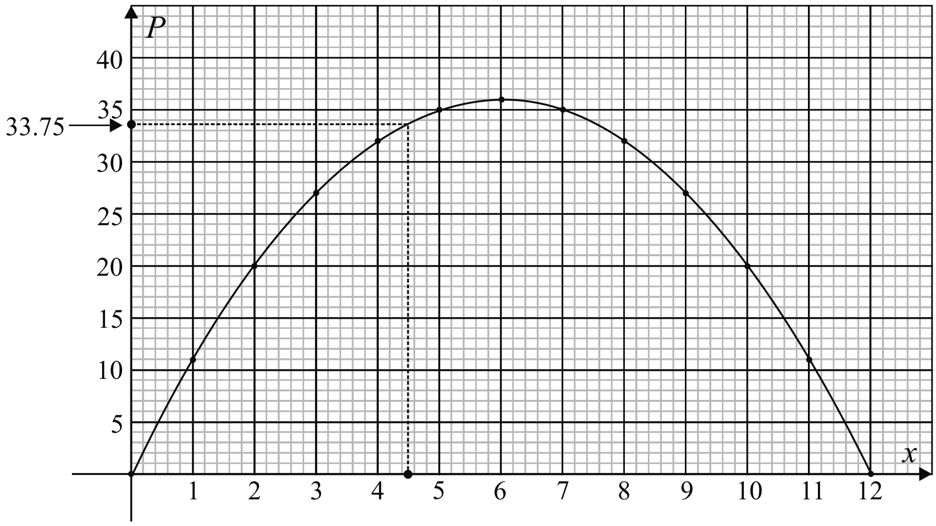
$x + y = 12$
$4 + y = 12$
$\therefore y = 8 \ 1$

QUESTION 9 (50 MARKS)

Question 9 (a)

x	0	1	2	3	4	5	6	7	8	9	10	11	12
y	12	11	10	9	8	7	6	5	4	3	2	1	0
$P = xy$	0	11	20	27	32	35	36	35	32	27	20	11	0

Question 9 (b)



Question 9 (c)

Maximum value of P = 36 (x = 6, y = 6)

Question 9 (d)

At x = 4.5, P = 33.75 (using graph)

$$P = xy \Rightarrow y = \frac{P}{x} = \frac{33.75}{4.5} = 7.5 \text{ (using algebra)}$$

Question 9 (e)

$$x + y = 12 \Rightarrow y = 12 - x$$

Question 9 (f)

$$P = xy = x(12 - x) = 12x - x^2$$

Question 9 (g)

$$P = 12x - x^2$$

$$\frac{dP}{dx} = 12 - 2x$$

$$\frac{dP}{dx} = 0 \Rightarrow 12 - 2x = 0$$

$$12 = 2x$$

$$\therefore x = 6$$

$$y = 12 - x = 12 - 6 = 6$$

Question 9 (h)

You obtained the same maximum value by using the graph or using calculus.

Sample Paper 1: Paper 2

Question 1 (25 marks)

Question 1 (a)

January		June
9	7	5 7 7 8 9
8 8 6 6 5 5 5 4 2 1 0	8	0 2 2 3 3 3 4 5 6
3 1	9	

Question 1 (b)

January		June
9	7	5 7 7 8 9
8 8 6 6 $\boxed{5\ \ 5}$ 5 4 2 1 0	8	0 $\boxed{2\ \ 2}$ 3 3 3 4 5 6
3 1	9	

Median in January = 85 kg
Median in June = 82 kg

Question 1 (c)

$$\% \text{ decrease in median weight} = \frac{3}{85} \times 100\% = 3.5\%$$

Question 2 (25 marks)

Question 2 (a) (i)

Possible outcomes: Head, Tail

Question 2 (a) (ii)

The coin is biased if the probability of tossing a head or a tail is **not** equal to 0.5.

Question 2 (b) (i)

Group B, because they tossed the coin more times than Group A, and so it is more likely that their estimate is closer to the true probability.

Question 2 (b) (ii)

$$\text{Expected number of heads} = 200 \times 0.44 = 88$$

Question 2 (b) (iii)

$$\text{Expected number of heads} = 500 \times 0.46 = 230$$

Total number of heads = 88 + 230 = 318

$$P(\text{Head}) = \frac{318}{700} = 0.454$$

Question 2 (c)

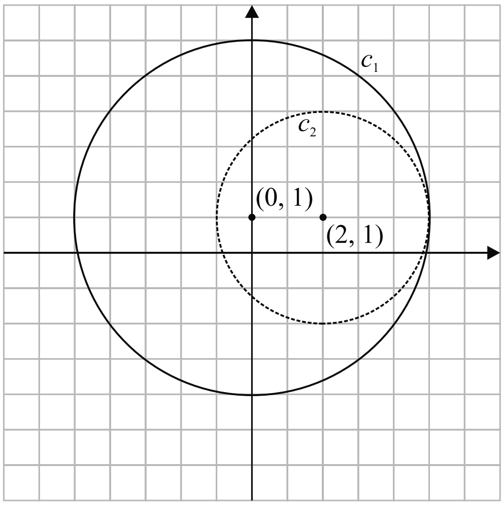
P(Head) = 0.54, P(Tail) = 0.546

$$\begin{aligned} &P(\text{At least 2 heads}) \\ &= P(3 \text{ heads, } 0 \text{ tails}) + P(2 \text{ heads, } 1 \text{ tail}) \\ &= (0.454)^3(0.546)^0 + 3(0.454)^2(0.546) \\ &= 0.43 \end{aligned}$$

Bernoulli Trials
$p = P(\text{Success})$, $q = P(\text{Failure})$
$P(r \text{ successes}) = {}^nC_r p^r q^{n-r}$

Question 3 (25 marks)

Question 3 (a)



Question 3 (b)

$$\boxed{(x-h)^2+(y-k)^2=r^2}$$

Centre (0, 1), $r = 5$

$(x-0)^2+(y-1)^2=5^2$

$x^2+(y-1)^2=25$

Question 3 (c)

$A(4,\ 4)\in c_1$?

$4^2+(4-1)^2=4^2+3^2=16+9=25$

A is on c_1 as it satisfies the equation of the circle.

$B(3, 5)$ is also on c_1.

$\therefore 3^2+(5-1)^2=3^2+4^2=9+16=25$

Question 3 (d)

Circle c_2: Centre(2, 1), $r = 3$

$(x-2)^2+(y-1)^2=9$

Question 4 (25 marks)

Question 4 (a)

$l: 4x - 3y + 12 = 0$

x-intercept: Put $y = 0 \Rightarrow 4x - 3(0) + 12 = 0$

$4x = -12$

$x = -3$

$\therefore A(-3, 0)$

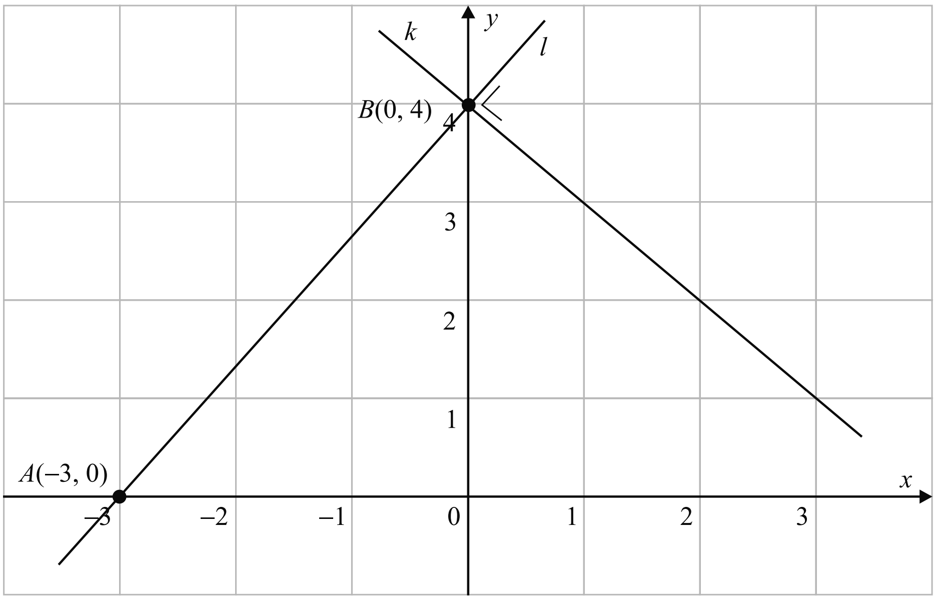
y-intercept: Put $x = 0 \Rightarrow 4(0) - 3y + 12 = 0$

$-3y = -12$

$y = 4$

$\therefore B(0, 4)$

Question 4 (b)



Question 4 (c)

Slope of l: $A(-3, 0)$, $B(0, 4)$

$$m = \frac{4-0}{0-(-3)} = \frac{4}{3} \qquad \boxed{m = \frac{y_2 - y_1}{x_2 - x_1}}$$

Equation of k: Point $B(0, 4)$, slope $m^{\perp} = -\dfrac{3}{4}$

$y - 4 = -\frac{3}{4}(x - 0)$ $\boxed{y - y_1 = m(x - x_1)}$

$4y - 16 = -3x$

$3x + 4y - 16 = 0$

Question 4 (d)

$(4t, -2t) \in k \Rightarrow 3(4t) + 4(-2t) - 16 = 0$

$12t - 8t = 16$

$4t = 16$

$\therefore t = 4$

Question 5 (25 marks)

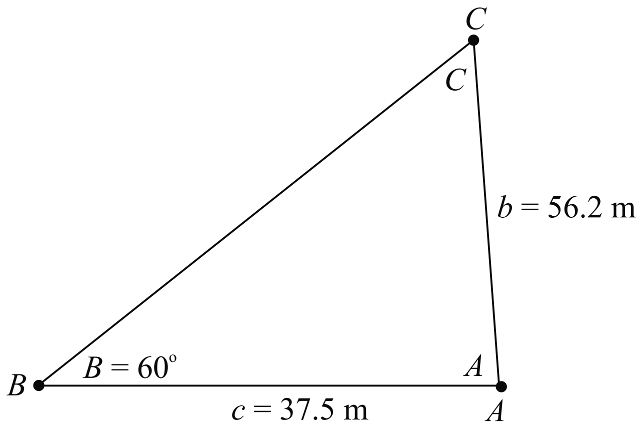
Question 5 (a) (i)

$$\boxed{\frac{\sin A}{a}=\frac{\sin B}{b}=\frac{\sin C}{c}}$$

$$\frac{\sin C}{37.5}=\frac{\sin 60^\circ}{56.2}$$

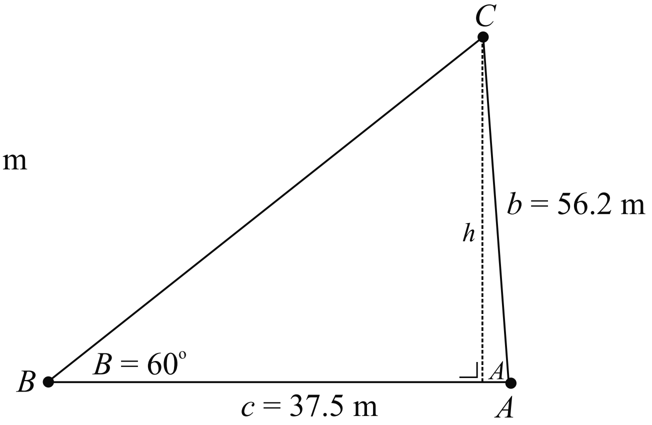
$$\therefore C=\sin^{-1}\left(\frac{37.5\sin 60^\circ}{56.2}\right)=35.3^\circ$$

$$|\angle CAB| = A = 180^\circ - 35.3^\circ - 60^\circ = 84.7^\circ$$



Question 5 (a) (ii)

$$\sin A=\frac{h}{56.2}\Rightarrow h=56.2\sin 84.7^\circ=55.96\text{ m}$$



Question 5 (b)

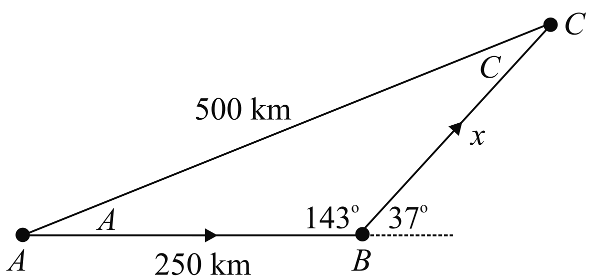
$$\frac{\sin C}{250}=\frac{\sin 143^\circ}{500}$$

$$\therefore C=\sin^{-1}\left(\frac{250\sin 143^\circ}{500}\right)=17.5^\circ$$

$$A=180^\circ-17.5^\circ-143^\circ=19.5^\circ$$

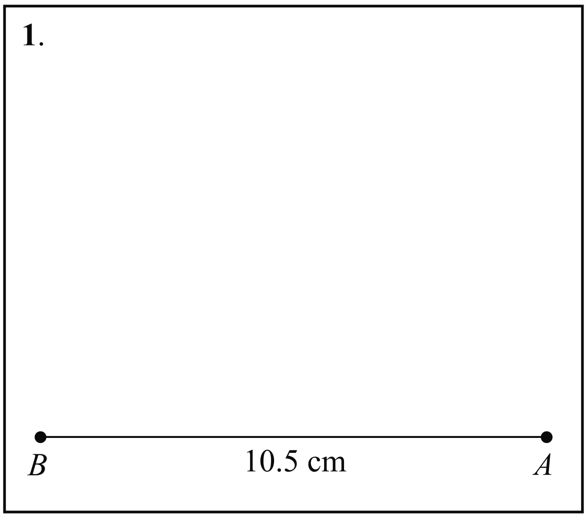
$$\frac{x}{\sin 19.5^\circ}=\frac{500}{\sin 143^\circ}$$

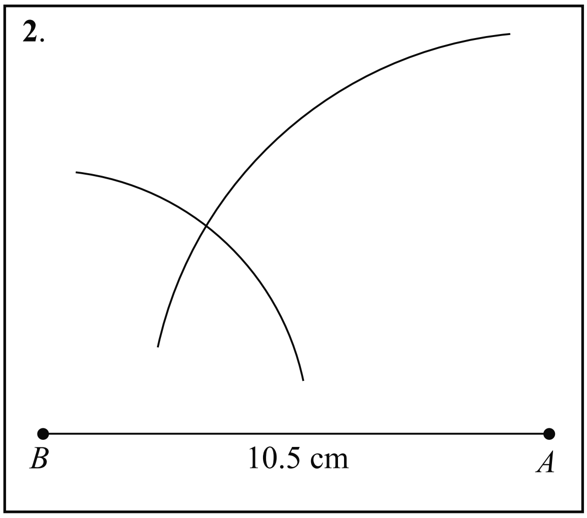
$$\therefore x=\frac{500\sin 19.5^\circ}{\sin 143^\circ}=277.3\text{ km}$$

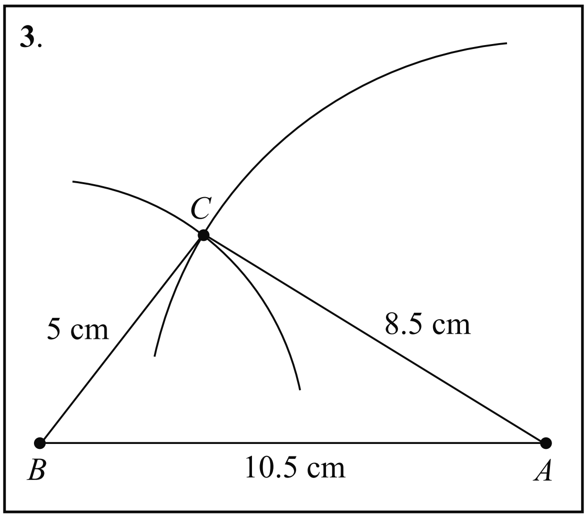


Question 6 (25 marks)

Question 6 (a)

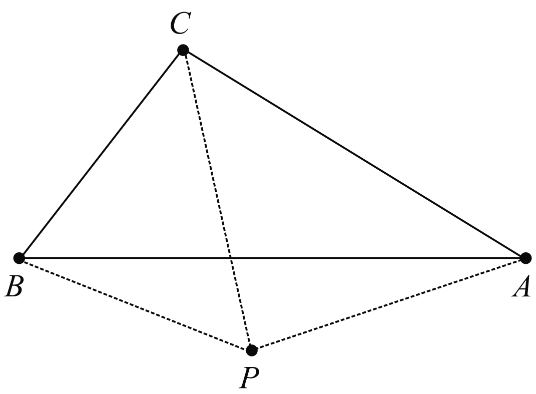




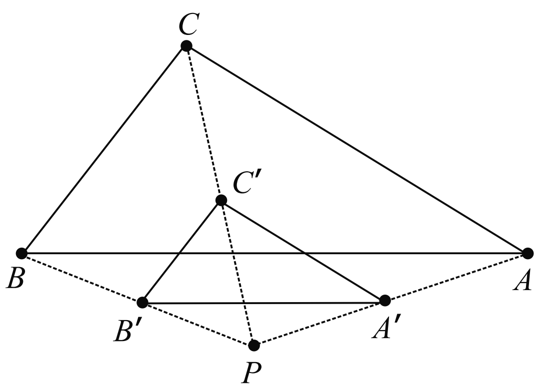


1. Draw a line AB of length 10.5 cm.
2. Use a compass to draw arcs of 5 cm and 8.5 cm.
3. Join A and B to the point of intersection of the arcs to form triangle ABC.

Question 6 (b)



1. Mark in a point P and draw lines from P to A, B and C.



2. Measure the length of each of these lines with a ruler. Multiply these distances by 0.4 to give points A', B' and C'.

Question 6 (c)

$k = 0.4$

Image area $= 3.36 \text{ cm}^2$

Object area = ?

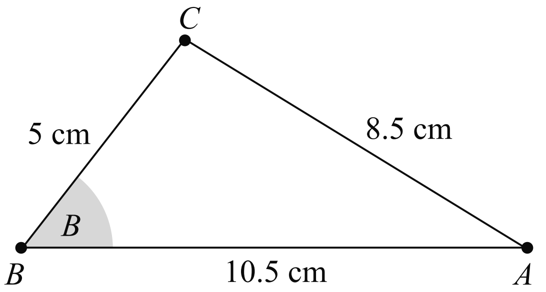
$$\boxed{k^2 = \frac{|\text{Image area}|}{|\text{Object area}|}}$$

$$0.4^2 = \frac{3.36}{\text{Object area}} \Rightarrow \text{Object area} = \frac{3.36}{0.4^2} = 21 \text{ cm}^2$$

Area $= \frac{1}{2}ac\sin B$

$21 = \frac{1}{2}(8.5)(10.5)\sin B$

$\therefore \sin B = \frac{42}{(5)(10.5)} = \frac{4}{5}$



Question 7 (75 marks)

Question 7 (a) (i)

60.3% voted yes. Therefore, 39.7% voted no.

$60.3\% \rightarrow 955\ 091$

$1\% \rightarrow \frac{955\ 091}{60.3}$

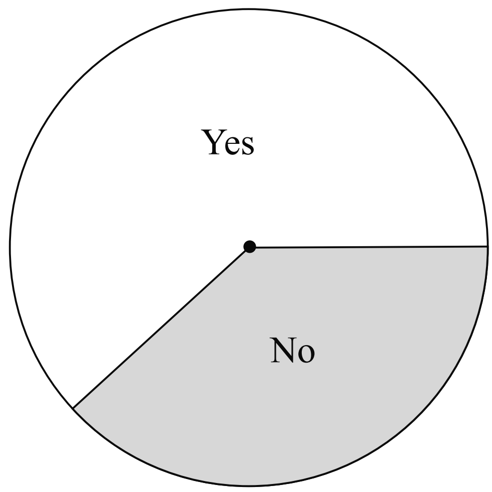
$39.7\% \rightarrow \frac{955\ 091}{60.3} \times 39.7 = 628\ 808$

Question 7 (a) (ii)

Total number of voters = 955 091 + 628 807 = 1 583 898

Yes vote: $\frac{60.3}{100} \times 360^\circ \approx 217^\circ$

No vote: $360^\circ - 217^\circ = 143^\circ$



Question 7 (b) (i)

People gave multiple answers to questions.

Question 7 (b) (ii)

28% of the 2000 people gave one of the reasons for voting ‘No’ as ‘Anti-Government’.

$2000 \times 0.28 = 560$ people

Question 7 (b) (iii)

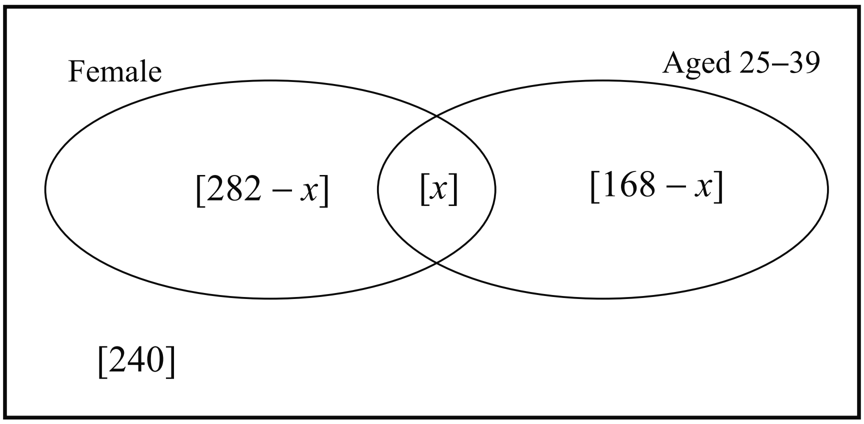
Pro-Eu: 18%
Dependent on EU: 16%
The maximum number of people that could have voted for **both** of these positions is 16%.
$2000 \times 0.16 = 320$ people

The minimum number of people is zero. In other words, none of the 18% who were ‘pro-EU’ were also ‘Dependent on EU’ and vice versa.

Question 7 (c) (i)

'Yes' voters: $1000 \times 0.6 = 600$
Females who voted 'Yes': $600 \times 0.47 = 282$
25–39-year-olds who voted 'Yes': $600 \times 0.28 = 168$
Males not aged 28-39 years old who voted 'Yes': $600 \times 0.40 = 240$

'Yes' voters: [600]



Question 7 (c) (ii)

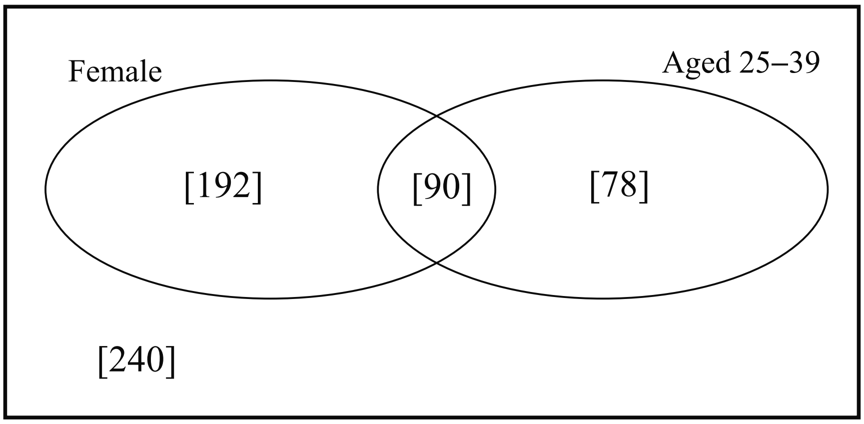
$282 - x + x + 168 - x + 240 = 600$

$690 - x = 600$

$690 - 600 = x$

$\therefore x = 90$

'Yes' voters: [600]



Question 7 (c) (iii)

P(Females aged 25–39 who votes 'Yes') $= \dfrac{90}{600} = \dfrac{3}{20}$

Question 7 (d) (i)

Expected number of females aged between 25 and 39 years of age $= 955\ 091 \times \dfrac{3}{20} = 143\ 264$

Question 7 (d) (ii)

Advantages of Telephone Survey

1. High Accessibility

Over 95% of the Irish population has a phone at their homes.

2. Good Quality Control

Trained interviewers can put the questions to the respondents in a uniform manner, promoting accuracy and precision in eliciting responses.

3. Anonymous Respondents

The telephone survey approach provides perhaps the highest level of anonymity for respondents who wish to keep their opinions confidential.

Disadvantages of Telephone Survey

1. Time-Constrained Interviews

Since telephone surveys may interrupt the personal time of the respondents, interviews conducted via phone are to be no longer than 15 minutes.

2. Hard-to-Reach Respondents

Many people use call screening to accept only calls that they are expecting.

3. Unseen Product

In market research, it is more ideal to conduct a face-to-face interview/survey rather than a telephone survey.

Question 7 (e)

Percentage turnout: $\frac{38\ 157}{81\ 587} \times 100\% = 46.8\%$

Valid poll: 38 157 – 187 = 37 970
No votes: 37 970 – 24 015 = 13 955

% no vote: $\frac{13\ 955}{37\ 970} \times 100\% = 36.75\%$

Question 7 (f) (i)

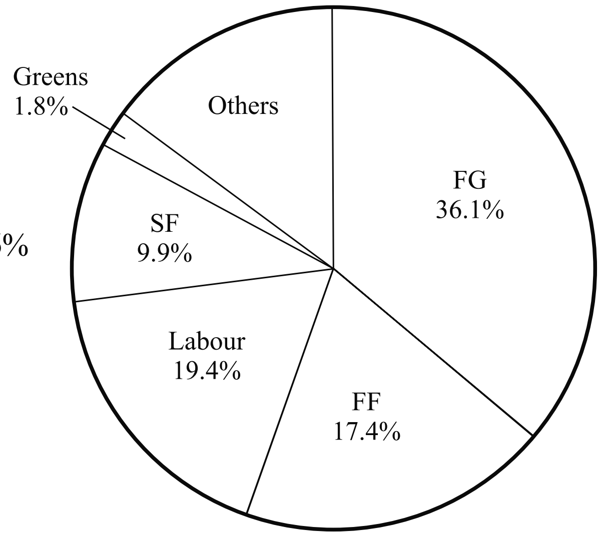
Date	Polling Company	FG	Lab	FF	SF	Others	Greens
07/01/2011	Red C	35%	21%	14%	14%	12%	4%
19/12/2010	Red C	34%	23%	17%	14%	10%	2%
16/12/2010	MRBI	30%	25%	17%	15%	11%	2%
03/12/2010	Red C	32%	24%	13%	16%	11%	3%
21/11/2010	Red C	33%	27%	17%	11%	8%	3%
	Average	32.8	24	15.6	14	10.4	2.8

Question 7 (f) (ii)

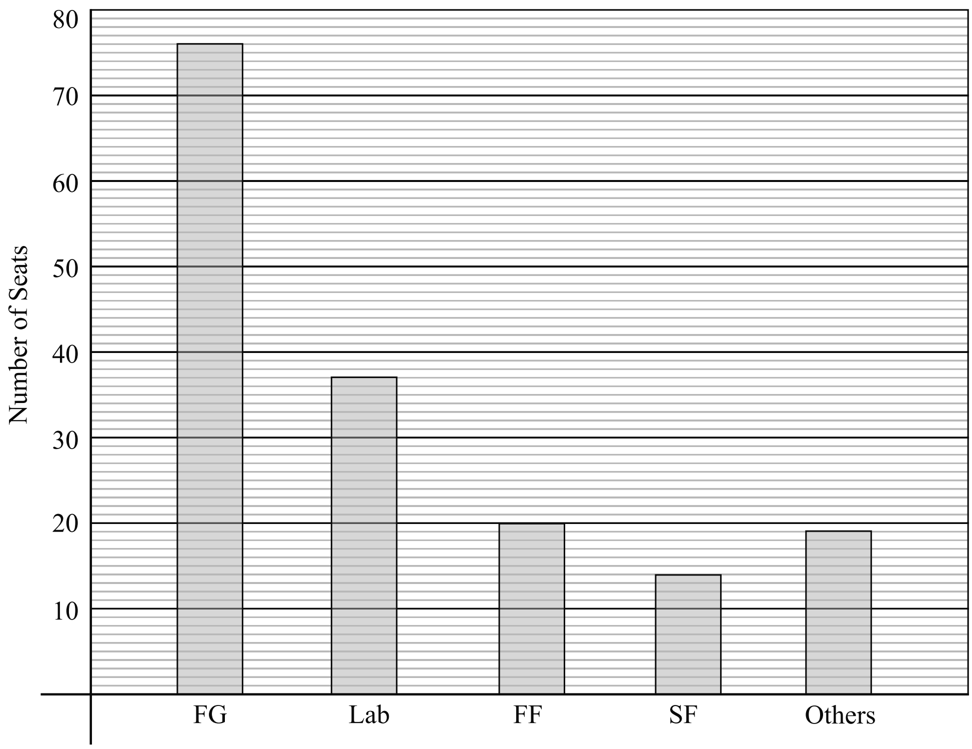
Others = 100% – 36.1% – 17.4% – 19.4% – 9.9% – 1.8% = 15.4%

Question 7 (f) (iii)

Error in FG's percentage = 36.1% – 32.8% = 3.3%
Error in FF's percentage = 17.4% – 15.6% = 1.8%
Error in Labour's percentage = 24% – 19.4% = 4.6%



Question 7 (g) (i)



Party	FG	Lab	FF	SF	Others
Number of seats	76	37	20	14	19

Total number of seats = 166

Question 7 (g) (ii)

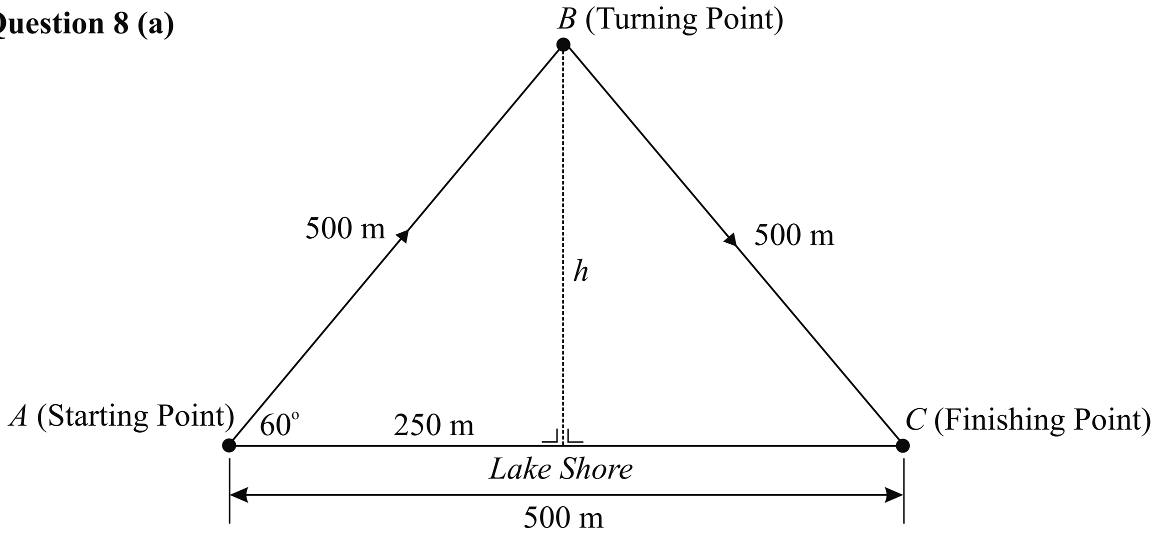
$36.1\% \to 76$ seats

$1\% \to \dfrac{76}{36.1} = 2.11$ seats

$1.8\% \to 2.11 \times 1.8 = 3.8$ (4 seats)

Question 8 (75 marks)

Question 8 (a)



(i) $|AB| = 500$ m, $|BC| = 500$ m,

(ii) Equilateral triangle

(iii) All angles in an equilateral triangle are 60°.

(iv) $250^2 + h^2 = 500^2$

$\therefore h = \sqrt{500^2 - 250^2} = 433 \text{ m}$

(v) Area $= \frac{1}{2}bh = \frac{1}{2}(500)(433) = 108\ 250 \text{ m}^2$

or

Area $= \frac{1}{2}ab\sin C = \frac{1}{2}(500)(500)\sin 60^\circ = 108\ 253 \text{ m}^2$

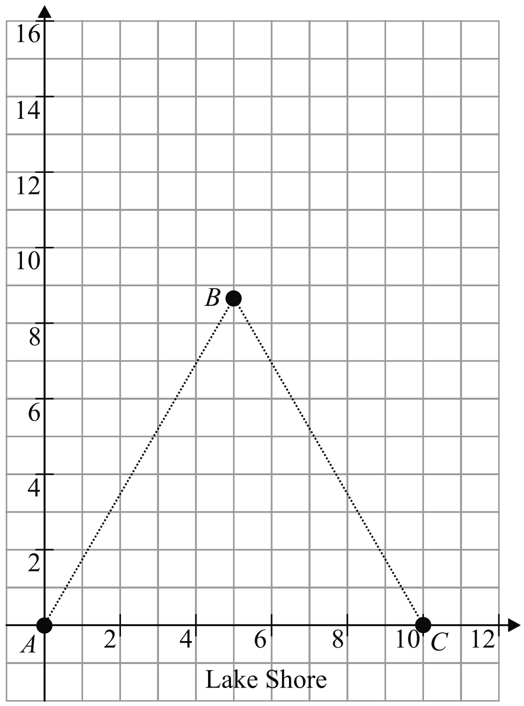
Question 8 (b)

Divide each distance by 50 to find its coordinates.

Starting point: $A(0, 0)$

Turning point: $B(5, 8.7)$

Finishing point: $C(10, 0)$



Question 8 (c) (i)

$$\boxed{\text{Speed} = \frac{\text{Distance}}{\text{Time}}}$$

Category 1: $1.5 \text{ km/h} = \frac{1 \text{ h}}{t} \Rightarrow t = \frac{1}{1.5} \text{ h} = \frac{2}{3} \text{h} = \frac{2}{3} \times 60 \text{ mins} = 40 \text{ mins}$

Category 2: $1.4 \text{ km/h} = \frac{1 \text{ h}}{t} \Rightarrow t = \frac{1}{1.4} \text{ h} = \frac{1}{1.4} \times 60 \text{ mins} \approx 43 \text{ mins}$

Category 3: $1.3 \text{ km/h} = \frac{1 \text{ h}}{t} \Rightarrow t = \frac{1}{1.3} \text{ h} = \frac{1}{1.3} \times 60 \text{ mins} \approx 46 \text{ mins}$

	Average Speed	Time
Category 1	1.5 km/h	40 mins
Category 2	1.4 km/h	43 mins
Category 3	1.3 km/h	46 mins

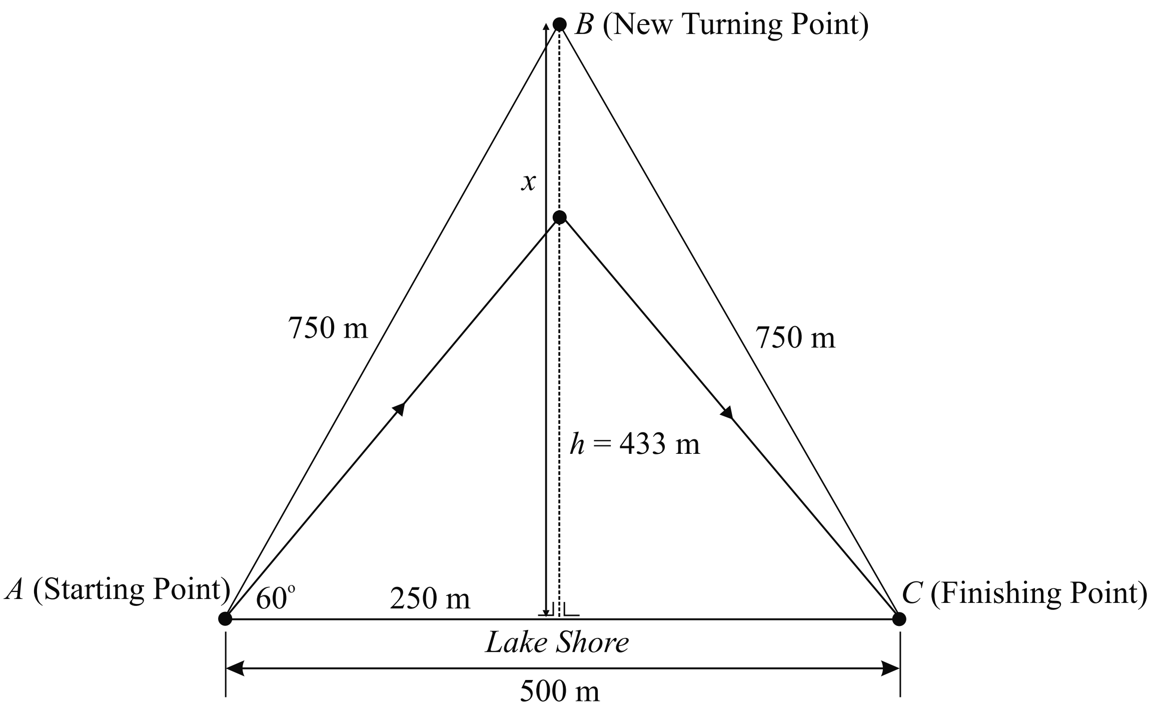
Question 8 (c) (ii)

Category 3 starts first and category 1 starts last. The race will be exciting, with a close finish between all swimmers.

Question 8 (c) (iii)

Starting time of race: 3:00 p.m.
Second starting time: 3:03 pm
Third starting time: 3:06 pm

Question 8 (d)

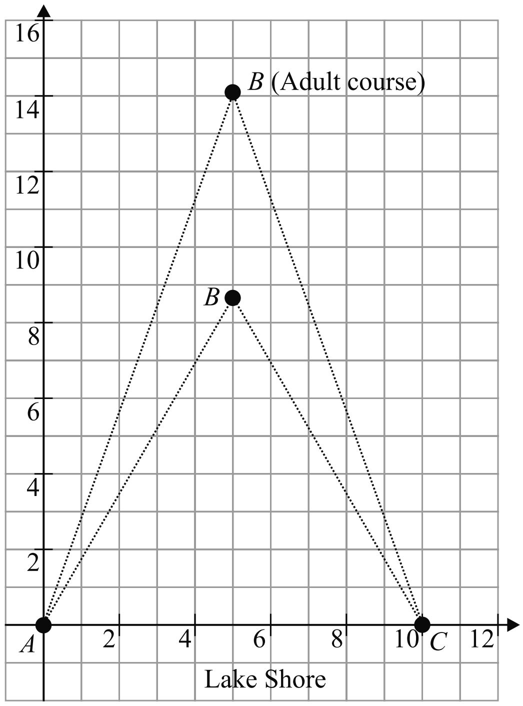


$250^2 + x^2 = 750^2$

$\therefore x = \sqrt{750^2 - 250^2} = 707$ m

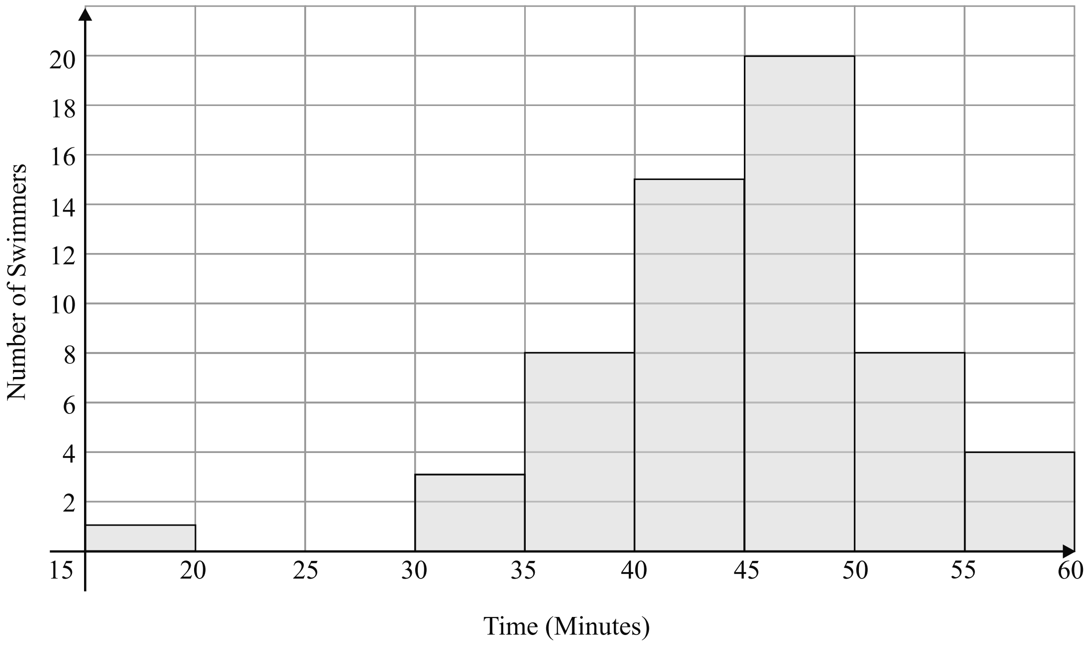
Extra distance $= 707 - 433 = 274$ m

New turning point: $B(5, 14.1)$



Question 8 (e) (i)

Time (Minutes)	Number of swimmers
15–20	1
20–25	0
25–30	0
30–35	3
35–40	8
40–45	15
45–50	20
50–55	8
55–60	4



(ii) An outlier is a result that is very different to the other results. It could have arisen because a top-class, professional swimmer took part.

(iii) The histogram has normal distribution if the outlier is excluded. Most of the swimmers were average swimmers who probably swam for recreation.

Sample Paper 2: Paper 1

Question 1 (25 marks)

Question 1 (a) (i)

$$f(x)=\frac{x-3}{2}$$

$$f(1)=\frac{1-3}{2}=\frac{-2}{2}=-1$$

$$f(2)=\frac{2-3}{2}=\frac{-1}{2}=-\frac{1}{2}$$

$$f(5)=\frac{5-3}{2}=\frac{2}{2}=1$$

Question 1 (a) (ii)

$$g(x)=\frac{2x-1}{3}$$

$$g(1)=\frac{2(1)-1}{3}=\frac{1}{3}$$

$$g(2)=\frac{2(2)-1}{3}=\frac{3}{3}=1$$

$$g(5)=\frac{2(5)-1}{3}=\frac{9}{3}=3$$

Question 1 (b)

$$\begin{aligned} f(x)+g(x) &= \frac{x-3}{2}+\frac{2x-1}{3} \\ &= \frac{3(x-3)+2(2x-1)}{6} \\ &= \frac{3x-9+4x-2}{6} \\ &= \frac{7x-11}{6} \end{aligned}$$

$$f(x)+g(x)=4$$

$$\therefore \frac{7x-11}{6}=4$$

$$7x-11=24$$

$$7x=24+11=35$$

$$x=\frac{35}{7}=5$$

Verify your answer: $f(5)+g(5)=1+3=4$

Question 1 (c)

$$6(f(x)+g(x))+11=x^2+12$$

$$6\left(\frac{7x-11}{6}\right)+11=x^2+12$$

$$7x-11+11=x^2+12$$

$$7x=x^2+12$$

$$x^2-7x+12=0$$

$$(x-3)(x-4)=0$$

$$\therefore x=3,\ 4$$

QUESTION 2 (25 MARKS)

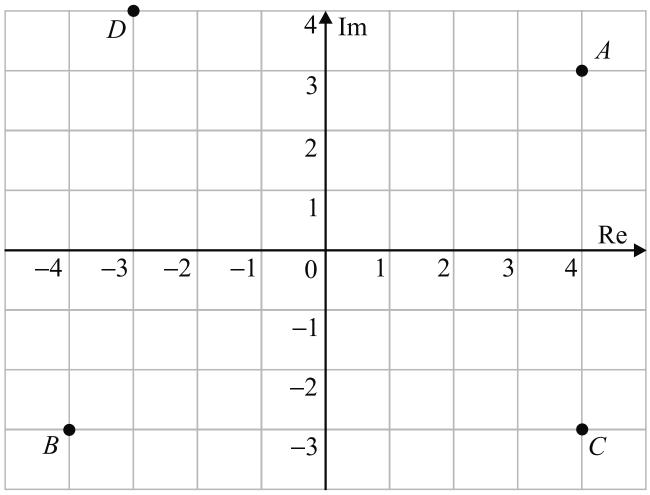
Question 2 (a)

$z = 4 + 3i \ (A)$

(i) $-z = -4 - 3i \ (B)$

(ii) $\bar{z} = 4 - 3i \ (C)$

(iii) $iz = 4i + 3i^2 = -3 + 4i \ (D)$



(iv) $O(0, 0)$, $A(4, 3)$

Slope of $OA = \frac{3-0}{4-0} = \frac{3}{4}$ $\boxed{m = \frac{y_2 - y_1}{x_2 - x_1}}$

$O(0, 0)$, $D(-3, 4)$

Slope of $OD = \frac{4-0}{-3-0} = -\frac{4}{3}$

$\frac{3}{4} \times -\frac{4}{3} = -1 \Rightarrow OA \perp OD$ $\boxed{\text{Perpendicular lines: } m_1 \times m_2 = -1}$

Question 2 (b) (i)

$w = 3 + 4i$

$w^2 = (3+4i)^2 = (3+4i)(3+4i)$

$= 9 + 12i + 12i + 16i^2$

$= 9 + 24i - 16$

$= -7 + 24i$

Question 2 (b) (ii)

$w = 3 + 4i \Rightarrow \bar{w} = 3 - 4i$

$(\bar{w})^2 = (3-4i)^2 = (3-4i)(3-4i)$

$= 9 - 12i - 12i + 16i^2$

$= 9 - 24i - 16$

$= -7 - 24i$

Question 2 (b) (iii)

$\frac{25}{w} = \frac{25}{3+4i}$

$= \frac{25}{(3+4i)} \times \frac{(3-4i)}{(3-4i)}$ [Multiply above and below by the conjugate of the denominator.]

$= \frac{25(3-4i)}{9 + 12i - 12i - 16i^2}$

$= \frac{25(3-4i)}{9+16} = \frac{25(3-4i)}{25}$

$= 3 - 4i = \bar{w}$

Question 3 (25 marks)

Question 3 (a)

Mass of sun $= 2.2\times10^{30}$ kg

Mass of earth $= 6\times10^{24}$ kg

$$\frac{6\times10^{24}}{2.2\times10^{30}}\times100\% = 2.73\times10^{-4}\%$$

Question 3 (b)

$$49^{-\frac{1}{2}} = \frac{1}{49^{\frac{1}{2}}} = \frac{1}{\sqrt{49}} = \frac{1}{7}$$

$$64^{\frac{2}{3}} = (64^{\frac{1}{3}})^2 = 4^2 = 16$$

Power Rules

$a^p a^q = a^{p+q}$

$\frac{a^p}{a^q} = a^{p-q}$

$(a^p)^q = a^{pq}$

$a^0 = 1$

$a^{-p} = \frac{1}{a^p}$

Question 3 (c)

$$\frac{a\sqrt{a}\times(2a)^3}{\sqrt{a^3}} = \frac{a^1\times a^{\frac{1}{2}}\times 8a^3}{(a^3)^{\frac{1}{2}}}$$

$$= \frac{8a^{\frac{9}{2}}}{a^{\frac{3}{2}}}$$

$$= 8a^{\frac{9}{2}-\frac{3}{2}}$$

$$= 8a^3$$

Question 3 (d)

$$25^{x+1} = 5^{x-1}$$

$$(5^2)^{x+1} = 5^{x-1}$$

$$5^{2x+2} = 5^{x-1}$$

$$\therefore 2x+2 = x-1$$

$$2x - x = -1-2$$

$$\therefore x = -3$$

Question 4 (25 marks)

Question 4 (a)

A prime number (or a prime) is a natural number greater than 1 that has no positive divisors other than 1 and itself.

List: 2, 3, 5, 7, 11, 13, 17

Question 4 (b)

Proper divisors of 6: 1, 2, 3

Proper divisors of 18: 1, 2, 3, 6, 9

2	18
3	9
3	3
	1

Question 4 (c)

6 = 3 + 2 + 1 (Perfect number)

$18 \neq 1+2+3+6+9$

Question 4 (d)

$$n = 2^{p-1}(2^p - 1)$$

$p = 2$:

$$n = 2^1(2^2-1) = 2(3) = 6$$

$$6 = 1+2+3$$

$$n = 2^{p-1}(2^p - 1)$$

$p = 3$:

$$n = 2^2(2^3-1) = 4(8-1) = 4(7) = 28$$

2	28
2	14
7	7
	1

Proper divisors of 28: 1, 2, 4, 7, 14

28 = 1 + 2 + 4 + 7 + 14

Question 5 (25 marks)

Question 5 (a)

$f(x) = x^3 - 6x^2 + 11x - 6$

$f(1) = (1)^3 - 6(1)^2 + 11(1) - 6 = 1 - 6 + 11 - 6 = 0$

$f(2) = (2)^3 - 6(2)^2 + 11(2) - 6 = 8 - 24 + 22 - 6 = 0$

$f(3) = (3)^3 - 6(3)^2 + 11(3) - 6 = 27 - 54 + 33 - 6 = 0$

Question 5 (b)

x	0.5	1.0	1.5	2.0	2.5	3.0	3.5
$y = f(x)$	–1.875	0	0.375	0	–0.375	0	1.875

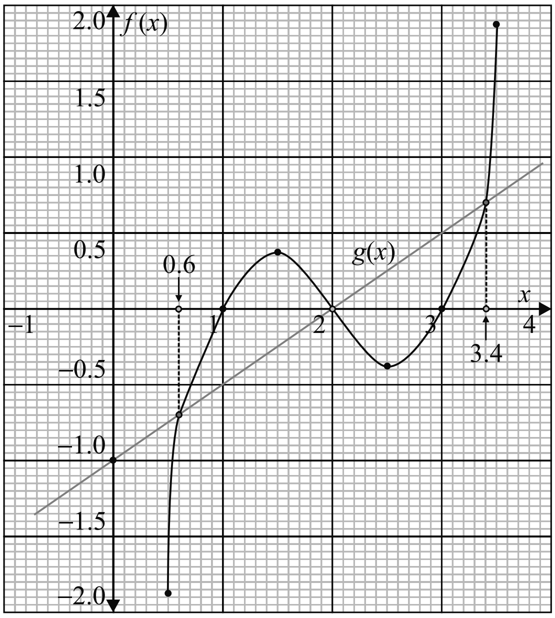
Question 5 (c) & (d)

$g(x)$ is a straight line. Find two points on this line so it can be drawn.

$g(x) = x - 2$

$g(1) = 1 - 2 = -1$

$g(2) = 2 - 2 = 0$



Question 5 (e)

$f(x) = g(x)$

$x^3 - 6x^2 + 11x - 6 = x - 2$

$x^3 - 6x^2 + 10x - 4 = 0$

Find solutions at which the two functions intersect.

$x = 0.6, 2, 3.4$

Question 6 (25 marks)

Question 6 (a)

$y = 2x^3 - 3x^2 - 12x + 4$

$\frac{dy}{dx} = 6x^2 - 6x - 12$

$\frac{dy}{dx} = 0 \Rightarrow 6x^2 - 6x - 12 = 0$ [The slope of the tangent is zero. Put the derivative equal to zero and solve for x.]

$x^2 - x - 2 = 0$

$(x+1)(x-2) = 0$

$\therefore x = -1, 2$

$x = -1 : y = 2(-1)^3 - 3(-1)^2 - 12(-1) + 4$
$= -2 - 3 + 12 + 4 = 11$

$x = 2 : y = 2(2)^3 - 3(2)^2 - 12(2) + 4$
$= 16 - 12 - 24 + 4 = -16$

Turning points: $(-1, 11)$, $(2, -16)$

Question 6 (b) (i)

$f(x) = x^2 + kx + 10$

$f'(x) = 2x + k$

Question 6 (b) (ii)

$f'(3) = 0 \Rightarrow 2(3) + k = 0$

$6 + k = 0$

$\therefore k = -6$

Question 6 (b) (iii)

$f(x) = x^2 - 6x + 10$

$f'(x) = 2x - 6$

$f(0) = 10 \Rightarrow (0, 10) \in t$

$f'(0) = m = -6$

Equation of t: Point (0, 10), $m = -6$

$y - 10 = -6(x - 0)$ $\boxed{y - y_1 = m(x - x_1)}$

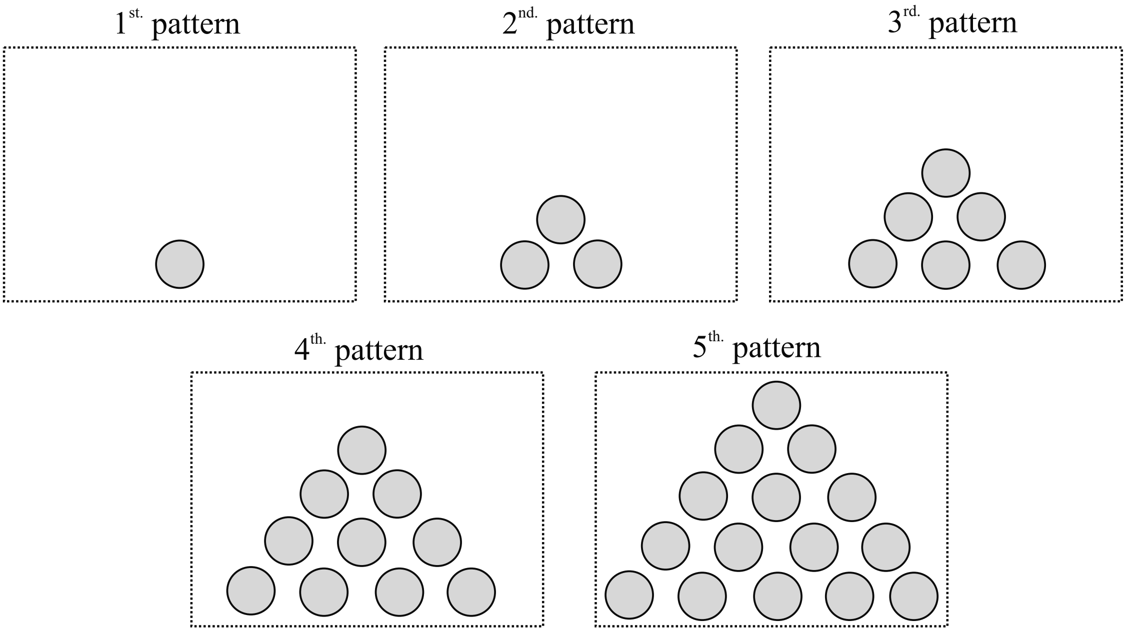
$y - 10 = -6x$

$6x + y - 10 = 0$

Sample 2 Paper 1

QUESTION 7 (50 MARKS)

Question 7 (a)



Question 7 (b)

Pattern	Number of counters
1st.	1
2nd.	3 = 1 + 2
3rd.	6 = 1 + 2 + 3
4th.	10 = 1 + 2 + 3 + 4
5th.	15 = 1 + 2 + 3 + 4 + 5

Question 7 (c)

15th triangular number = 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 + 11 + 12 + 13 + 14 + 15 = 120

Question 7 (d) (i)

$S_n = 1 + 2 + 3 + ... + n$

$a = 1,\ d = 1$

$S_n = \frac{n}{2}[2(1) + (n-1)1] = \frac{n}{2}[2 + n - 1]$

$\therefore S_n = \frac{n}{2}(n+1)$

Question 7 (d) (ii)

$S_{15} = \frac{15}{2}(15+1) = \frac{15}{2}(16) = 15 \times 8 = 120$

Question 7 (d) (iii)

$S_{100} = \frac{100}{2}(100+1) = 50(101) = 5050$

Question 7 (e)

$666 = \frac{n}{2}(n+1)$

$1332 = n(n+1)$

$1332 = n^2 + n$

$0 = n^2 + n - 1332$

$0 = (n-36)(n+37)$

$n = 36$

Finding the factors of 1332:

2	1332
2	666
3	333
3	111
	37

Question 7 (f) (i)

$$
\begin{array}{ccccccccccccccc}
 & & & & & & & 1 & & & & & & & \\
 & & & & & & 1 & & 1 & & & & & & \\
 & & & & & 1 & & 2 & & 1 & & & & & \\
 & & & & 1 & & 3 & & 3 & & 1 & & & & \\
 & & & 1 & & 4 & & 6 & & 4 & & 1 & & & \\
 & & 1 & & 5 & & 10 & & 10 & & 5 & & 1 & & \\
 & 1 & & 6 & & 15 & & 20 & & 15 & & 6 & & 1 & \\
1 & & \boxed{7} & & \boxed{21} & & \boxed{35} & & \boxed{35} & & \boxed{21} & & \boxed{7} & & 1
\end{array}
$$

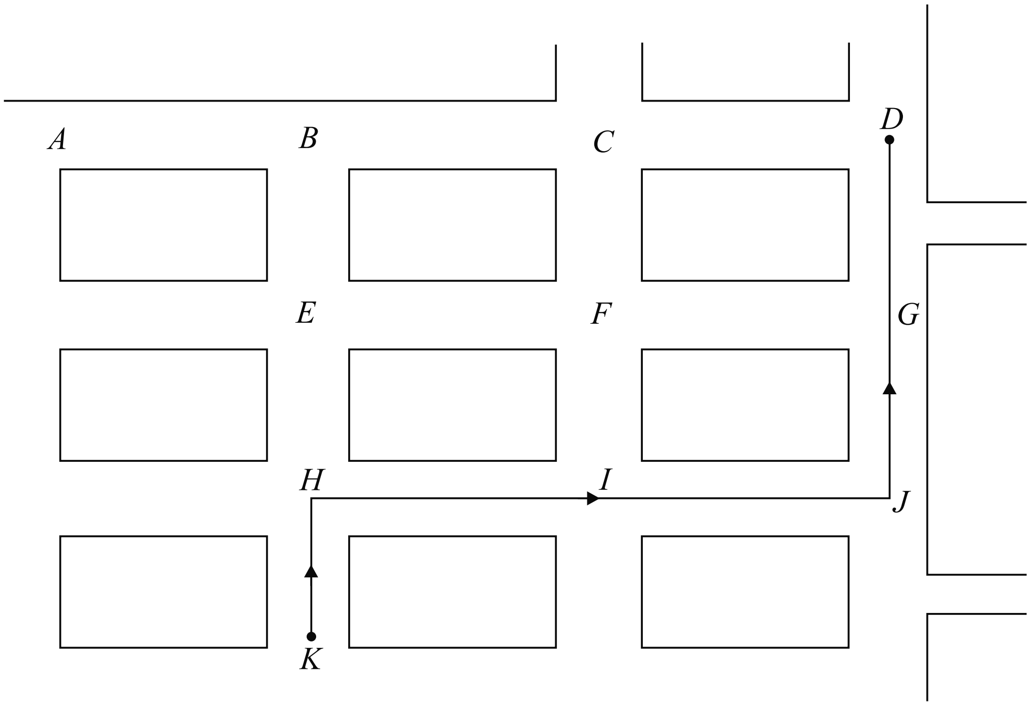
Question 7 (f) (ii)

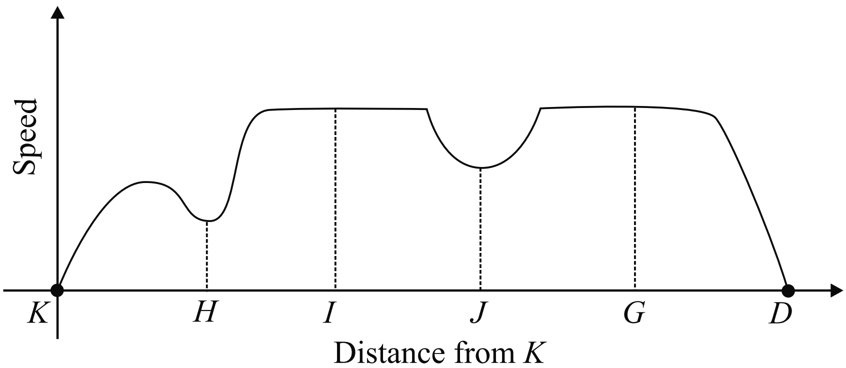
$$
\begin{array}{ccccccccccccccc}
 & & & & & & & 1 & & & & & & & \\
 & & & & & & 1 & & 1 & & & & & & \\
 & & & & & 1 & & 2 & & 1 & & & & & \\
 & & & & 1 & & 3 & & 3 & & 1 & & & & \\
 & & & 1 & & 4 & & 6 & & 4 & & 1 & & & \\
 & & 1 & & 5 & & 10 & & 10 & & 5 & & 1 & & \\
 & 1 & & 6 & & 15 & & 20 & & 15 & & 6 & & 1 & \\
1 & & \boxed{7} & & \boxed{21} & & \boxed{35} & & \boxed{35} & & \boxed{21} & & \boxed{7} & & 1
\end{array}
$$

Sum of the first 5 triangular numbers = 35

QUESTION 8 (50 MARKS)

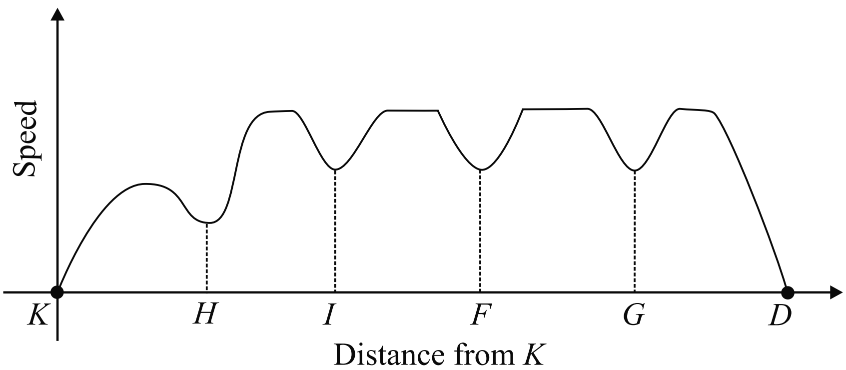
Question 8 (a) (i)



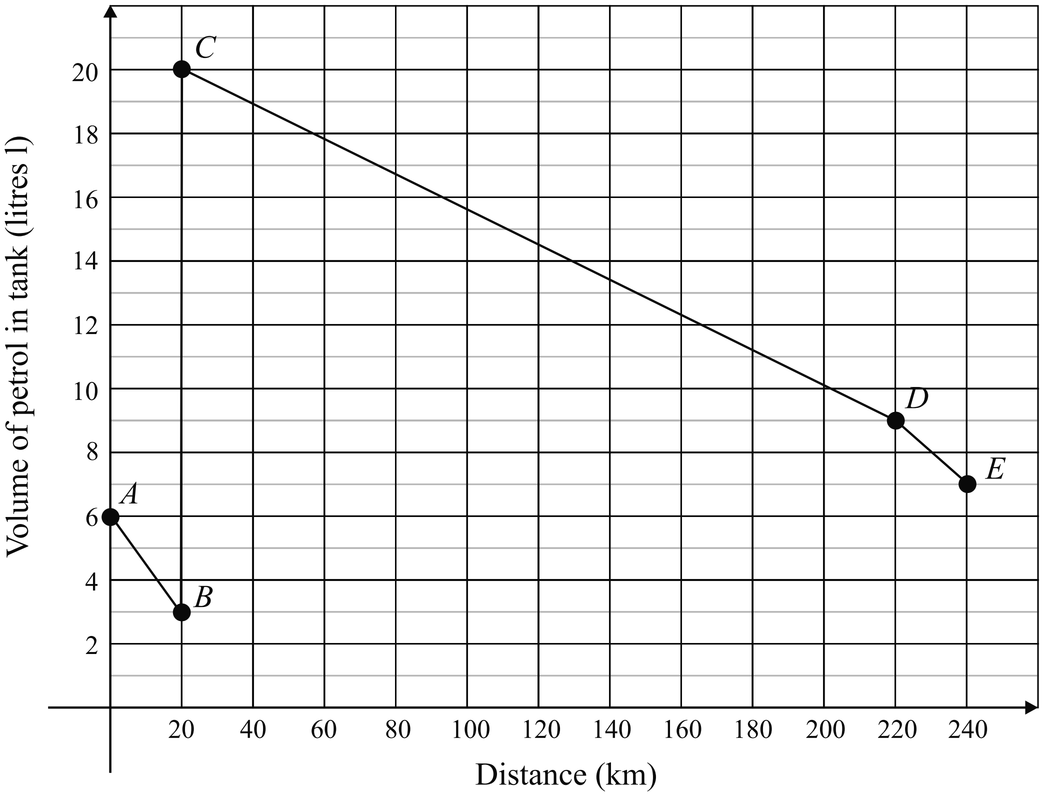


Route taken: *KHIJGD*

Question 8 (a) (ii)



Question 8 (b) (i)



AB: Car is travelling in town for 20 km. There is a rapid decline in petrol from 6 l to 3 l.
BC: Car pulls into a petrol station and fills the tank with petrol from 3 l to 20 l.
CD: Car travels 200 km on the motorway using only 11 l of petrol.
DE: Car is travelling through town using up 2 l of petrol in 20 km.

Question 8 (b) (ii)

Volume = 6 litres

Question 8 (b) (iii)

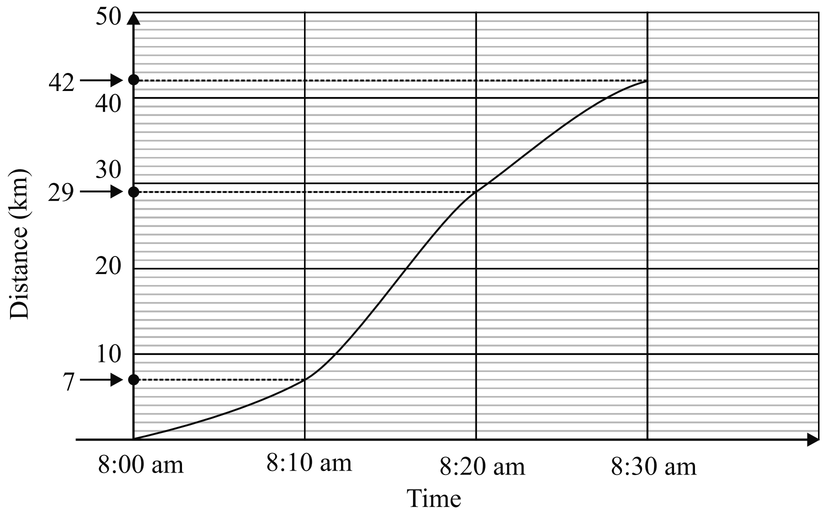
$$\text{Consumption rate} = \frac{\text{Number of litres of petrol}}{\text{Distance travelled (km)}}$$

$$AB: \text{Consumption rate} = \frac{6-3}{20-0} = \frac{3}{20} = 0.15 \text{ l/km}$$

$$CD: \text{Consumption rate} = \frac{20-9}{220-20} = \frac{11}{200} = 0.055 \text{ l/km}$$

Sample 2 Paper 1

Question 8 (c) (i)



$$\text{Average Speed} = \frac{7-0}{\frac{1}{6}} = \frac{7 \text{ km}}{\frac{1}{6} \text{ h}} = 42 \text{ km/h}$$

$$\boxed{\text{Average Speed} = \frac{\text{Distance}}{\text{Time}}}$$

Question 8 (c) (ii)

He accelerates up to a maximum speed as he turns onto the motorway. The car covers a greater distance in a shorter time as it speeds up. His average speed on the motorway is 132 km/h.

$$\text{Average Speed} = \frac{29-7}{\frac{1}{6}} = \frac{22 \text{ km}}{\frac{1}{6} \text{ h}} = 132 \text{ km/h}$$

Question 8 (c) (iii)

$$\text{Average Speed} = \frac{42-29}{\frac{1}{6}} = \frac{13 \text{ km}}{\frac{1}{6} \text{ h}} = 78 \text{ km/h}$$

Question 8 (c) (iv)

Yes, he broke the speed limit on the motorway where he was travelling on average at 132 km/h. The speed limit is 120 km/h.

Question 8 (c) (v)

$$\text{Average Speed} = \frac{42-0}{\frac{1}{2}} = \frac{42 \text{ km}}{\frac{1}{2} \text{ h}} = 84 \text{ km/h}$$

Question 9 (50 marks)

Question 9 (a)

Distance (km)	1	2	3	4	5	6	7	8	9	10
Price (€)	7	9	11	13	15	17	19	21	23	25

Question 9 (b)

$p = 5 + 2d$

Question 9 (c)

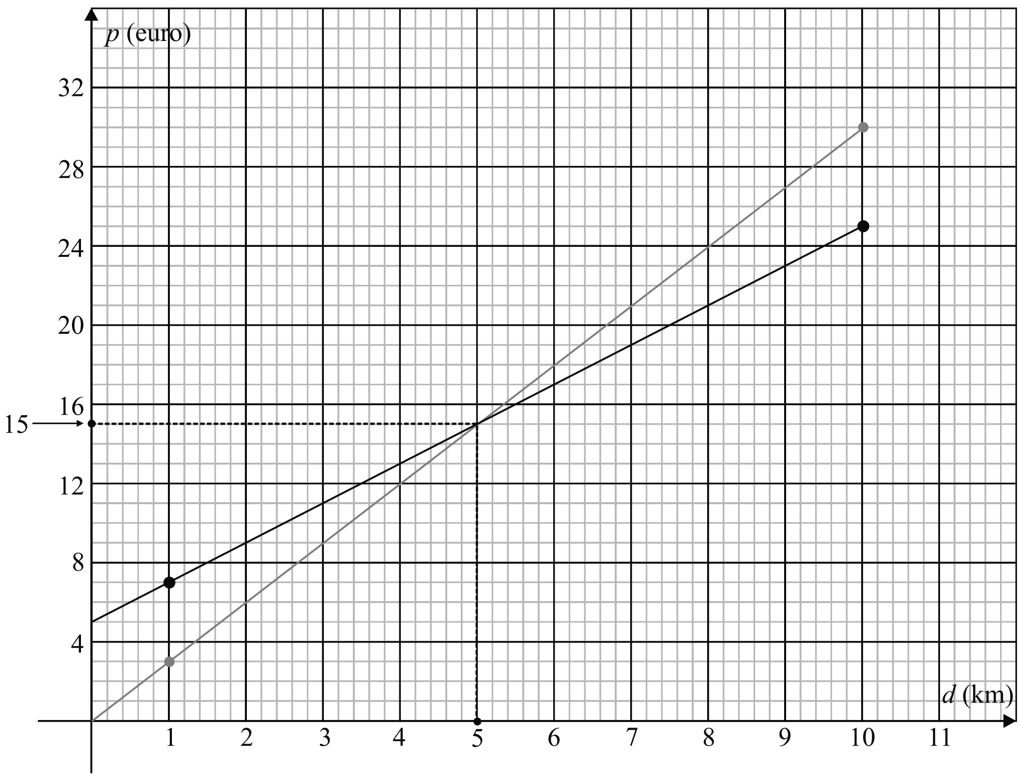
$$\begin{aligned} p &= 5 + 2d \\ &= 5 + 2(15.5) \\ &= 5 + 31 \\ &= €36 \end{aligned}$$

Question 9 (d)

Fixed charge = €0

Distance (km)	1	2	3	4	5	6	7	8	9	10
Price (€)	3	6	9	12	15	18	21	24	27	30

Question 9 (e)



Question 9 (f)

Graph: $d = 5$ km , $p = €15$

Algebra: $5 + 2d = 3d$

$\therefore d = 5$ km

$p = 3d = €15$

Question 9 (g)

Average distance per job = 5.6 km

Total distance travelled $= 15 \times 5.6 = 84$ km

Income from fares $p = 15(5 + 2(5.6)) = €243$

Petrol consumption: 7 litres $\rightarrow 100$ km

$\frac{7}{100}$ litres $\rightarrow 1$ km

$\frac{7}{100} \times 84$ litres $\rightarrow 84$ km

$= 5.88$ litres

Cost of petrol = $5.88 \times 1.45 = €8.53$

Profit = €243 – €25 – €8.53 = €209.47

Sample Paper 2: Paper 2

Question 1 (25 marks)

Question 1 (a)

$l : x + 3y = 0 \Rightarrow m_1 = -\frac{1}{3}$

$k : ax + y = 0 \Rightarrow m_2 = -a$

$m_1 \times m_2 = -1 \Rightarrow (-\frac{1}{3})(-a) = -1$

$\boxed{\text{Perpendicular lines: } m_1 \times m_2 = -1}$

$\frac{1}{3}a = -1$

$\therefore a = -3$

Question 1 (b)

$(0,\ 0) \in l : x + 3y = 0?$

$(0) + 3(0) = 0$

$(0,\ 0) \in k : -3x + y = 0?$

$-3(0) + (0) = 0$

Question 1 (c)

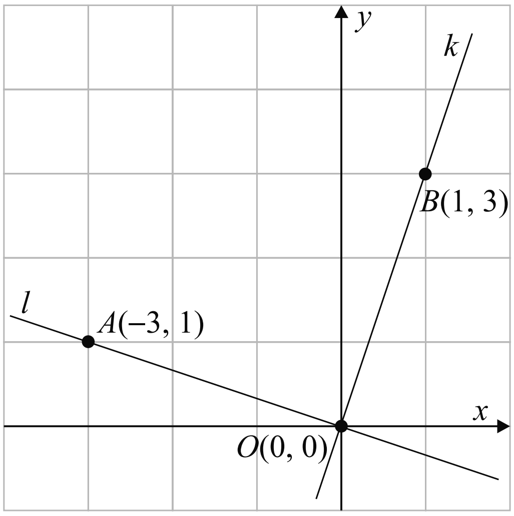
$A(-3,\ 1) \in l : x + 3y = 0?$

$(-3) + 3(1) = -3 + 3 = 0$

$B(1,\ 3) \in k : -3x + y = 0?$

$-3(1) + (3) = -3 + 3 = 0$

Question 1 (d)



Question 1 (e)

Find D by a translation:

$O(0,\ 0) \to B(1,\ 3)$

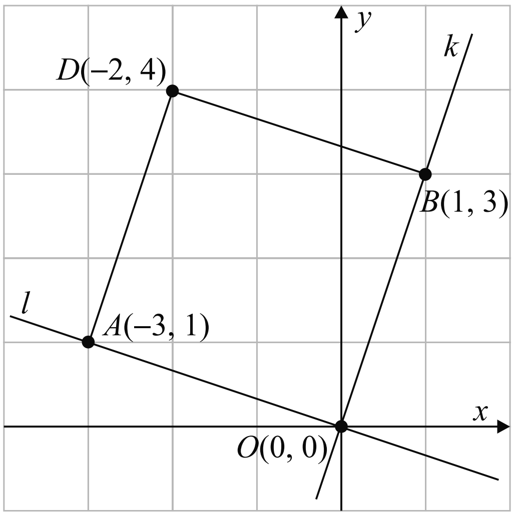
$A(-3,\ 1) \to D(-2,\ 4)$

$$\boxed{|PQ| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}}$$

$O(0,\ 0),\ B(1,\ 3)$

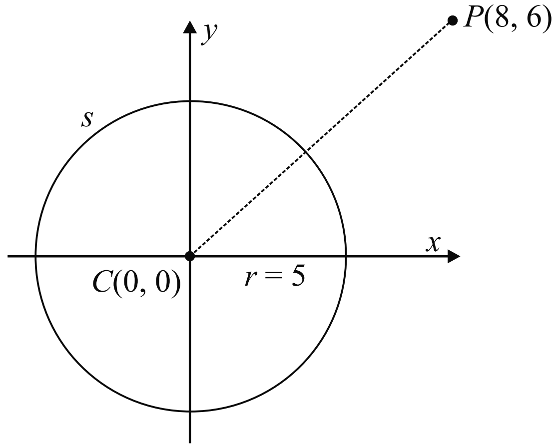
$|OB| = \sqrt{(3-0)^2 + (1-0)^2} = \sqrt{9+1} = \sqrt{10}$

Area of $AOBD = \sqrt{10} \times \sqrt{10} = 10$



Question 2 (25 marks)

Question 2 (a)



$$\boxed{\begin{array}{l}\text{Circle: } x^2 + y^2 = r^2 \\ \text{Centre}(0, 0), \text{ Radius} = r\end{array}}$$

Circle s: Centre (0, 0), $r = 5$

Question 2 (b)

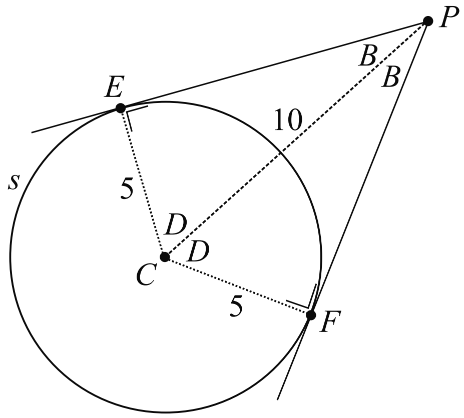
$$\boxed{|PQ| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}}$$

$C(0, 0)$, $P(8, 6)$

$$|CP| = \sqrt{(8-0)^2 + (6-0)^2} = \sqrt{64+36} = \sqrt{100} = 10$$

Question 2 (c)

Triangles PCF and PCE are congruent right-angled triangles. The hypotenuse is CP.



$$\boxed{\sin A = \frac{\text{Opposite}}{\text{Hypotenuse}}}$$

$$\sin B = \frac{5}{10} = \frac{1}{2} \Rightarrow B = \sin^{-1}(\tfrac{1}{2}) = 30^\circ$$

$$\angle B + \angle D + 90^\circ = 180^\circ$$

$$30^\circ + \angle D = 90^\circ$$

$$\therefore \angle D = 60^\circ$$

Question 2 (d)

$$r = 6400 \text{ km}, \ \theta = 2(\angle D) = 120^\circ$$

$$l = 2\pi(6400)\left(\frac{120^\circ}{360^\circ}\right) = 13404 \text{ km}$$

$$\boxed{l = 2\pi r\left(\frac{\theta}{360^\circ}\right)}$$

$$|CP| = 2r \Rightarrow h = r$$

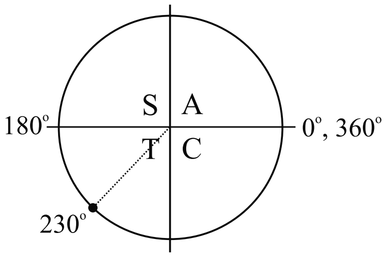
Height $h = 6400$ km

Question 3 (25 marks)

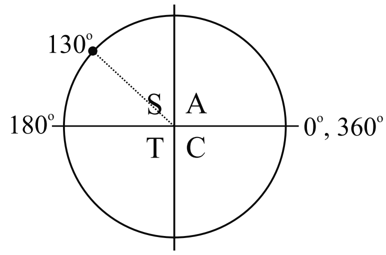
Question 3 (a)

$\sin 50^\circ = 0.766$
$\cos 50^\circ = 0.643$
$\tan 50^\circ = 1.192$

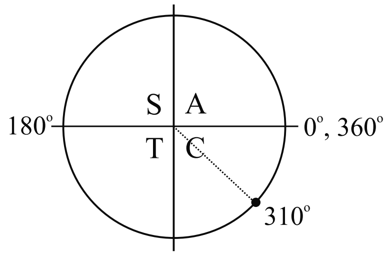
Question 3 (b)



$$\sin 230^\circ$$
$$= \sin(180^\circ + 50^\circ)$$
$$= -\sin 50^\circ$$
$$= -0.766$$



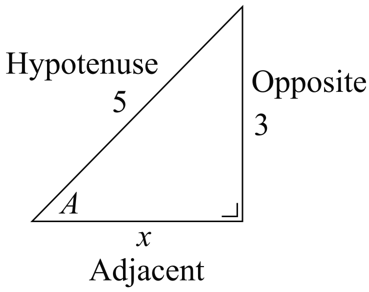
$$\cos 130^\circ$$
$$= \cos(180^\circ - 50^\circ)$$
$$= -\cos 50^\circ$$
$$= -0.643$$



$$\tan 310^\circ$$
$$= \tan(360^\circ - 50^\circ)$$
$$= -\tan 50^\circ$$
$$= -1.192$$

Question 3 (c) (i)

$$\sin A = \frac{3}{5} = \frac{\text{Opposite}}{\text{Hypotenuse}}$$
$$x^2 + 3^2 = 5^2$$
$$x^2 = 25 - 9 = 16$$
$$x = \sqrt{16} = 4$$



$$\cos A = \frac{\text{Adjacent}}{\text{Hypotenuse}} = \frac{4}{5}$$

$$\tan A = \frac{\text{Opposite}}{\text{Adjacent}} = \frac{3}{4}$$

Question 3 (c) (ii)

$$\cos(180^\circ + A) = -\cos A = -\tfrac{4}{5}$$

$$\tan(180^\circ + A) = +\tan A = \tfrac{3}{4}$$

QUESTION 4 (25 MARKS)

Question 4 (a)

(i) Modal class: 8–10

(ii) Median class: 8–10

(iii) Mean $= \dfrac{5\times2+7\times4+9\times8+11\times5+13\times1}{20} = 8.9$

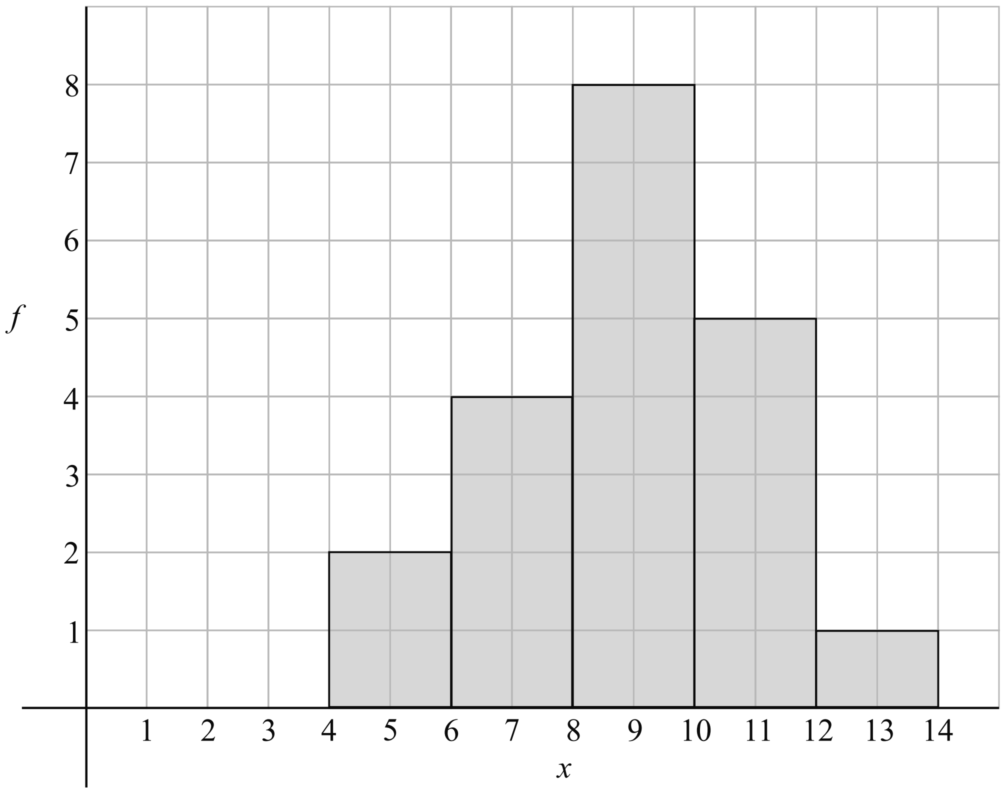
x	f
4–6	2
6–8	4
8–10	**8**
10–12	5
12–14	1
	20

10th. and 11th. results in here

(iv) $\boxed{\text{Range = Mid-interval value of highest interval − Mid-interval value of lowest interval}}$

Range $= 13 - 5 = 8$

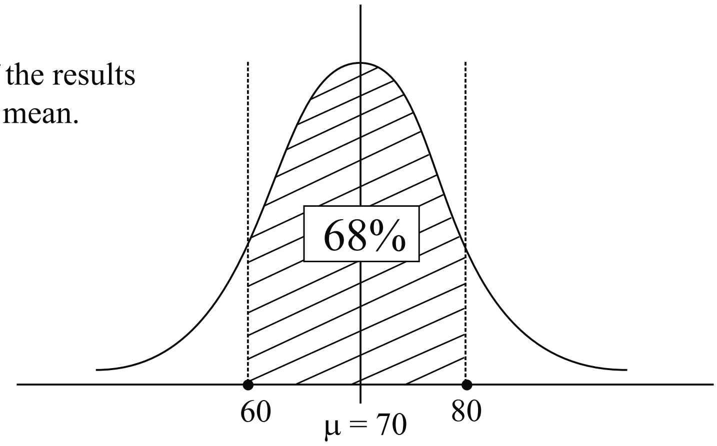
(v)



Question 4 (b)

According to the empirical rule, 68% of the results are within one standard deviation of the mean.

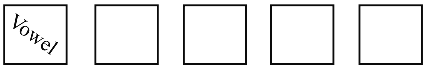
Lowest C: 60
Highest C: 80



Question 5 (25 marks)

Question 5 (a) (i)

2 × 4 × 3 × 2 × 1 = 48

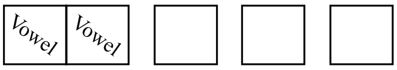


Number of arrangements = 48

There are 2 ways to fill the first box (an A or E).
Once this box has been filled, there are 4 ways to fill the second box.
Once the first 2 boxes have been filled, there are 3 ways to fill the third box, and so on.

Question 5 (a) (ii)

4 × 3 × 2 × 1 = 24



2 ways

Number of arrangements = 24 × 2 = 48

Glue the 2 vowels together and treat them as a single unit.
There are 4 ways to fill the first box.
Once this box has been filled, there are 3 ways to fill the second box, and so on.
The number of arrangements needs to be multiplied by 2, as the two vowels can be swapped.

Question 5 (b) (i)

$$P(\text{Double}) = \frac{\text{Number of doubles}}{\text{Number of outcomes}} = \frac{6}{36} = \frac{1}{6}$$

Die 2

Die 1	1	2	3	4	5	6
1	(1, 1)	(1, 2)	(1, 3)	(1, 4)	(1, 5)	(1, 6)
2	(2, 1)	(2, 2)	(2, 3)	(2, 4)	(2, 5)	(2, 6)
3	(3, 1)	(3, 2)	(3, 3)	(3, 4)	(3, 5)	(3, 6)
4	(4, 1)	(4, 2)	(4, 3)	(4, 4)	(4, 5)	(4, 6)
5	(5, 1)	(5, 2)	(5, 3)	(5, 4)	(5, 5)	(5, 6)
6	(6, 1)	(6, 2)	(6, 3)	(6, 4)	(6, 5)	(6, 6)

Question 5 (b) (ii)

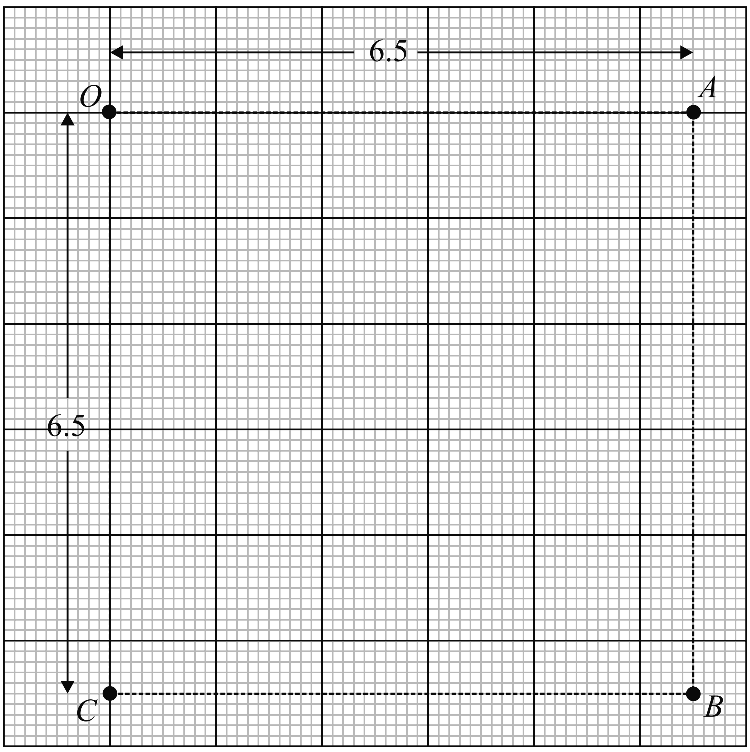
$$E(\text{Doubles}) = \frac{1}{6} \times 1872 = 312$$

Question 5 (c)

The likelihood that the last roll is a six is $\frac{1}{6}$ as all events are independent of each other.

Question 6 (25 marks)

Question 6 (a)



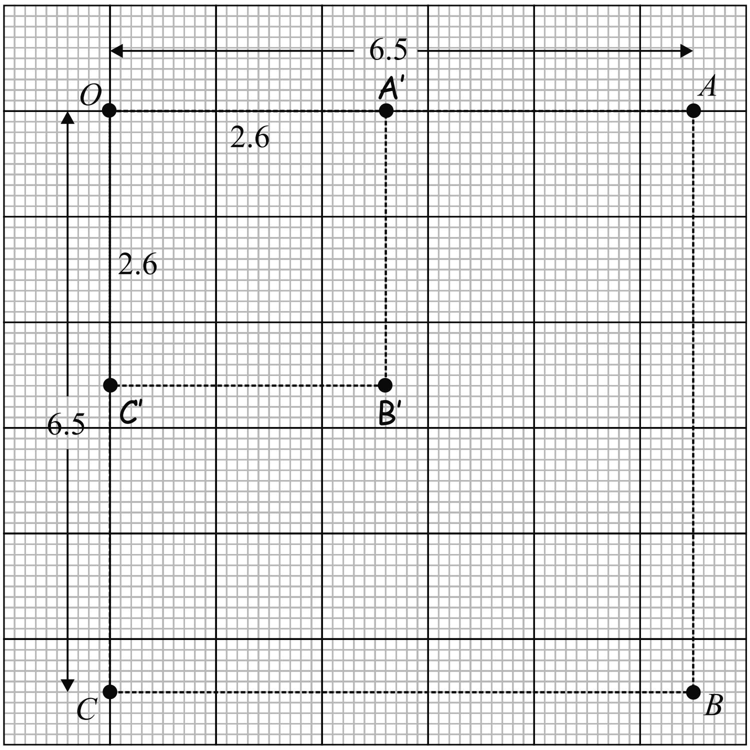
Question 6 (b)

$k = 0.4$

Length of side $= 6.5$

$$\text{Scale factor } k = \frac{|\text{Image length}|}{|\text{Object length}|}$$

$$k = \frac{|\text{Image length}|}{6.5} \Rightarrow |\text{Image length}| = 0.4 \times 6.5 = 2.6 \text{ units}$$



Question 6 (c)

Area $= 2.6 \times 2.6 = 6.76$ square units

Question 6 (d)

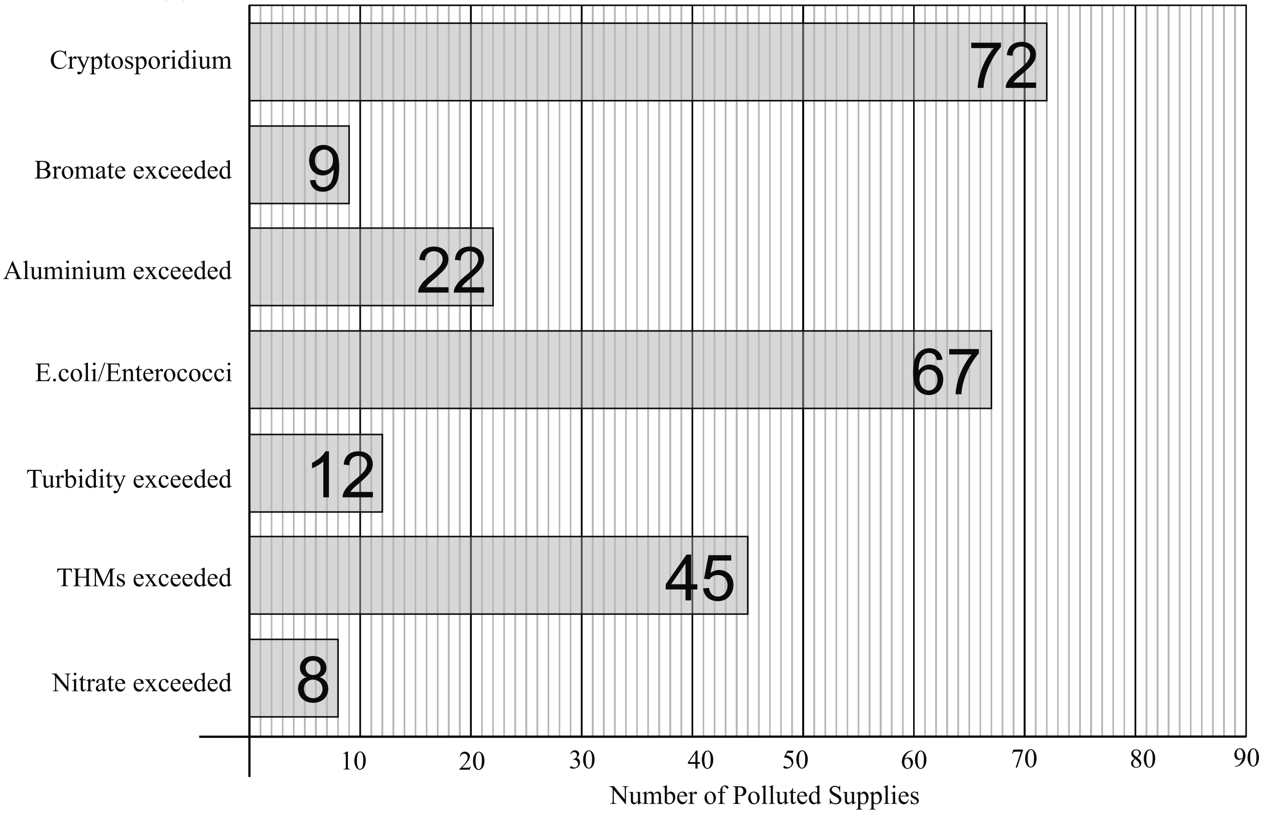
$$k^2 = \frac{95.0625}{6.5 \times 6.5} = \frac{9}{4}$$

$$k^2 = \frac{|\text{Image area}|}{|\text{Object area}|}$$

$$\therefore k = \sqrt{\frac{9}{4}} = \frac{3}{2} = 1.5$$

Question 7 (45 marks)

Question 7 (a)



Maximum number of supplies = 72 + 9 + 22 + 67 + 12 + 45 + 8 = 235

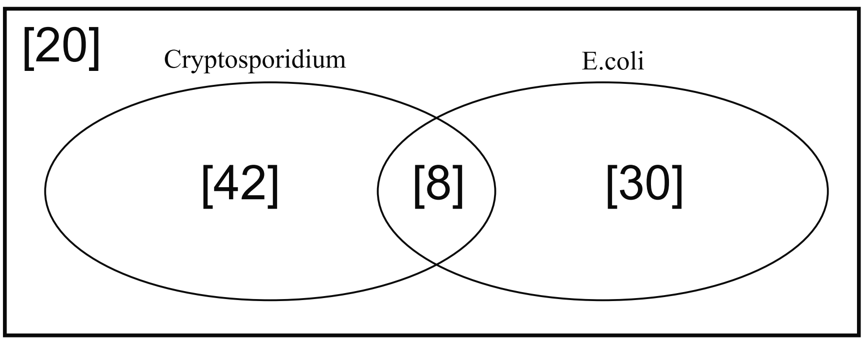
Question 7 (b)

$\frac{72}{300} \times 100\% = 24\%$ [72 water supplies had Cryptosporidium. They may have had other pollutants but they had Cryptosporidium at least.]

Question 7 (c)

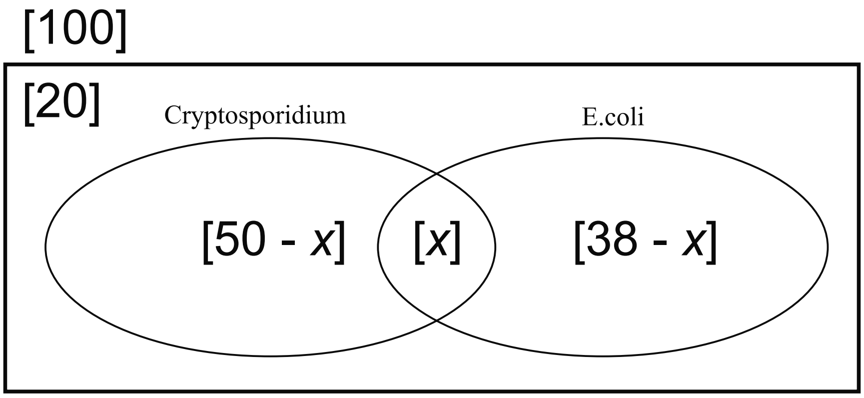
The name of the pollutants are too large to put on the horizontal axis, making them much harder to read.

Question 7 (d)



Total number of supplies = 100
Number of unpolluted supplies = 20
Number of polluted supplies = 80
Number of supplies polluted with Cryptosporidium = 50
Number of supplies polluted with E.coli = 38
Number of supplies polluted with Cryptosporidium **or** E.coli = 50 + 38 = 88
Number of supplies polluted with Cryptosporidium **and** E.coli = 88 − 80 = 8

Or



$$50 - x + x + 38 - x + 20 = 100$$
$$108 - x = 100$$
$$108 - 100 = x$$
$$\therefore x = 8$$

Question 7 (e) (i)

$P(\text{Supply is polluted with Crytosporidium and E.coli}) = \dfrac{8}{100} = 0.08 = 8\%$

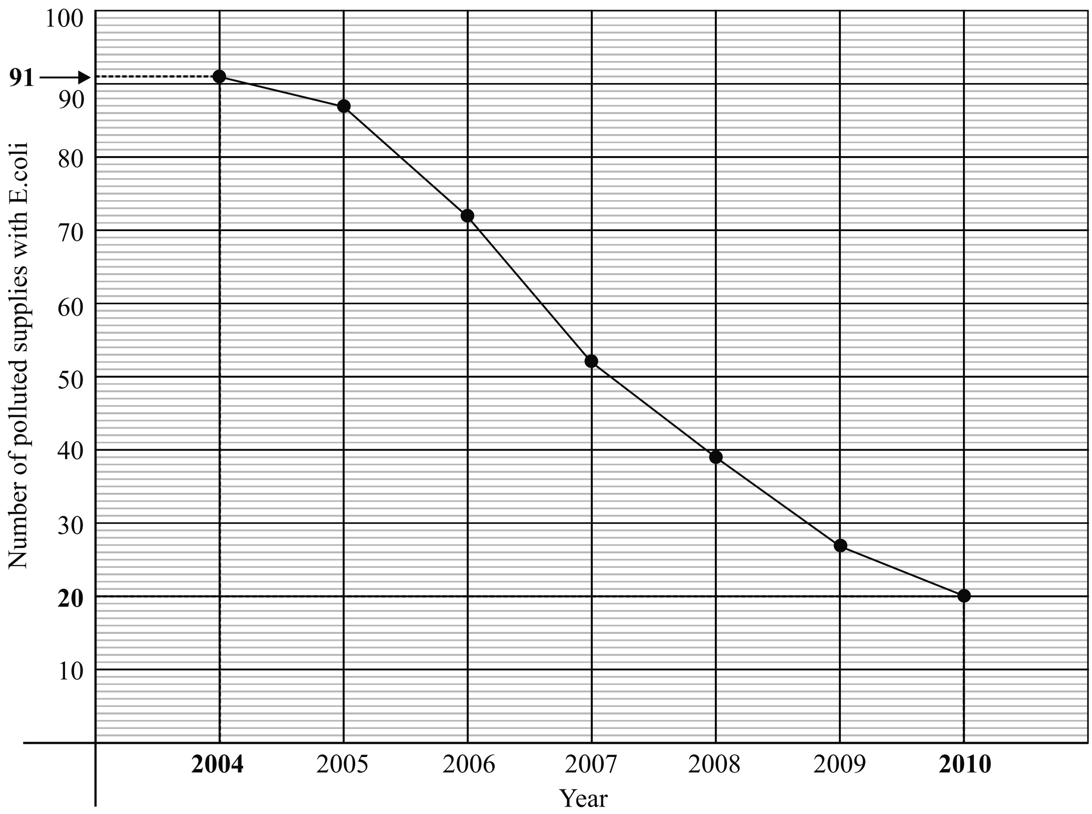
Question 7 (e) (ii)

$P(\text{Supply is polluted with E.coli only}) = \dfrac{30}{100} = 0.3 = 30\%$

Question 7 (f)

$P(\text{3 supplies are polluted with Crytosporidium and E.coli}) = 0.08 \times 0.08 \times 0.08 = \dfrac{8}{15625}$

Question 7 (g) (i)



% of supplies in 2010 which had E.coli $= \dfrac{20}{945} \times 100\% = 2.12\%$

Question 7 (g) (ii)

$$\% \text{ decrease in polluted supplies from 2004 to 2010} = \frac{91-20}{945} \times 100\% = 7.51\%$$

Question 7 (g) (iii)

Private supplies are inferior in terms of pollution.

Question 8 (30 marks)

Question 8 (a)

A simple random sample is one where every member of the population has an equal chance of being included in the sample.

Question 8 (b)

This would not give you a simple random sample because people without a telephone or who are ex-directory have no chance of being included in the sample.

Question 8 (c)

The null hypothesis H_o states that the Government does not call an election. $P \leq 50\%$

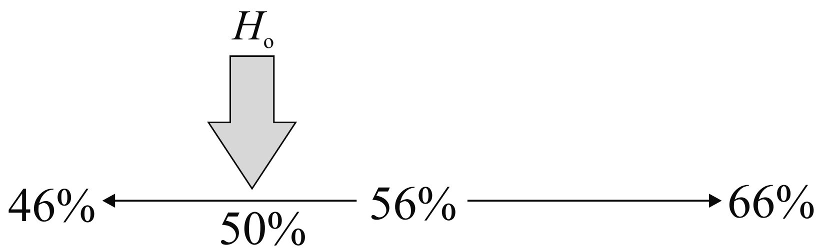
Question 8 (d)

$n = 100$

$$\text{Margin of error} = \pm\frac{1}{\sqrt{n}} = \pm\frac{1}{\sqrt{100}} = \pm 0.1 = \pm 10\%$$

Question 8 (e)

Proportion of those in the sample in favour of the Government = 56%
Test (using the sample proportion): 46% ← 56% → 66%
Since 50% is inside this interval, accept the null hypothesis which means the Government should not call an election.



Question 8 (f)

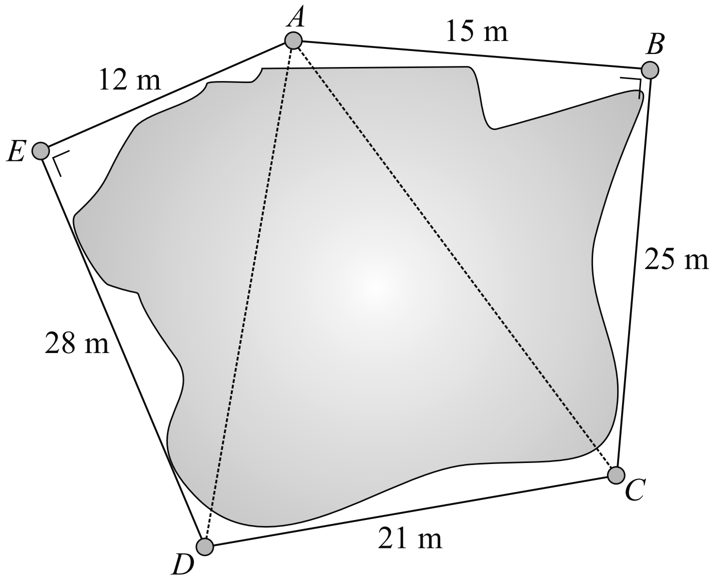
$n = ?$

$$\text{Margin of error} = \pm\frac{1}{\sqrt{n}} = \pm 5\% = \pm 0.05$$

$$\frac{1}{n} = (\pm 0.05)^2 = 0.0025$$

$$\therefore n = \frac{1}{0.0025} = 400$$

QUESTION 9 (75 MARKS)



Question 9 (a) (i)

No, a regular pentagon has five **equal** sides, whereas a pentagon is a 5-sided figure.

Question 9 (a) (ii)

Perimeter = 15 + 25 + 21 + 28 + 12 = 101 m

Question 9 (b) (i)

The pedometer needs to know the length of Joan's stride.
Distance walked = Number of clicks × length of stride

Question 9 (b) (ii)

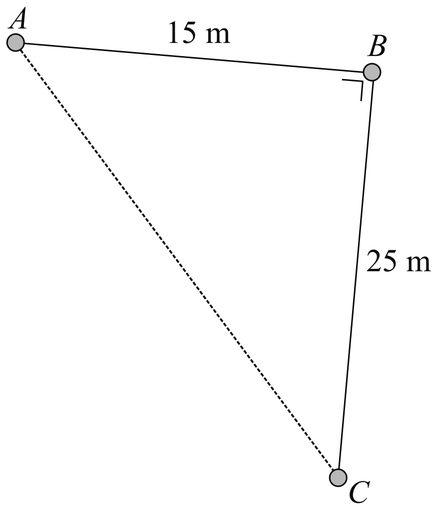
She will find the sum of the areas of triangles *AED*, *ADC* and *ABC*.

Question 9 (c) (i)

Name: Hypotenuse

Question 9 (c) (ii)

$$\text{Area} = \tfrac{1}{2} \times \text{Base} \times \text{Height} = \tfrac{1}{2}(15)(25) = 187.5 \text{ m}^2$$



Question 9 (c) (iii)

$$|AC|^2 = 15^2 + 25^2$$

$$\therefore |AC| = \sqrt{15^2 + 25^2} = 29.2 \text{ m}$$

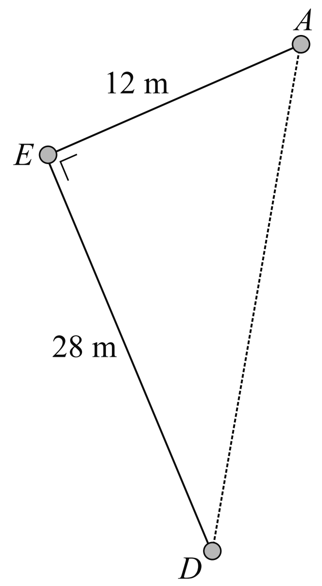
Question 9 (c) (iv)

$$\tan(|\angle BAC|) = \frac{25}{15} \Rightarrow |\angle BAC| = \tan^{-1}(\tfrac{5}{3}) = 59^\circ$$

$$|\angle BCA| = 180^\circ - 90^\circ - 59^\circ = 31^\circ$$

Question 9 (d) (i)

$$\text{Area} = \tfrac{1}{2} \times \text{Base} \times \text{Height} = \tfrac{1}{2}(12)(28) = 168 \text{ m}^2$$



Question 9 (d) (ii)

$$|AD|^2 = 12^2 + 28^2$$

$$\therefore |AC| = \sqrt{12^2 + 28^2} = 30.5 \text{ m}$$

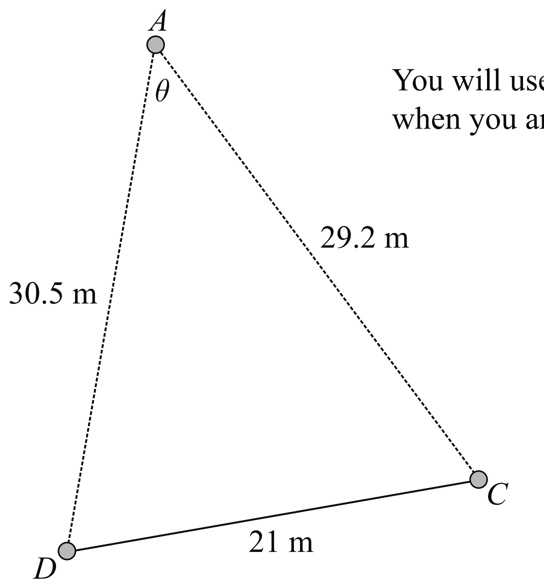
Question 9 (d) (iii)

No, for a triangle, base times height does not depend on the choice of base. $(A = \tfrac{1}{2}bh = \tfrac{1}{2}hb)$

Question 9 (e) (i)

Type of triangle: Scalene

Question 9 (e) (ii)



You will use the Cosine rule. This rule is used when you are given 3 sides and no angle.

Question 9 (e) (iii)

$$\boxed{a^2 = b^2 + c^2 - 2bc\cos A}$$

$$21^2 = 30.5^2 + 29.2^2 - 2(30.5)(29.2)\cos\theta$$

$$2(30.5)(29.2)\cos\theta = 30.5^2 + 29.2^2 - 21^2$$

$$1781.2\cos\theta = 1341.89$$

$$\theta = \cos^{-1}\left(\frac{1341.89}{1781.2}\right) = 41.1^\circ$$

Question 9 (e) (iv)

$$\boxed{\text{Area} = \tfrac{1}{2}ab\sin C}$$

$$\text{Area} = \tfrac{1}{2}(30.5)(29.2)\sin 41.1^\circ = 292.7 \text{ m}^2$$

Question 9 (f) (i)

$$\text{Area } |\Delta ABC| + \text{Area } |\Delta DEA| + \text{Area } |\Delta ADC|$$

$$= 187.5 + 168 + 292.7 = 648.2 \text{ m}^2 \approx 648 \text{ m}^2$$

Question 9 (f) (ii)

$$\%\text{ Error} = \frac{648 - 610}{610} = 6\%$$

Sample Paper 3: Paper 1

Question 1 (25 marks)

Question 1 (a)

$123\% = €738$

$1\% = \dfrac{€738}{123} = €6$

$100\% = €6 \times 100 = €600$

Cost price = €600
VAT paid = €738 – €600 = €138

Question 1 (b)

$\$1 = \dfrac{1}{1.32}€$

$\$500 = €\left(\dfrac{1}{1.32}\right) \times 500 = €378.79$

$\text{Loss} = €738 - €378.79 = €359.21$

$\%\ \text{Loss} = \dfrac{359.21}{738} \times 100\% = 48.7\%$

Question 1 (c) (i)

$€36\ 000 \times (0.8)^3 = €18\ 432$

Question 1 (c) (ii)

$€36\ 000 \times (0.4) = €14\ 400$

Question 1 (c) (iii)

Sells for the depreciation value:

$\text{Loss} = €36\ 000 - €18\ 432 = €17\ 568$

$\%\ \text{Loss} = \dfrac{€17\ 568}{€36\ 000} \times 100\% = 48.8\%$

Sells for the scrap value:

$\text{Loss} = €36\ 000 - €14\ 400 = €21\ 600$

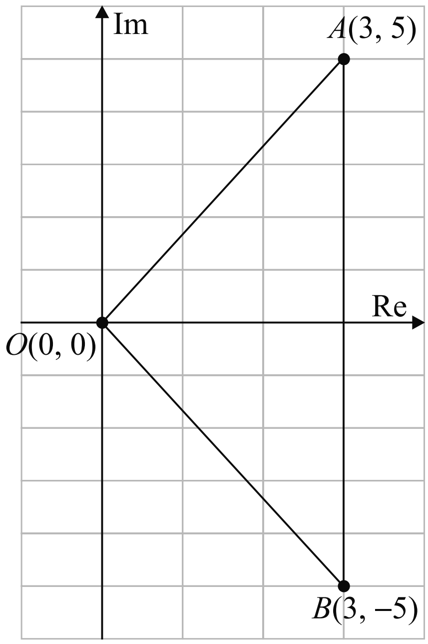
$\%\ \text{Loss} = \dfrac{€21\ 600}{€36\ 000} \times 100\% = 60\%$

Question 2 (25 marks)

Question 2 (a)

$z = 3 + 5i : A(3,\ 5)$

$\bar{z} = 3 - 5i : B(3,\ -5)$



$$\boxed{|PQ| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}}$$

$O(0,\ 0),\ A(3,\ 5)$

$|OA| = \sqrt{(3-0)^2 + (5-0)^2} = \sqrt{9+25} = \sqrt{34}$

$O(0,\ 0),\ B(3,\ -5)$

$|OB| = \sqrt{(3-0)^2 + (-5-0)^2} = \sqrt{9+25} = \sqrt{34}$

$\therefore |OA| = |OB|$ (Isosceles triangle)

Area $= \frac{1}{2} \times \text{Base} \times \text{Height} = \frac{1}{2}(10)(3) = 15$

Question 2 (b)

$z = 4 - i$

$z + 4i = 4 - i + 4i = 4 + 3i$

$|z + 4i| = |4 + 3i| = \sqrt{16+9} = \sqrt{25} = 5$ $\boxed{|a+ib| = \sqrt{a^2+b^2}}$

$w = \dfrac{1}{z+4i} = \dfrac{1}{4+3i}$ [Multiply above and below by the conjugate of the denominator.]

$= \dfrac{1}{4+3i} \times \dfrac{4-3i}{4-3i} = \dfrac{4-3i}{16 - 12i + 12i - 9i^2}$ $\boxed{z = a + ib \Rightarrow \bar{z} = a - ib}$

$= \dfrac{4-3i}{16+9} = \dfrac{4-3i}{25}$

$= \dfrac{4}{25} - \dfrac{3}{25}i$

LHS	*RHS*
$25w$	$\overline{z+4i}$
$25\left(\dfrac{4}{25} - \dfrac{3}{25}i\right)$	$\overline{4+3i}$
$4 - 3i$	$4 - 3i$

QUESTION 3 (25 MARKS)

Question 3 (a)

$\frac{1}{x} = x + 1$ [Multiply across by x.]

$1 = x^2 + x$

$0 = x^2 + x - 1$

$a = 1, b = 1, c = -1$ $\boxed{x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}}$

$$x = \frac{-1 \pm \sqrt{1^2 - 4(1)(-1)}}{2(1)} = \frac{-1 \pm \sqrt{1+4}}{2} = \frac{-1 \pm \sqrt{5}}{2}$$

Question 3 (b)

$\frac{1}{u} = \frac{1}{v} + \frac{1}{2}$

$\frac{2}{u} = \frac{3}{v} - 2$

Let $x = \frac{1}{u}$, $y = \frac{1}{v}$

$x = y + \frac{1}{2}$ $(\times 2)$

$2x = 3y - 2$

$2x - 2y = 1$

$2x - 3y = -2$

$y = 3$

$2x - 2(3) = 1$

$2x - 6 = 1$

$2x = 7$

$\therefore x = \frac{7}{2}$

$x = \frac{7}{2} \Rightarrow u = \frac{2}{7}$

$y = 3 \Rightarrow v = \frac{1}{3}$

Question 4 (25 marks)

Question 4 (a)

$\frac{125}{\sqrt{5}} = \frac{5^3}{5^{\frac{1}{2}}} = 5^{3-\frac{1}{2}} = 5^{\frac{5}{2}}$

$5^{3x-1} = \frac{125}{\sqrt{5}}$

$5^{3x-1} = 5^{\frac{5}{2}}$

$\therefore 3x - 1 = \frac{5}{2}$

$3x = \frac{7}{2}$

$\therefore x = \frac{7}{6}$

Power Rules

$a^p a^q = a^{p+q}$

$\frac{a^p}{a^q} = a^{p-q}$

$(a^p)^q = a^{pq}$

$a^0 = 1$

$a^{-p} = \frac{1}{a^p}$

Question 4 (b)

$x - 10 \le 2(x-1) < x$

$x - 10 \le 2x - 2$

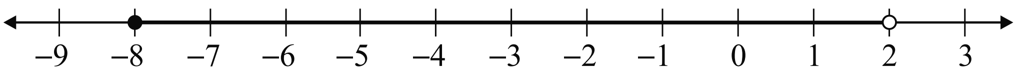
$-8 \le x$

$x - 10 \le 2(x-1) < x$

$2x - 2 < x$

$x < 2$

Answer: $-8 \le x < 2$



Question 4 (c)

$\frac{1}{x-1} - \frac{1}{x} = \frac{1}{2}$ [Multiply across by the denominators.]

$\frac{2x(x-1)1}{x-1} - \frac{2x(x-1)1}{x} = \frac{2x(x-1)1}{2}$

$2x - 2(x-1) = x(x-1)$

$2x - 2x + 2 = x^2 - x$

$0 = x^2 - x - 2$

$0 = (x+1)(x-2)$

$\therefore x = -1, 2$

Question 5 (25 marks)

Question 5 (a) (i)

$y = 2x^3 - 7x^2 + 5x - 3$

$\frac{dy}{dx} = 6x^2 - 14x + 5$

Question 5 (a) (ii)

$y = (2x-1)(2x+1)$

$= 4x^2 - 1$

$\frac{dy}{dx} = 8x$

Question 5 (b) (i)

$v = 6 : v = 8t - 2t^2 = 6$

$2t^2 - 8t + 6 = 0$

$t^2 - 4t + 3 = 0$

$(t-3)(t-1) = 0$

$t = 1 \text{ s}, 3 \text{ s}$

$a = \frac{dv}{dt} = 8 - 4t$

$t = 1 \text{ s}: a = 8 - 4(1) = 4 \text{ m s}^{-2}$

$t = 3 \text{ s}: a = 8 - 4(3) = -4 \text{ m s}^{-2}$

Question 5 (b) (ii)

$a = 0 : a = 8 - 4t = 0$

$8 = 4t$

$\therefore t = 2 \text{ s}$

$v = 8t - 2t^2 = 8(2) - 2(2)^2 = 16 - 8 = 8 \text{ m s}^{-1}$

Question 5 (b) (iii)

$s = 4t^2 - \frac{2}{3}t^3$

$v = \frac{ds}{st} = 8t - 2t^2$

Question 6 (25 marks)

Question 6 (a)

$y = f(x) = 9 - x^2$

$x = -3: f(-3) = 9 - (-3)^2 = 9 - 9 = 0$

$x = -2: f(-2) = 9 - (-2)^2 = 9 - 4 = 5$

$x = -1: f(-1) = 9 - (-1)^2 = 9 - 1 = 8$

$x = 0: f(0) = 9 - (0)^2 = 9 - 0 = 9$

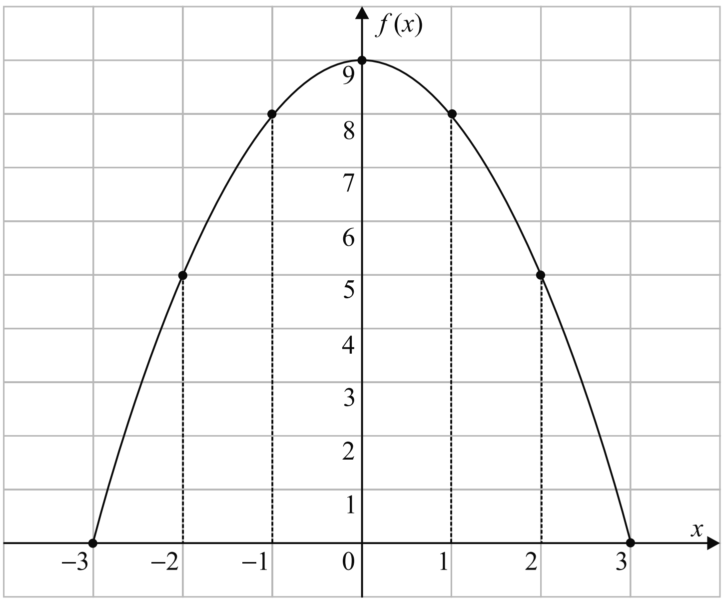
$x = 1: f(1) = 9 - (1)^2 = 9 - 1 = 8$

$x = 2: f(2) = 9 - (2)^2 = 9 - 4 = 5$

$x = 3: f(3) = 9 - (3)^2 = 9 - 9 = 0$

x	-3	-2	-1	0	1	2	3
$f(x)$	0	5	8	9	8	5	0

Question 6 (b)



Question 6 (c)

$A = \frac{1}{2}\{0 + 0 + 2(5 + 8 + 9 + 8 + 5)\} = 35$

Question 7 (50 marks)

Question 7 (a)

MAY Bealtaine 2012

Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
		1	2	3	4	5
6 ○	7	8	9	10	11	12 ◑
13	14	15	16	17	18	19
20	21 ●	22	23	24	25	26
27	28 ◐	29	30	31		

(i) The difference between corresponding numbers in consecutive rows is 15 – 8 = 7.

(ii) The difference between corresponding numbers in consecutive columns is 9 – 8 = 1.

Question 7 (b)

MAY Bealtaine 2012

Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
		1	2	3	4	5
6 ○	7	8	9	10	11	12 ◑
13	14	15	16	17	18	19
20	21 ●	22	23	24	25	26
27	28 ◐	29	30	31		

(i) The difference between corresponding numbers in consecutive rows is 27 – 20 = 7.

(ii) The difference between corresponding numbers in consecutive columns is 21 – 20 = 1.

Question 7 (c)

Calendar Magic Number $= BL \times TR - BR \times TL$
$= 22 \times 10 - 24 \times 8 = 220 - 192 = 28$

Top Left (TL) Top Right (TR)

8	9	10
15	16	17
22	23	24

Bottom Left (BL) Bottom Right (BR)

Question 7 (d) (i)

Top Left (TL) Top Right (TR)

9	10	11
16	17	18
23	24	25

Bottom Left (BL) Bottom Right (BR)

Calendar Magic Number $= BL \times TR - BR \times TL$
$= 23 \times 11 - 25 \times 9 = 253 - 225 = 28$

Question 7 (d) (ii)

Top Left (TL) Top Right (TR)

1	2	3
8	9	10
15	16	17

Bottom Left (BL) Bottom Right (BR)

Calendar Magic Number $= BL \times TR - BR \times TL$
$= 15 \times 3 - 17 \times 1 = 45 - 17 = 28$

Question 7 (e) (i)

Top Left (TL) Top Right (TR)

a	$a + 1$	$a + 2$
$a + 7$	$a + 8$	$a + 9$
$a + 14$	$a + 15$	$a + 16$

Bottom Left (BL) Bottom Right (BR)

Question 7 (e) (ii)

Calendar Magic Number $= BL \times TR - BR \times TL$

$= (a+14) \times (a+2) - (a+16) \times a$

$= a^2 + 2a + 14a + 28 - a^2 - 16a$

$= 28$

Question 7 (e) (iii)

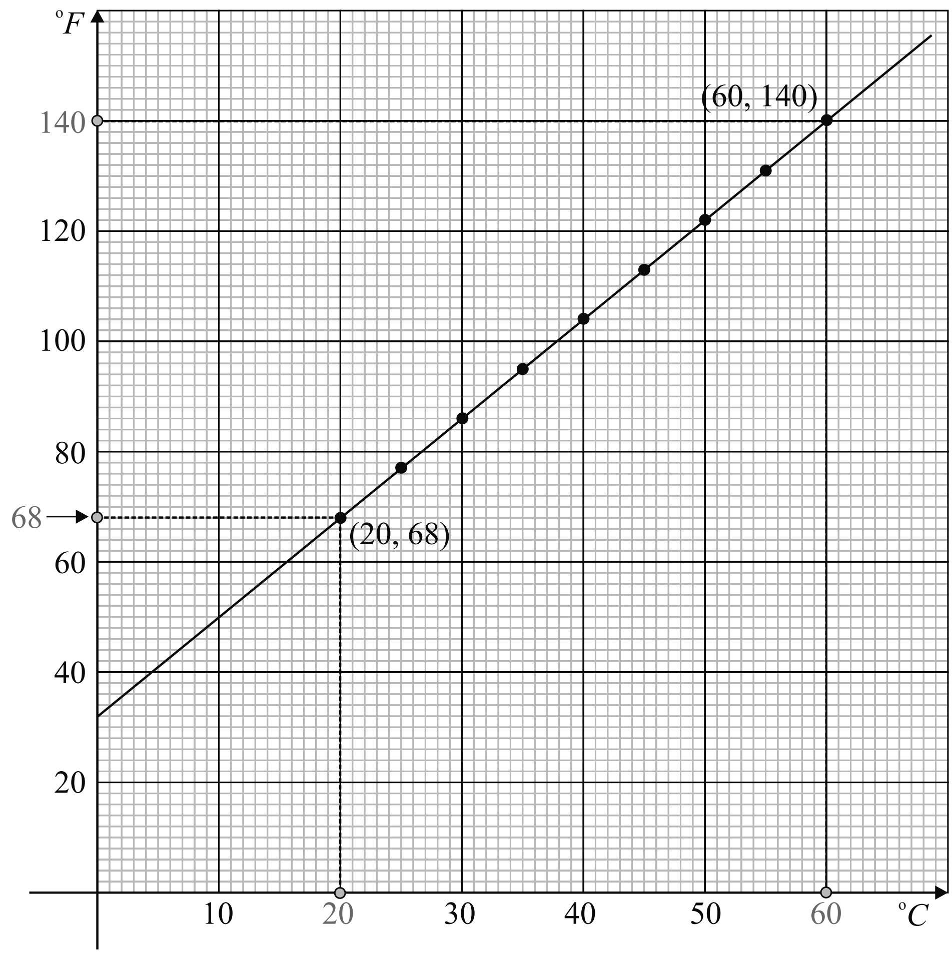
All of the magic numbers are the same. The situation was generalised by allowing any number a to be the top left number. When the formula was applied in this general situation, 28 was produced. So it makes no difference what part of the page the 3 by 3 rectangle moves to.

Sample 3 Paper 1

Question 8 (50 marks)

Question 8 (a)

^{o}C	20	25	30	35	40	45	50	55	60
^{o}F	68	77	86	95	104	113	122	131	140



Question 8 (b)

Choose two points on the graph: (20, 68) and (60, 140)

$$m = \frac{140-68}{60-20} = \frac{9}{5} \qquad \boxed{m = \frac{y_2 - y_1}{x_2 - x_1}}$$

Equation of line: Point(20, 68), $m = \frac{9}{5}$

$$y - 68 = \tfrac{9}{5}(x-20) \qquad \boxed{y - y_1 = m(x - x_1)}$$

$$F - 68 = \tfrac{9}{5}(C-20)$$

$$F = \tfrac{9}{5}C - \tfrac{9}{5}(20) + 68$$

$$F = \tfrac{9}{5}C - 36 + 68$$

$$\therefore F = \tfrac{9}{5}C + 32$$

Question 8 (c)

$F = \frac{9}{5}C + 32$

$F - 32 = \frac{9}{5}C$

$\therefore C = \frac{5}{9}(F - 32)$

Question 8 (d)

(i) $F = \frac{9}{5}C + 32$

$F = \frac{9}{5}(0) + 32 = 32^\circ\text{F}$

(ii) $F = \frac{9}{5}C + 32$

$F = \frac{9}{5}(100) + 32 = 180 + 32 = 212^\circ\text{F}$

(iii) $C = \frac{5}{9}(F - 32)$

$C = \frac{5}{9}(100 - 32) = \frac{5}{9}(68) \approx 38^\circ\text{C}$

(iv) $C = \frac{5}{9}(F - 32)$

$C = \frac{5}{9}(-22 - 32) = \frac{5}{9}(-54) = -30^\circ\text{C}$

Question 8 (e)

$F = \frac{9}{5}C + 32$

$C = \frac{5}{9}(F - 32)$

$F = C \Rightarrow \frac{9}{5}C + 32 = \frac{5}{9}(C - 32)$

$\frac{9}{5}C + 32 = \frac{5}{9}C - \frac{160}{9}$

$\frac{9}{5}C - \frac{5}{9}C = -\frac{160}{9} - 32$

$\frac{56}{45}C = -\frac{448}{9}$

$$\therefore C = \frac{-448 \times 45}{9 \times 56} = -40^\circ\text{C}$$

$\therefore -40^\circ\text{C} = -40^\circ\text{F}$

Question 8 (f) (i)

$h_1 = 3 \text{ cm},\ h_2 = 23 \text{ cm},\ h_3 = 17 \text{ cm}$

$$C = 100\left(\frac{h_3 - h_1}{h_2 - h_1}\right) = 100\left(\frac{17-3}{23-3}\right) = 100\left(\frac{14}{20}\right) = 70^\circ\text{C}$$

Question 8 (f) (ii)

$C = \frac{5}{9}(F - 32)$

$F = 161^\circ\text{F}:\ C = \frac{5}{9}(161 - 32) = \frac{215}{3}^\circ\text{C}$

$\text{Error} = \frac{215}{3}^\circ\text{C} - 70^\circ\text{C} = \frac{5}{3}^\circ\text{C}$

$$\%\ \text{error} = \frac{\frac{5}{3}}{\frac{215}{3}} \times 100\% = 2.33\%$$

Sample 3 Paper 1

Question 9 (50 marks)

Question 9 (a) (i)

$A = 12t^2 - \frac{2}{3}t^3$

$\frac{dA}{dt} = 24t - 2t^2$

$\left(\frac{dA}{dt}\right)_{t=1} = 24(1) - 2(1)^2$

$= 24 - 2 = 22 \text{ m}^2/\text{month}$

After one month, the area of the algae is increasing.

Question 9 (a) (ii)

$\left(\frac{dA}{dt}\right)_{t=14} = 24(14) - 2(14)^2$

$= 24 - 2 = -56 \text{ m}^2/\text{month}$

After 14 months, the area of the algae is decreasing.

Question 9 (b)

$\frac{dA}{dt} = 0 \Rightarrow 24t - 2t^2 = 0$

$12t - t^2 = 0$

$t(12 - t) = 0$

$\therefore t = 12 \text{ months}$

Question 9 (c)

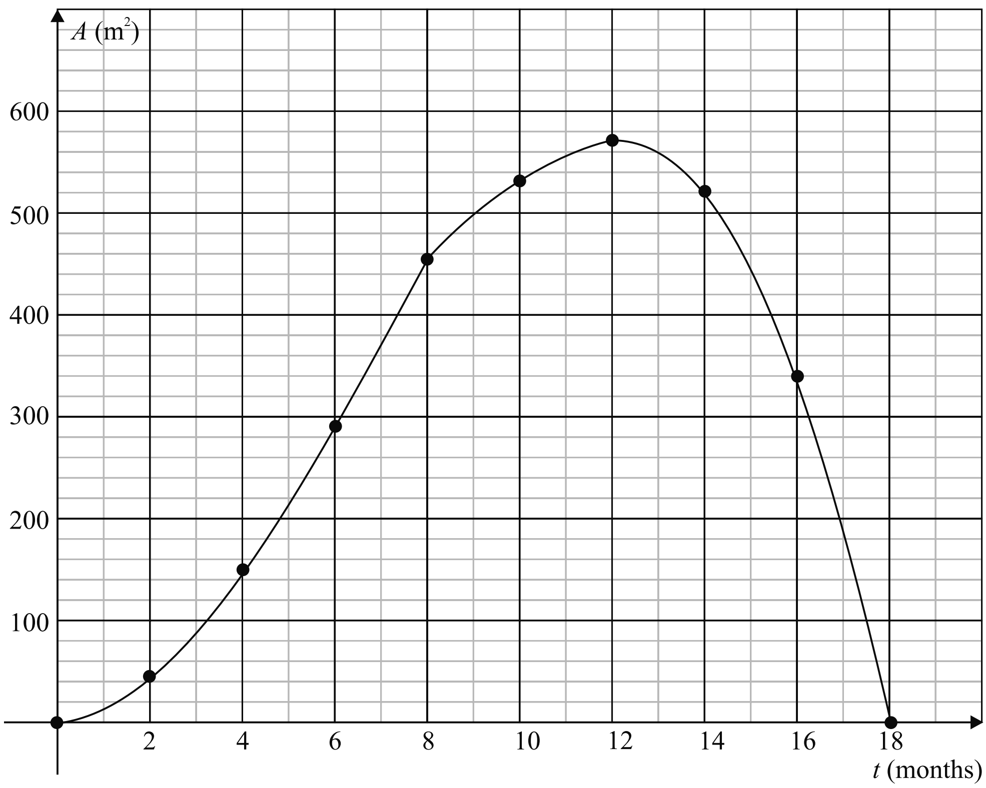
$A = 12t^2 - \frac{2}{3}t^3$

$t = 12 \text{ months: } A_{\text{Max.}} = 12(12)^2 - \frac{2}{3}(12)^3 = 576 \text{ m}^2$

Question 9 (d)

t (Months)	0	2	4	6	8	10	12	14	16	18
A (m^2)	0	43	149	288	427	533	576	523	341	0

Question 9 (e)



Question 9 (f)

The algae grows slowly to a maximum area of 576 m^2 over the first 12 months, and then rapidly decreases to zero over the last six months.

Question 9 (g)

$A = 12t^2 - \tfrac{2}{3}t^3$

$t = 15$ months: $A = 12(15)^2 - \tfrac{2}{3}(15)^3 = 450 \text{ m}^2$

$A = \pi r^2 \Rightarrow r = \sqrt{\dfrac{A}{\pi}} = \sqrt{\dfrac{450}{\pi}} \approx 12 \text{ m}$

Sample 3
Paper 1

Sample Paper 3: Paper 2

Question 1 (25 marks)

Question 1 (a)

58	3	4	8						
59	0	0	2	3	7	8	8		
60	0	2	4	5	$\boxed{\mathbf{7}}$	9			
61	1	3	3	5	6	6	8	8	9
62	0	1	3	4					

Key: 61|1 means 61.1 kg

There are 29 students. Arrange the weights in order. The median weight is now the weight of the middle student. This is the weight of the $15^{th.}$ student.
Median weight = 60.7 kg

Question 1 (b)

Range R = 62.4 – 58.3 = 4.1 kg

58	**3**	4	8						
59	0	0	2	3	7	8	8		
60	0	2	4	5	$\boxed{7}$	9			
61	1	3	3	5	6	6	8	8	9
62	0	1	3	**4**					

Question 1 (c)

58	3	4	8						
59	0	0	2	$\boxed{\mathbf{3}}$	$\boxed{\mathbf{7}}$	8	8		
60	0	2	4	5	$\boxed{7}$	9			
61	1	3	3	5	6	$\boxed{\mathbf{6}}$	$\boxed{\mathbf{8}}$	8	9
62	0	1	3	4					

First quartile $Q_1 = \dfrac{59.3 + 59.7}{2} = 59.5 \text{ kg}$

Third quartile $Q_3 = \dfrac{61.6 + 61.8}{2} = 61.7 \text{ kg}$

Interquartile range = $Q_3 - Q_1$ = 61.7 kg – 59.5 kg = 2.2 kg

Question 1 (d)

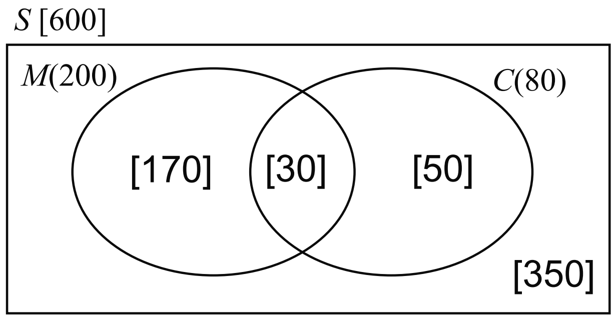
(i) The data is univariate as the only variable is weight.
(ii) The data is continuous as weight can be measured to any decimal place.

Question 2 (25 marks)

Question 2 (a)

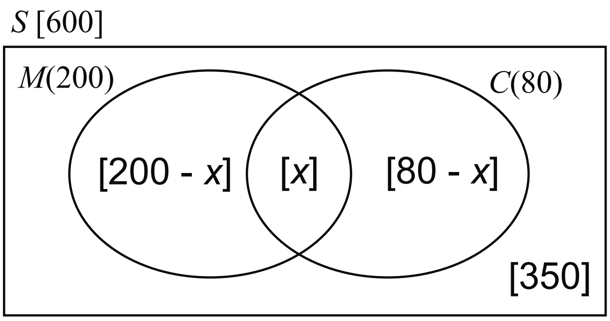
Number of students who neither sing nor play an instrument $= 600 \times \frac{7}{12} = 350$

Question 2 (b)



Total number of students = 600
Number of students who neither sing nor play an instrument = 350
Number of students who sing **or** play an instrument = 600 – 350 = 250
Number of students who play a musical instrument = 200
Number of students who sing in the choir = 80
Number of students who sing **and** play an instrument = 280 – 250 = 30

Or



$$200 - x + x + 80 - x + 350 = 600$$
$$630 - x = 600$$
$$630 - 600 = x$$
$$\therefore x = 30$$

Question 2 (c) (i)

P(Student sings in choir and plays an instrument)

$$= \frac{\text{Number of students who sing in choir and play an instrument}}{\text{Number of students}} = \frac{30}{600} = \frac{1}{20}$$

Question 2 (c) (ii)

P(Student does not sing)

$$= \frac{\text{Number of students who do not sing}}{\text{Number of students}} = \frac{600-80}{600} = \frac{520}{600} = \frac{13}{15}$$

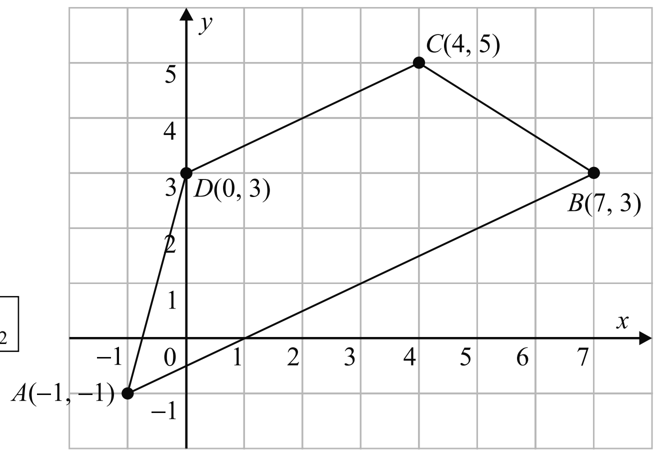
Question 2 (c) (iii)

P(Student plays a musical instrument only)

$$= \frac{\text{Number of students who play an instrument only}}{\text{Number of students}} = \frac{170}{600} = \frac{17}{60}$$

Question 3 (25 marks)

Question 3 (a) (i)



$A(-1, -1)$, $B(7, 3)$

$$m_1 = \frac{3-(-1)}{7-(-1)} = \frac{4}{8} = \frac{1}{2}$$

$$\boxed{m = \frac{y_2 - y_1}{x_2 - x_1}}$$

$D(0, 3)$, $C(4, 5)$

$$m_2 = \frac{5-3}{4-0} = \frac{2}{4} = \frac{1}{2}$$

$m_1 = m_2 \Rightarrow AB \parallel DC$ $\boxed{\text{Parallel lines: } m_1 = m_2}$

Question 3 (a) (ii)

$A(-1, -1)$, $D(0, 3)$

$$|AD| = \sqrt{(0-(-1))^2 + (3-(-1))^2} = \sqrt{1+16} = \sqrt{17}$$

$$\boxed{|PQ| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}}$$

$B(7, 3)$, $C(4, 5)$

$$|BC| = \sqrt{(4-7)^2 + (5-3)^2} = \sqrt{9+4} = \sqrt{13}$$

$\therefore |AD| \neq |BC|$

Question 3 (b)

Equation of line DB: $y = 3$

Equation of line AC:
Find slope of AC

$A(-1, -1)$, $C(4, 5)$

$$\boxed{m = \frac{y_2 - y_1}{x_2 - x_1}}$$

$$m = \frac{5-(-1)}{4-(-1)} = \frac{6}{5}$$

$y - 5 = \frac{6}{5}(x - 4)$ $\boxed{y - y_1 = m(x - x_1)}$

$5y - 25 = 6x - 24$

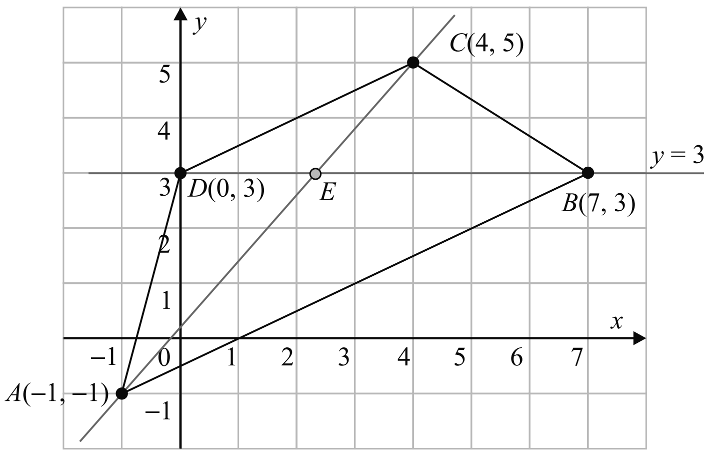
$0 = 6x - 5y + 1$

$AC \cap DB : 6x - 5y + 1 = 0 \cap y = 3$

$6x - 5(3) + 1 = 0$

$6x = 14 \Rightarrow x = \frac{7}{3}$

$\therefore E(\frac{7}{3}, 3)$



Question 3 (c)

Midpoint of DB:

$D(0, 3)$, $B(7, 3)$

Midpoint $= \left(\frac{0+7}{2}, \frac{3+3}{2}\right) = (\frac{7}{2}, 3)$ $\boxed{\text{Midpoint} = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)}$

Midpoint of AC:

$A(-1, -1)$, $C(4, 5)$

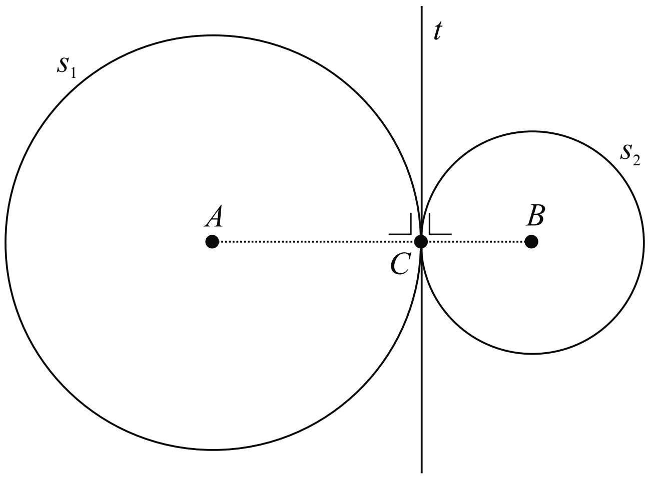
Midpoint $= \left(\frac{-1+4}{2}, \frac{-1+5}{2}\right) = (\frac{3}{2}, 2)$

The midpoint of each diagonal is not equal to E, the point where the diagonals intersect. Therefore, neither diagonal bisects the other.

Question 4 (25 marks)

Question 4 (a)

The tangent t is perpendicular to the lines joining the point of contact C to the centres, A and B.
Therefore, ACB is a straight line and points A, B and C are collinear.



Question 4 (b) (i)

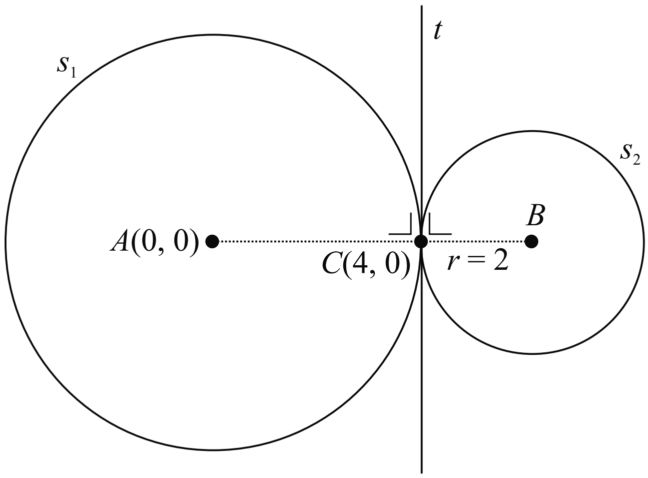
Radius of $s_1 = |AC|$

$= \sqrt{(4-0)^2+(0-0)^2}$

$= \sqrt{16} = 4$

Or

The line AC lies along a horizontal line.
Radius of $s_1 = 4$



Question 4 (b) (ii)

Centre $B(6, 0)$

Question 4 (c)

Equation of s_1:
Centre $A(0, 0)$, $r = 4$

$(x-0)^2+(y-0)^2 = 4^2$ $\boxed{(x-h)^2+(y-k)^2 = r^2}$

$s_1 : x^2 + y^2 = 16$

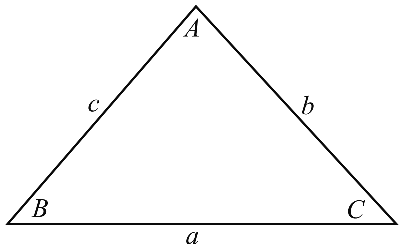
Equation of s_2:
Centre $B(6, 0)$, $r = 2$

$(x-6)^2+(y-0)^2 = 2^2$

$s_2 : (x-6)^2 + y^2 = 4$

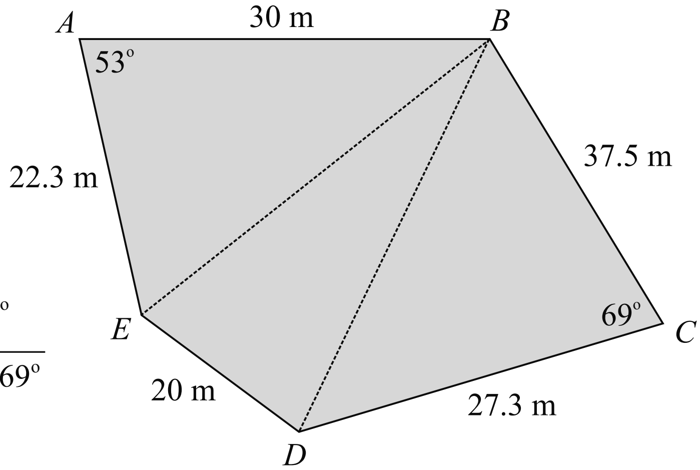
Question 5 (25 marks)

Question 5 (a)



Formula: $a^2 = b^2 + c^2 - 2bc\cos A$

Question 5 (b) (i)



$a^2 = b^2 + c^2 - 2bc\cos A$

$|BE|^2 = 30^2 + 22.3^2 - 2(30)(22.3)\cos 53^\circ$

$\therefore |BE| = \sqrt{30^2 + 22.3^2 - 2(30)(22.3)\cos 53^\circ}$

$= 24.3 \text{ m}$

Question 5 (b) (ii)

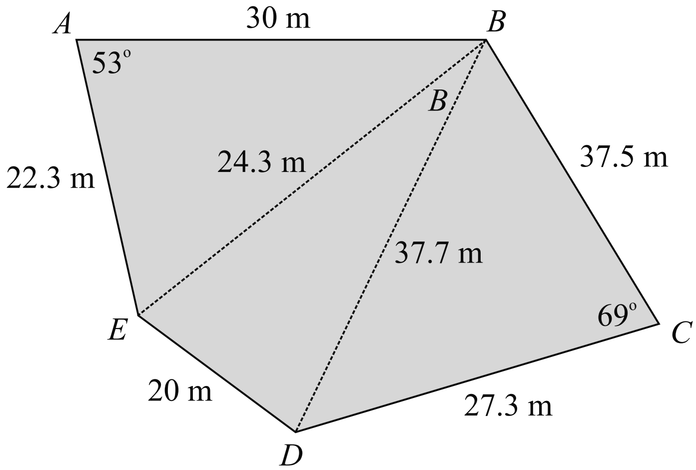
$a^2 = b^2 + c^2 - 2bc\cos A$

$|BD|^2 = 37.5^2 + 27.3^2 - 2(37.5)(27.3)\cos 69^\circ$

$\therefore |BE| = \sqrt{37.5^2 + 27.3^2 - 2(37.5)(27.3)\cos 69^\circ}$

$= 37.7 \text{ m}$

Question 5 (b) (iii)



$a^2 = b^2 + c^2 - 2bc\cos A$

$20^2 = 24.3^2 + 37.7^2 - 2(24.3)(37.7)\cos B$

$2(24.3)(37.7)\cos B = 24.3^2 + 37.7^2 - 20^2$

$$\cos B = \frac{24.3^2 + 37.7^2 - 20^2}{2(24.3)(37.7)}$$

$$B = \cos^{-1}\left(\frac{24.3^2 + 37.7^2 - 20^2}{2(24.3)(37.7)}\right) = 28.4^\circ$$

Question 5 (b) (iv)

$$\boxed{\text{Area} = \tfrac{1}{2}ab\sin C}$$

Area of lake = Area of $|\Delta ABE|$ + Area of $|\Delta BED|$ + Area of $|\Delta BDC|$

$= \tfrac{1}{2}(30)(22.3)\sin 53^\circ + \tfrac{1}{2}(24.3)(37.7)\sin 28.4^\circ + \tfrac{1}{2}(27.3)(37.5)\sin 69^\circ$

$= 963 \text{ m}^2$

Question 6 (25 marks)

Question 6 (a)

If a theorem states that p implies q, then the converse of the theorem states that q implies p.
Converse: $q \Rightarrow p$

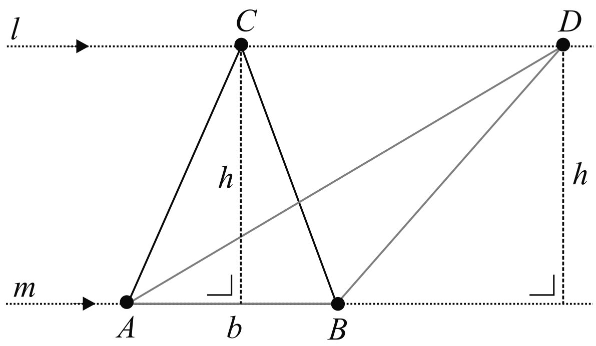
Question 6 (b)

Converse: 'If a number is divisible by 3, then it is also divisible by 6.'
The converse is false.
Example: 27 is divisible by 3 but not by 6.

Question 6 (c)

Converse: 'Triangles with equal areas are congruent.'
This statement is false.

Explain:



Draw two parallel lines l and m.
Draw a line [AB] which forms the base of two triangles, ABC and ABD.
Each triangle drawn has the same base b and height h, and therefore, each triangle has the same area.
However, it is obvious that these triangles are **not** congruent.

Question 7 (75 marks)

Question 7 (a)

Boys

Age	Number	Overweight (%)	Obese (%)
4	650	19	7
5	800	17	5
6	700	13	5
7	820	13	7
8	950	17	7
9	800	13	9
10	950	14	10
11	750	18	6
12	800	19	6
	7220		

Number of boys = 7220

Girls

Age	Number	Overweight (%)	Obese (%)
4	600	22	7
5	800	22	7
6	900	22	7
7	650	19	11
8	730	22	8
9	**900**	**22**	**9**
10	800	20	8
11	940	19	6
12	760	19	6
	7080		

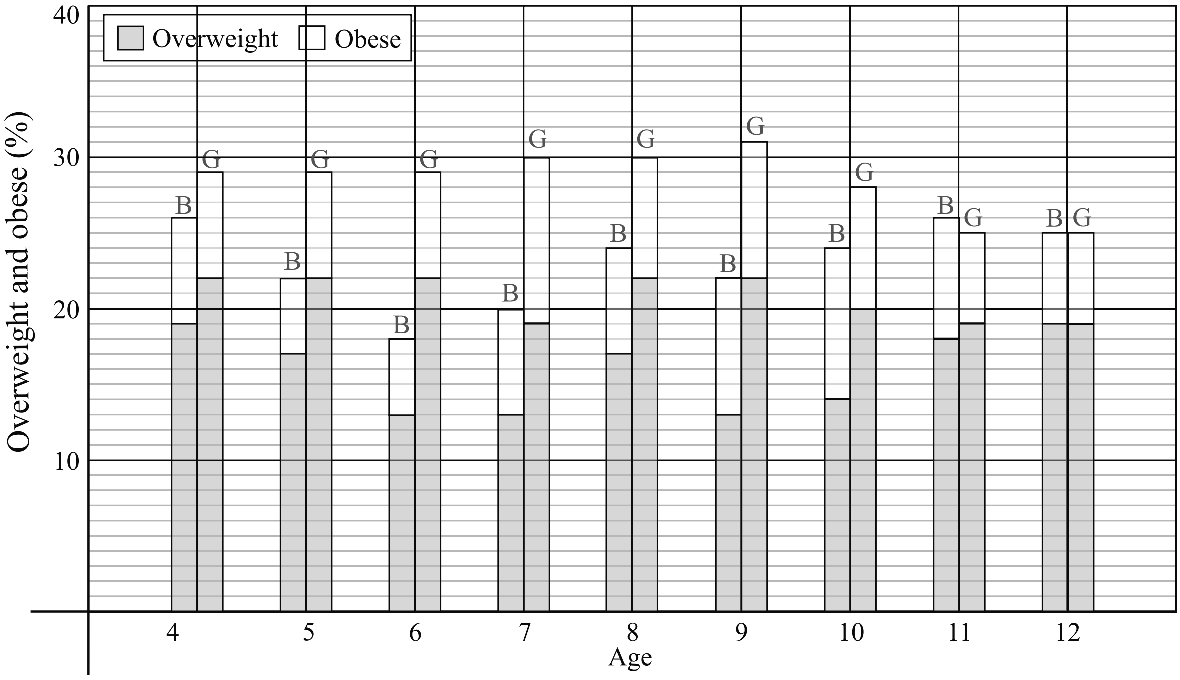
Number of girls = 7080

Number of children = 7220 + 7080 = 14 300

Question 7 (b)

Girls aged nine years have the highest percentage of being overweight and obese.
Percentage of being obese and overweight = 22% + 9% = 31%

Question 7 (c)



Girls on average are more obese than boys. Girls seem to get less overweight as they get older.

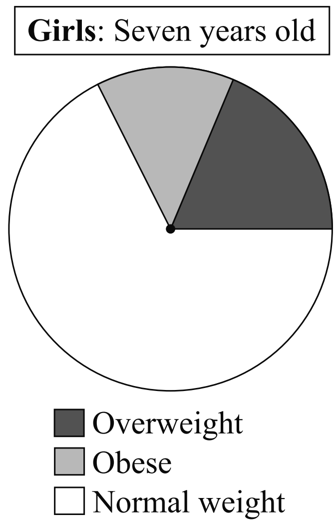
Question 7 (d) (i)

Seven-year-old girls:

Overweight 19% $\rightarrow \frac{19}{100} \times 360^\circ \approx 68^\circ$

Obese 11% $\rightarrow \frac{11}{100} \times 360^\circ \approx 40^\circ$

Normal weight 70% $\rightarrow \frac{70}{100} \times 360^\circ = 252^\circ$



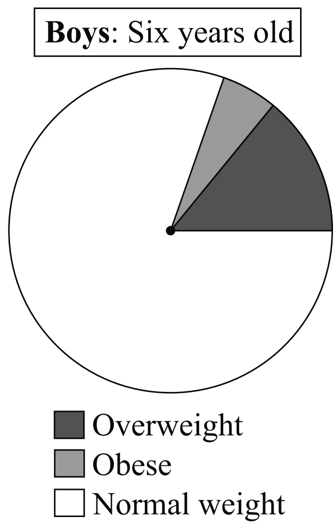
Question 7 (d) (ii)

Six-year-old boys:

Overweight 13% $\rightarrow \frac{13}{100} \times 360^\circ \approx 47^\circ$

Obese 5% $\rightarrow \frac{5}{100} \times 360^\circ = 18^\circ$

Normal weight 82% $\rightarrow \frac{82}{100} \times 360^\circ \approx 295^\circ$



Question 7 (e) (i)

Number of boys who are overweight or obese:

$650 \times 0.26 + 800 \times 0.22 + 7000 \times 0.18 + 820 \times 0.2 + 950 \times 0.24$
$+ 800 \times 0.22 + 950 \times 0.24 + 750 \times 0.24 + 800 \times 0.25$
$= 1647$

Question 7 (e) (ii)

Number who will be overweight as an adult = $1647 \times 0.3 = 494.1$

Percentage of boys who will be overweight as adults $= \dfrac{494.1}{7220} \times 100\% = 6.8\%$

Question 7 (f)

You can work this out by counting how many times you need to add 10,000 on to 300,000 to reach 500,000 and you get 20 years.

Or

$$\text{Number of years} = \frac{500\,000 - 300\,000}{10\,000} = \frac{200\,000}{10\,000} = 20$$

Question 7 (g) (i)

$\text{Height} = 150 \text{ cm} = 1.5 \text{ m}$

$\text{Weight} = 37 \text{ kg}$

$\text{BMI} = \dfrac{37}{1.5^2} = 16.4 \text{ kg/m}^2$

Question 7 (g) (ii)

$\text{Height} = 1.78 \text{ m}$

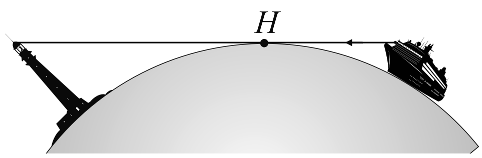
$\text{BMI} = 35 \text{ kg/m}^2$

$35 = \dfrac{\text{Weight}}{1.78^2} \Rightarrow \text{Weight} = 35 \times 1.78^2 = 110.9 \text{ kg}$

His weight must be less that 111 kg.

QUESTION 8 (75 MARKS)

Question 8 (a) (i)



Question 8 (a) (ii)

The first part of the lighthouse that Julie sees is the top of the lighthouse, because the other points are below the horizon.

Question 8 (b) (i)

$d = 3.57\sqrt{h}$

$\therefore d = 3.57\sqrt{1.7} = 4.65 \text{ km}$

Question 8 (b) (ii)

$d = 3.57\sqrt{h}$

$\therefore d = 3.57\sqrt{20} = 15.97 \text{ km}$

Question 8 (b) (iii)

$d = 3.57\sqrt{h}$

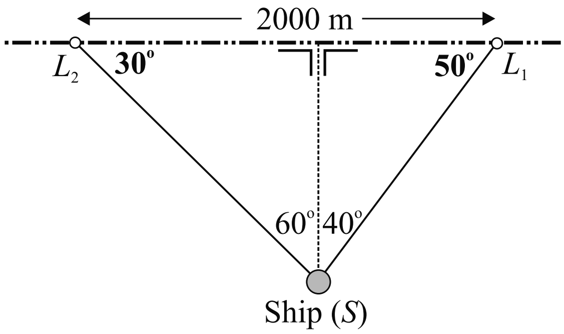
$\therefore d = 3.57\sqrt{35} = 21.12 \text{ km}$

Question 8 (b) (iv)

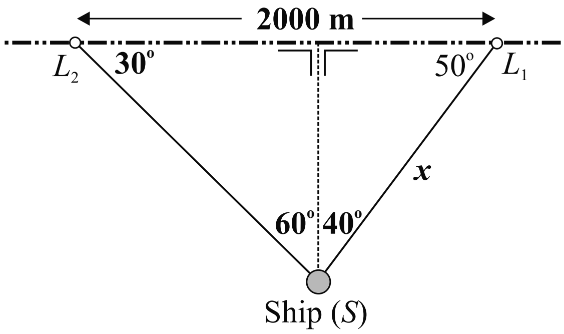
Distance of ship to Paradise Island

$= 15.97 \text{ km} + 21.12 \text{ km} = 37.09 \text{ km} \approx 37 \text{ km}$

Question 8 (c) (i)



Question 8 (c) (ii)

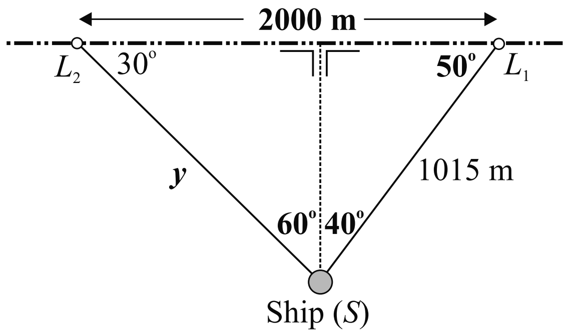


$$\boxed{\frac{a}{\sin A} = \frac{b}{\sin B}}$$

$$\frac{x}{\sin 30^\circ} = \frac{2000}{\sin 100^\circ}$$

$$x = \frac{2000 \sin 30^\circ}{\sin 100^\circ} \approx 1015 \text{ m}$$

Question 8 (c) (iii)



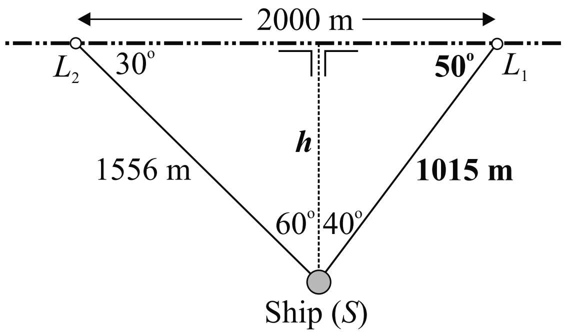
$$\boxed{\frac{a}{\sin A} = \frac{b}{\sin B}}$$

$$\frac{y}{\sin 50^\circ} = \frac{2000}{\sin 100^\circ}$$

$$y = \frac{2000 \sin 50^\circ}{\sin 100^\circ} \approx 1556 \text{ m}$$

Sample 3 Paper 2

Question 8 (c) (iv)



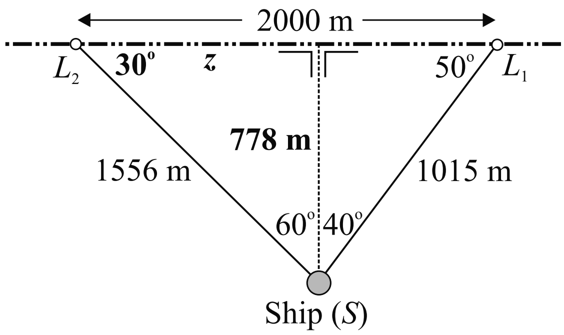
$$\boxed{\sin A = \frac{\text{Opposite}}{\text{Hypotenuse}}}$$

$$\sin 50^\circ = \frac{h}{1015}$$

$$\therefore h = 1015 \sin 50^\circ \approx 778 \text{ m}$$

Yes, it is further than 500 m from the shore.

Question 8 (d) (i)



$$\boxed{\tan A = \frac{\text{Opposite}}{\text{Adjacent}}}$$

$$\tan 30^\circ = \frac{778}{z}$$

$$\therefore z = \frac{778}{\tan 30^\circ} = 1347.5 \text{ m} = 1.3475 \text{ km}$$

Speed = 20 km/h

Distance = 1.3475 km

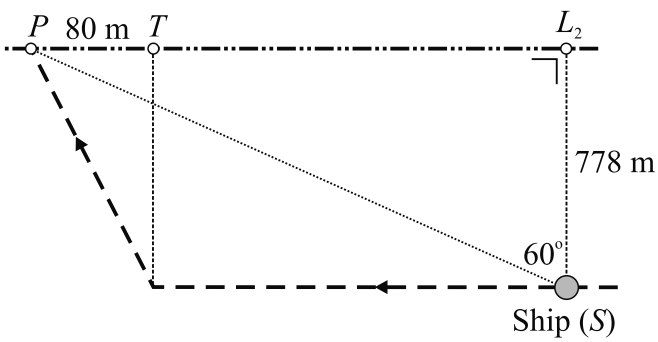
Time = t = ?

$$\boxed{\text{Speed} = \frac{\text{Distance}}{\text{Time}}}$$

$$20 = \frac{1.3475}{t} \Rightarrow t = \frac{1.3475}{20} \times 60 \approx 4 \text{ minutes}$$

The ship is directly opposite the second lighthouse at 12.04 pm.

Question 8 (d) (ii)

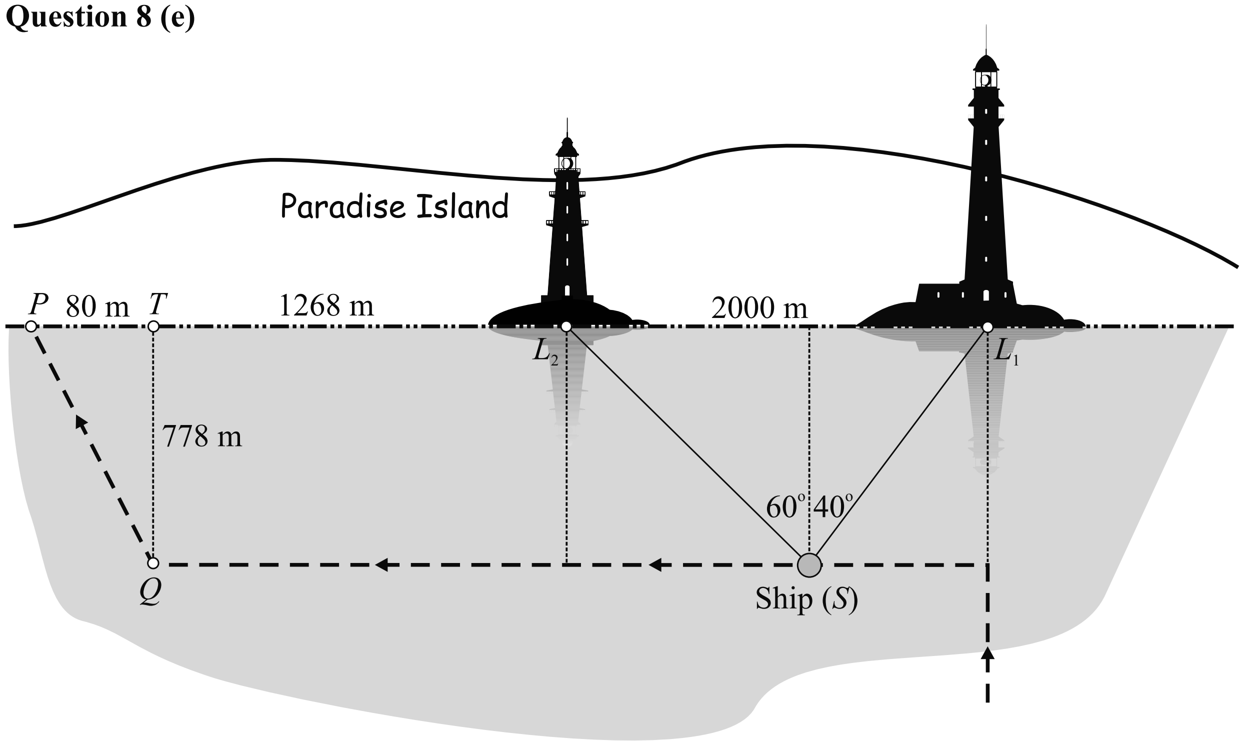


$$\boxed{\tan A = \frac{\text{Opposite}}{\text{Adjacent}}}$$

$$\tan 60^\circ = \frac{|PL_2|}{778}$$

$$\therefore |PL_2| = 778 \tan 60^\circ \approx 1348 \text{ m}$$

Question 8 (e)



$|TL_2| = 1348 - 80 = 1268$ m

$|PQ|^2 = 80^2 + 778^2$

$\therefore |PQ| = \sqrt{80^2 + 778^2} \approx 782$ m

Total distance travelled by ship parallel to coast and into the port = 2000 + 1268 + 782 = 4050 m

Sample Paper 4: Paper 1

Question 1 (25 marks)

Question 1 (a) (i)

$p = 2 + \sqrt{3},\ q = 2 - \sqrt{3}$

$p + q = 2 + \sqrt{3} + 2 - \sqrt{3} = 4$

$pq = (2 + \sqrt{3})(2 - \sqrt{3})$

$\quad = 4 - 2\sqrt{3} + 2\sqrt{3} - 3$

$\quad = 1$

Question 1 (a) (ii)

Roots: $x = p,\ x = q$

$\therefore (x - p)(x - q) = 0$

$x^2 - px - qx + pq = 0$

$x^2 - (p + q)x + pq = 0$

$x^2 - 4x + 1 = 0$

Question 1 (b)

$l : 3x + y = 25 \Rightarrow y = 25 - 3x$

$s : x^2 + y^2 = 65$

$\therefore x^2 + (25 - 3x)^2 = 65$

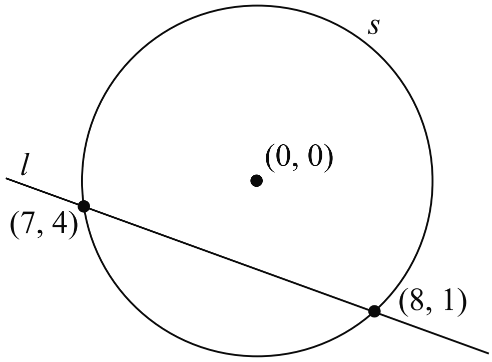
$x^2 + 625 - 150x + 9x^2 - 65 = 0$

$10x^2 - 150x + 560 = 0$

$x^2 - 15x + 56 = 0$

$(x - 7)(x - 8) = 0$

$\therefore x = 7, 8$



$x = 7 : y = 25 - 3(7) = 25 - 21 = 4$

$x = 8 : y = 25 - 3(8) = 25 - 24 = 1$

$\therefore$ (7, 4) and (8, 1) are the points of intersection.

The line cuts the circle at two points whose centre is (0, 0). There the line l is a chord.

Question 2 (25 marks)

Question 2 (a)

$$3+2i(5+8i)-11i$$
$$=3+10i+16i^2-11i$$
$$=3+10i-16-11i$$
$$=-13-i$$

Question 2 (b) (i)

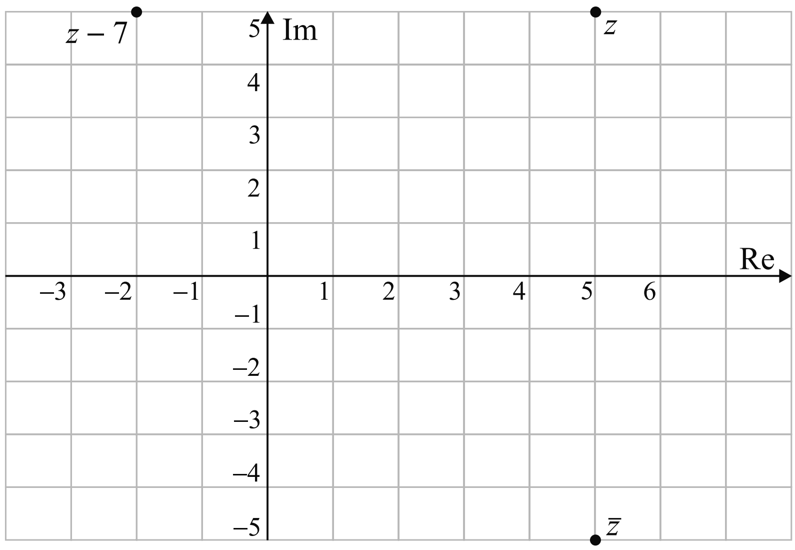
$$z=(1+3i)(-i+2)$$
$$=-i+2-3i^2+6i$$
$$=-i+2-3(-1)+6i$$
$$=-i+2+3+6i$$
$$=5+5i$$

Question 2 (b) (ii)

$$z=5+5i$$
$$z-7=5+5i-7$$
$$=-2+5i$$

Question 2 (b) (iii)

$$z=5+5i \Rightarrow \bar{z}=5-5i$$



Question 2 (c)

$$z=\frac{2+4i}{1-i}$$ [Multiply above and below by the conjugate of the denominator.]

$$\boxed{z=a+ib \Rightarrow \bar{z}=a-ib}$$

$$=\frac{(2+4i)}{(1-i)}\times\frac{(1+i)}{(1+i)}=\frac{2+2i+4i+4i^2}{1+i-i-i^2}$$
$$=\frac{2+2i+4i-4}{1+i-i+1}=\frac{-2+6i}{2}$$
$$=-1+3i$$

$$z=-1+3i$$
$$|z|=\sqrt{(-1)^2+3^3}=\sqrt{10}$$

$$\boxed{|a+ib|=\sqrt{a^2+b^2}}$$

$$\bar{z}=-1-3i$$

$$z\bar{z} = |z|^2$$
$$(-1+3i)(-1-3i) = (\sqrt{10})^2$$
$$1+3i-3i-9i^2 = 10$$
$$1+9 = 10$$

QUESTION 3 (25 MARKS)

Question 3 (a)

Arithmetic sequence: $x+1,\ 2x-1,\ 5x+3$

$\therefore 5x+3-(2x-1)=2x-1-(x+1)$

$5x+3-2x+1=2x-1-x-1$

$3x+4=x-2$

$2x=-6$

$\therefore x=-3$

Arithmetic sequence: $(-3)+1,\ 2(-3)-1,\ 5(-3)+3=-2,\ -7,\ -12$

$-2,\ -7,\ -12$

$a=-2,\ d=-7-(-2)=-5$ $\boxed{T_n=a+(n-1)d}$

$T_n=-2+(n-1)(-5)$

$=-2-5n+5$

$=-5n+3$

Question 3 (b)

$2,\ 4,\ 6,\ 8,\ 10,\ 12, ...100$

$a=2,\ d=2$

$T_n=100=2+(n-1)2$

$\therefore 100=2+2n-2$

$100=2n$

$\therefore n=50$

Question 3 (c)

$2,\ 4,\ 6,\ 8,\ 10,\ 12, ...$ $\boxed{S_n=\tfrac{n}{2}[2a+(n-1)d]}$

$a=2,\ d=2,\ n=100$

$S_{100}=\tfrac{100}{2}[2(2)+(100-1)2]$

$=50[4+99(2)]$

$=50[202]$

$=10\ 100$

QUESTION 4 (25 MARKS)

Question 4 (a)

Gross amount at the end of 5 years = €10 000 × 1.01^5 = €10 510.10
Interest earned = €510.10
Tax is payable on the interest at 30%, which means you get to keep 70% of the interest.
Amount of interest retained = €510.10 × 0.7 = €357.07
Amount payable after 5 years = €10 357.07

Question 4 (b)

Gross amount at the end of 5 years = €10 000 × 1.01^5 = €10 510.10

Bonus payment = €10 510.10 × 0.1 = €1051.01

Total interest earned = €510.10 + €1051.01 = €1561.11

Amount of interest retained = €1561.11 × 0.7 = €1092.78

Amount payable after 5 years = €11 092.78

Question 5 (25 marks)

Question 5 (a)

$f(x) = 3x^2 - 7x$

$f'(x) = 6x - 7$

$f'(3) = 6(3) - 7 = 18 - 7 = 11$

Question 5 (b) (i)

$y = ax^2 + bx$

$(3,\ 4) \in y = ax^2 + bx \Rightarrow 4 = a(3)^2 + b(3)$

$4 = 9a + 3b....(\mathbf{1})$

$y = ax^2 + bx$

$\dfrac{dy}{dx} = 2ax + b$

$\left(\dfrac{dy}{dx}\right)_{x=1} = 0 \Rightarrow 2a(1) + b = 0$

$2a + b = 0....(\mathbf{2})$

Solve equations (**1**) and (**2**) simultaneously to find a and b (see right).

$a = \frac{4}{3},\ b = -\frac{8}{3}$

$y = \frac{4}{3}x^2 - \frac{8}{3}x$

$9a + 3b = 4....(\mathbf{1})$

$2a + b = 0......(\mathbf{2})(\times -3)$

$$\begin{array}{l} 9a + 3b = 4 \\ \underline{-6a - 3b = 0} \\ 3a \qquad\ = 4 \Rightarrow a = \frac{4}{3} \end{array}$$

$2a + b = 0......(\mathbf{2})$

$2(\frac{4}{3}) + b = 0$

$\frac{8}{3} + b = 0$

$\therefore b = -\frac{8}{3}$

Question 5 (b) (ii)

As the tangent is parallel to the x-axis, its slope is zero. Turning points are points whose tangents have zero slope.

Local minimum at $x = 1$

$y = \frac{4}{3}x^2 - \frac{8}{3}x = \frac{4}{3}(1)^2 - \frac{8}{3}(1) = \frac{4}{3} - \frac{8}{3} = -\frac{4}{3}$

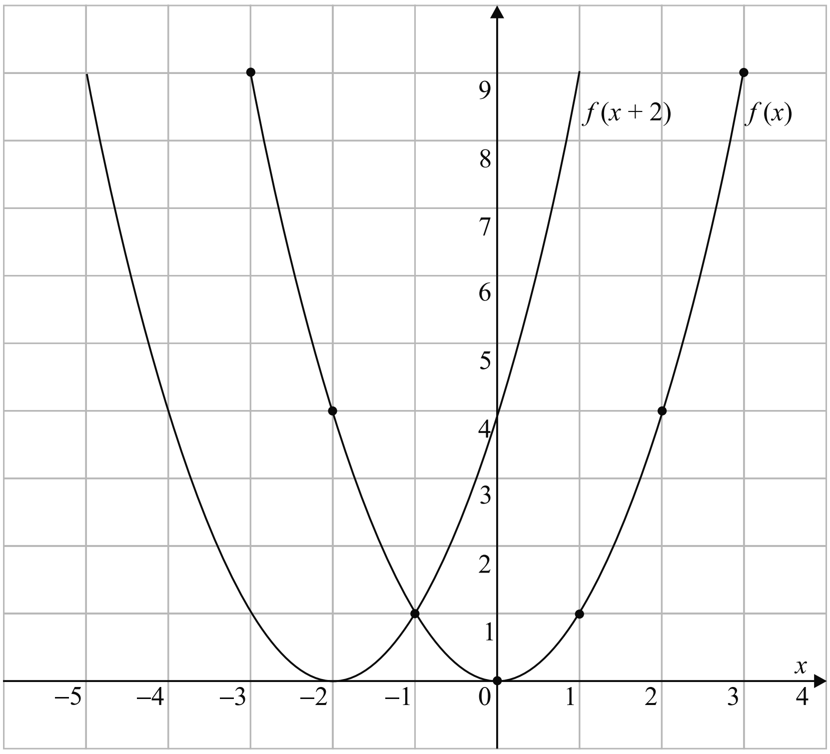
Local minimum: $(1,\ -\frac{4}{3})$

Sample 4 Paper 1

Question 6 (25 marks)

Question 6 (a)

To draw the curve $f(x+2)$ given the curve $f(x)$, move the $f(x)$ curve 2 units to the left along the x-axis.



Question 6 (b)

$f(x) = f(x+2)$

$x^2 = (x+2)^2$

$x^2 = x^2 + 4x + 4$

$0 = 4x + 4$

$-4 = 4x$

$\therefore x = -1$

Question 6 (c)

$y = (x+2)^2 = x^2 + 4x + 4$

$\frac{dy}{dx} = 2x + 4$

$\left(\frac{dy}{dx}\right)_{x=-1} = 2(-1) + 4 = 2 = m_1$

$y = x^2$

$\frac{dy}{dx} = 2x$

$\left(\frac{dy}{dx}\right)_{x=-1} = 2(-1) = -2 = m_2$

$m_1 + m_2 = 2 - 2 = 0$

Question 7 (50 marks)

Question 7 (a)

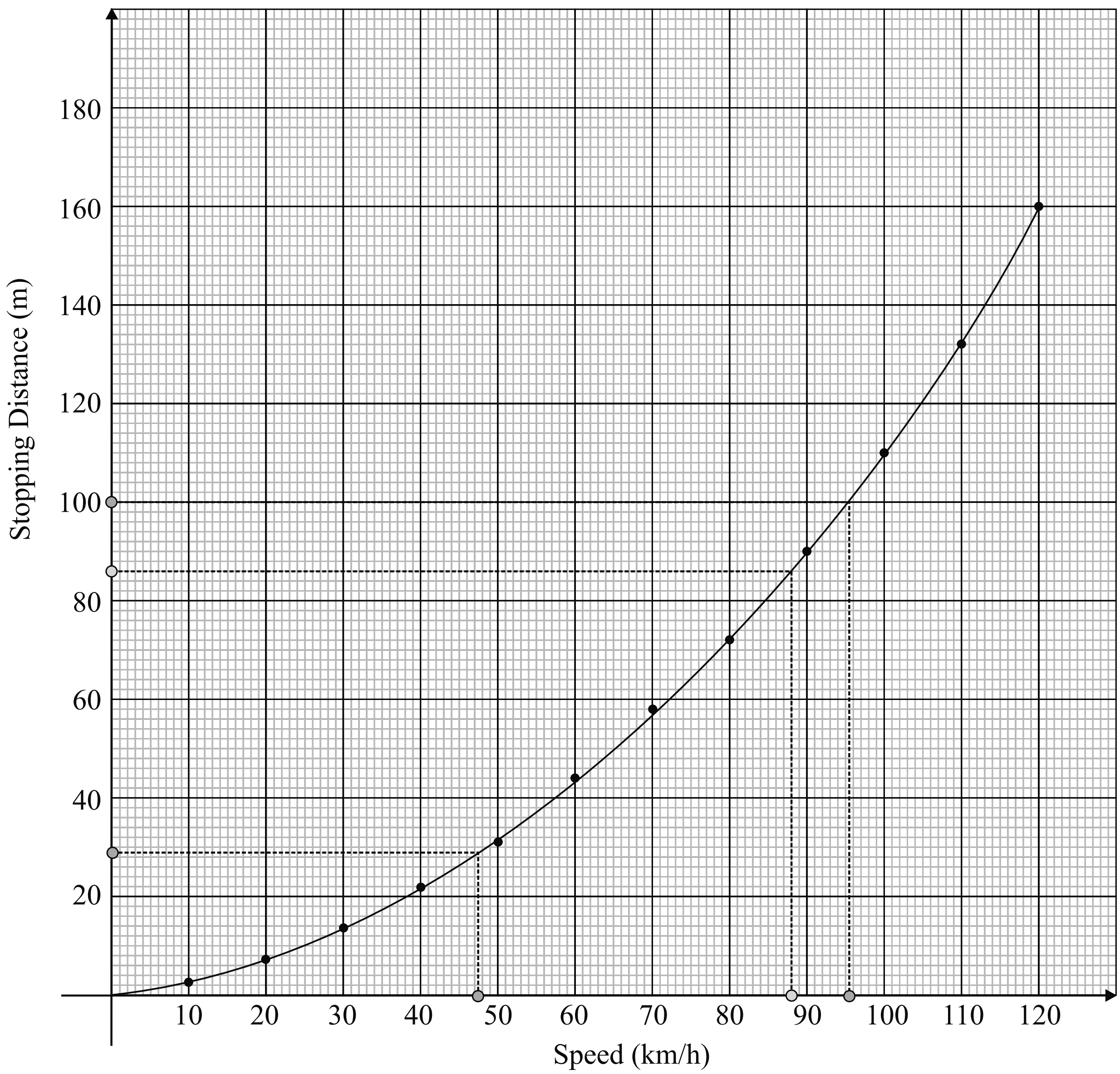
$1 \text{ mile} = 1.6 \text{ km}$

$1 \text{ mph} = 1.6 \text{ km/h}$

$55 \text{ mph} = 55 \times 1.6 \text{ km/h} = 88 \text{ km/h}$

Go to 88 km/h on the horizontal axis. Draw a line straight up to the graph and then straight across to the vertical axis. Read off the distance.

Answer: 86 m



Question 7 (b)

Go to 100 m on the vertical axis. Its corresponding speed is 95 km/h.
Half of this speed is 47.5 km/h.
Go to 47.5 km/h on the horizontal axis. Its corresponding distance is 29 m.

Question 7 (c)

$y = 0.014x^2 + 0.065x$

$x = 20: y = 0.014(20)^2 + 0.065(20) = 6.9 \approx 7 \text{ m}$

$x = 30: y = 0.014(30)^2 + 0.065(30) = 14.55 \approx 15 \text{ m}$

$x = 50: y = 0.014(50)^2 + 0.065(50) = 38.25 \approx 38 \text{ m}$

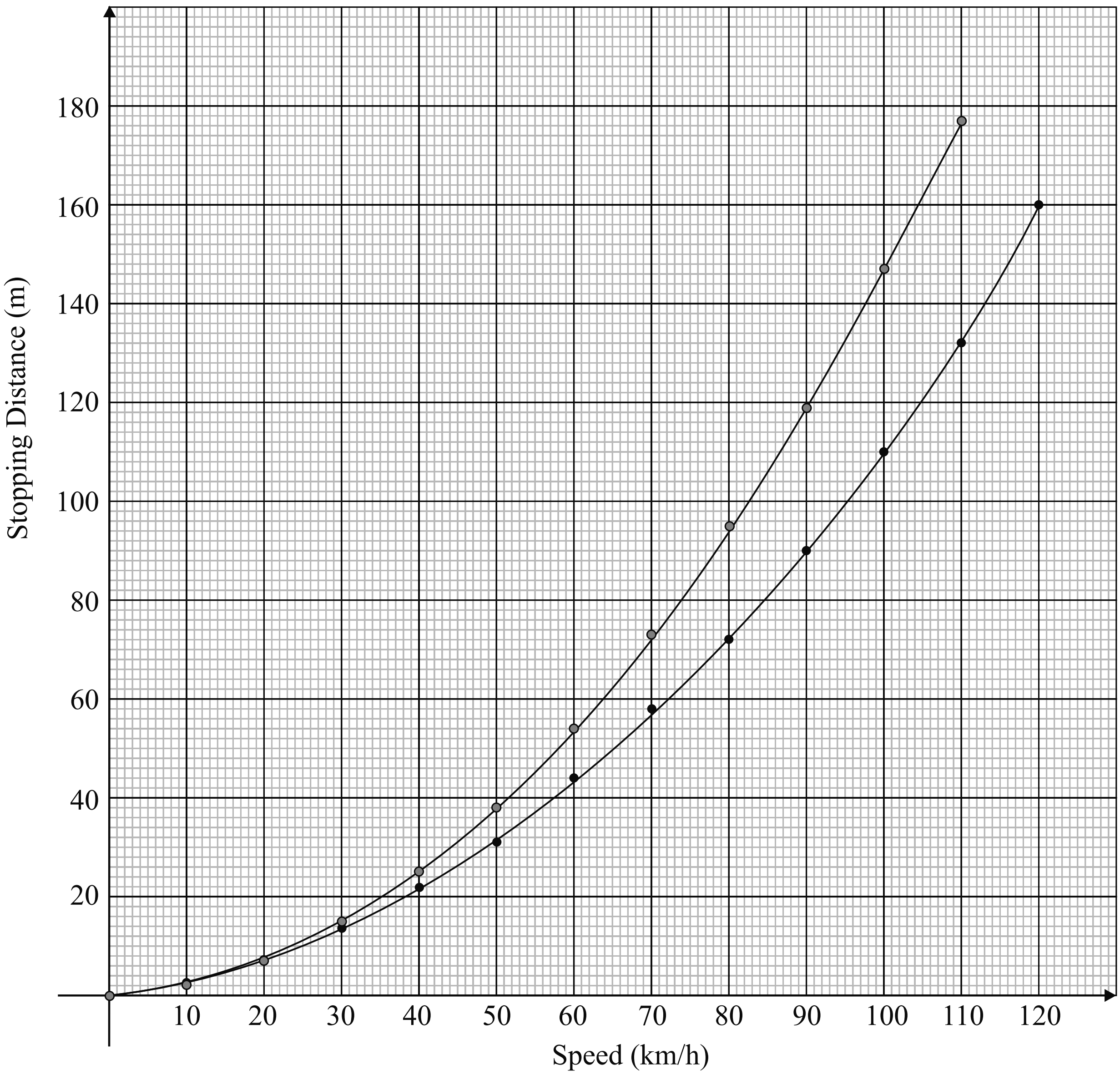
$x = 60: y = 0.014(60)^2 + 0.065(60) = 54.3 \approx 54 \text{ m}$

$x = 70: y = 0.014(70)^2 + 0.065(70) = 73.15 \approx 73 \text{ m}$

$x = 100: y = 0.014(100)^2 + 0.065(100) = 146.5 \approx 147 \text{ m}$

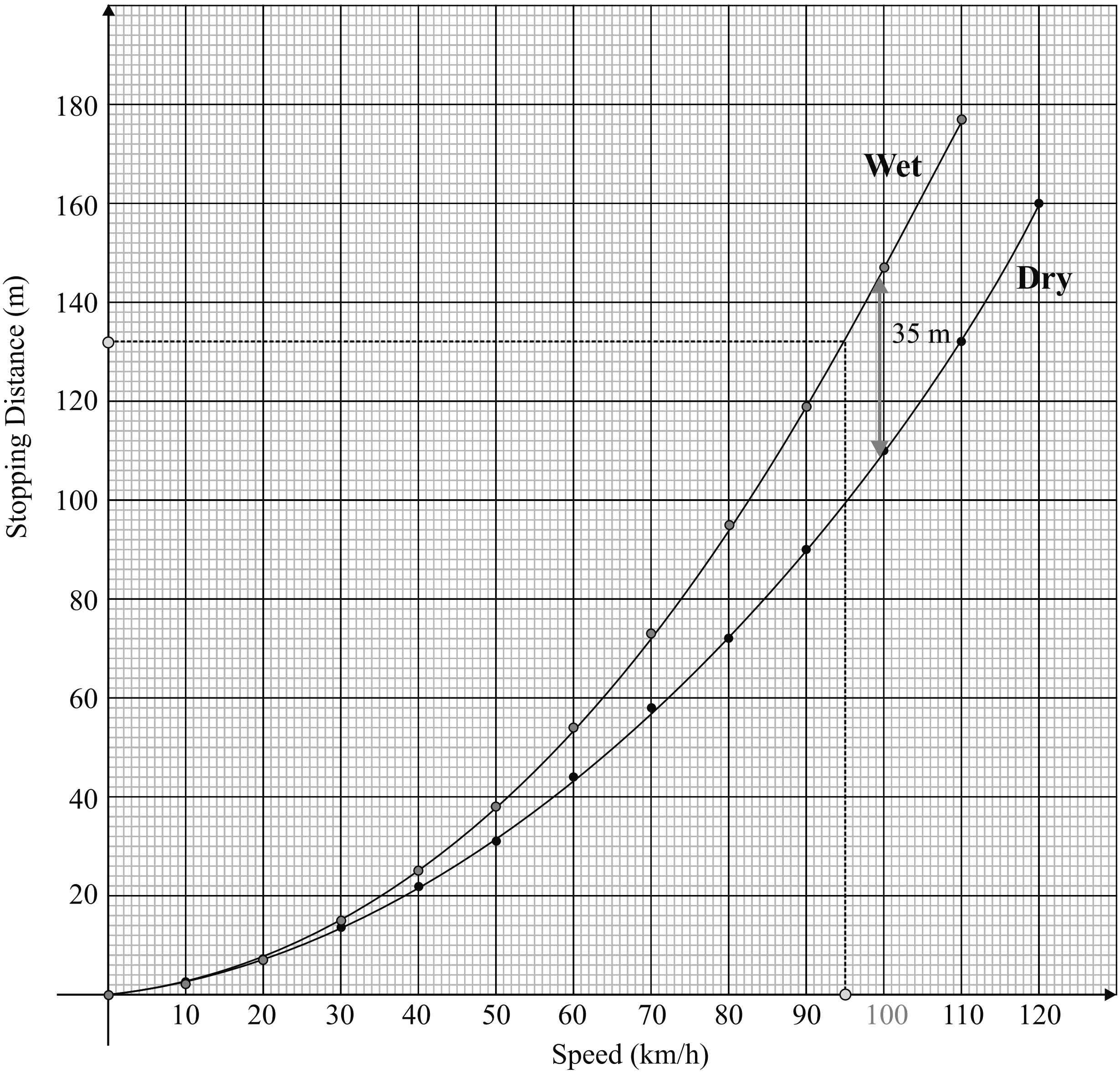
x	0	10	20	30	40	50	60	70	80	90	100	110
y	0	2	7	15	25	38	54	73	95	119	147	177

Question 7 (d)



Question 7 (e)

Go to 132 m on the vertical axis. Draw a line straight across until this line meets the wet graph. The corresponding value on the horizontal axis is 95 km/h.

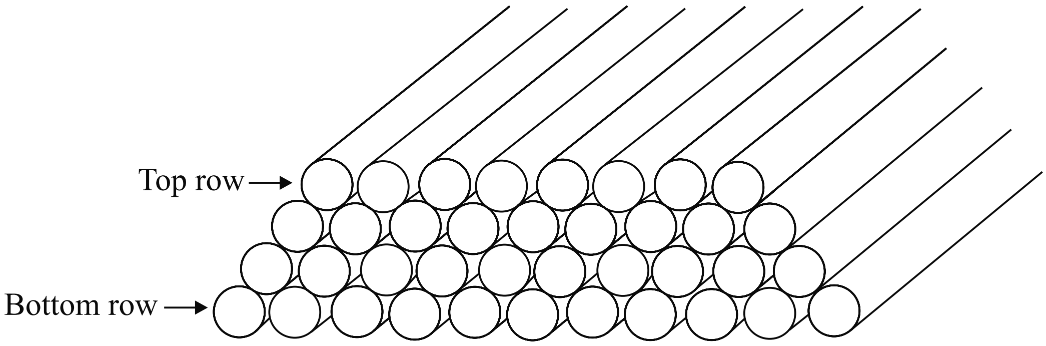


Question 7 (f)

Find out where there is a vertical gap of 35 m between the wet road and dry road graphs. This occurs approximately at a speed of 100 km/h.

Question 8 (50 marks)

Question 8 (a)



$$N = \frac{(b+t)(b-t+1)}{2}$$

$b = 20,\ t = 12$

$$\therefore N = \frac{(20+12)(20-12+1)}{2} = \frac{(32)(9)}{2} = 144$$

Question 8 (b)

$$N = \frac{(b+t)(b-t+1)}{2}$$

$b = 37,\ N = 598,\ t = ?$

$$598 = \frac{(37+t)(37-t+1)}{2}$$

$1196 = (37+t)(38-t)$

$1196 = 1406 - 37t + 38t - t^2$

$t^2 - t - 210 = 0$

$(t-15)(t+14) = 0$

$\therefore t = 15$

3	210
7	70
5	10
2	2
	1

Percentage of pipes on the top $= \frac{15}{598} \times 100\% \approx 2.5\%$

Question 8 (c) (i)

$\boxed{\text{Cylinder: } V = \pi r^2 h}$

$r = 3\text{cm},\ h = 30\ \text{cm},\ t = 15$

$V = 15 \times \pi \times 3^2 \times 30 = 4050\pi\ \text{cm}^3$

Question 8 (c) (ii)

$r = 3\text{cm},\ h = 30\ \text{cm},\ b = 37$

$V = 37 \times \pi \times 3^2 \times 30 = 9990\pi\ \text{cm}^3$

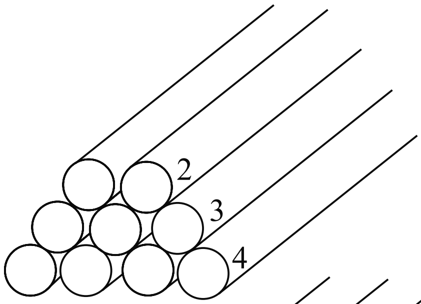
Question 8 (c) (iii)

Number of pipes in other rows $n = N - t - b = 598 - 15 - 37 = 546$

$r = 3\text{cm},\ h = 30\ \text{cm},\ n = 546$

$V = 546 \times \pi \times 3^2 \times 30 = 147\ 420\pi\ \text{cm}^3$

Question 8 (d) (i)

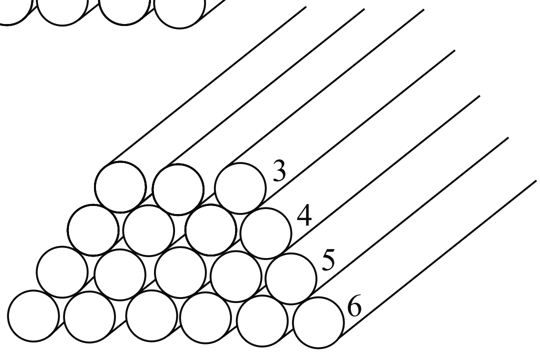


$$N = \frac{(b+t)(b-t+1)}{2}$$

$$b = 2t$$

$$N = \frac{(2t+t)(2t-t+1)}{2} = \frac{(3t)(t+1)}{2}$$

Question 8 (d) (ii)



$$t = 2$$

$$N = \frac{(3t)(t+1)}{2} = \frac{(3(2))(2+1)}{2} = \frac{(6)(3)}{2} = 9$$

Question 8 (d) (iii)

$$t = 3$$

$$N = \frac{(3t)(t+1)}{2} = \frac{(3(3))(3+1)}{2} = \frac{(9)(4)}{2} = 18$$

Question 8 (e)

$t = 10,\ b = 20$

Total number of pipes $= 10+11+12+13+14+15+16+17+18+19+20$

$a = 10,\ d = 1,\ n = 11$

$S_{11} = \frac{11}{2}[2(10)+(11-1)1] = \frac{11}{2}[20+10] = \frac{11}{2}[30] = 165$ $\boxed{S_n = \frac{n}{2}[2a+(n-1)d]}$

Question 8 (f)

Use the result in **8 (d) (i)**:

$$b = 2t \Rightarrow N = \frac{(3t)(t+1)}{2}$$

$$\therefore 513 = \frac{(3t)(t+1)}{2}$$

$$1026 = 3t^2 + 3t$$

$$0 = t^2 + t - 342$$

$$0 = (t+19)(t-18)$$

$$\therefore t = 18$$

2	342
3	171
3	57
19	19
	1

Yes, this is possible. There are 18 pipes in the top row and 36 in the bottom row. The total number of pipes will be 513.

QUESTION 9 (50 MARKS)

Question 9 (a)

$S = 2200 - 2p \geq 0$ $\qquad$ $D = 4p - 320 \geq 0$

$2200 \geq 2p$ $\qquad$ $4p \geq 320$

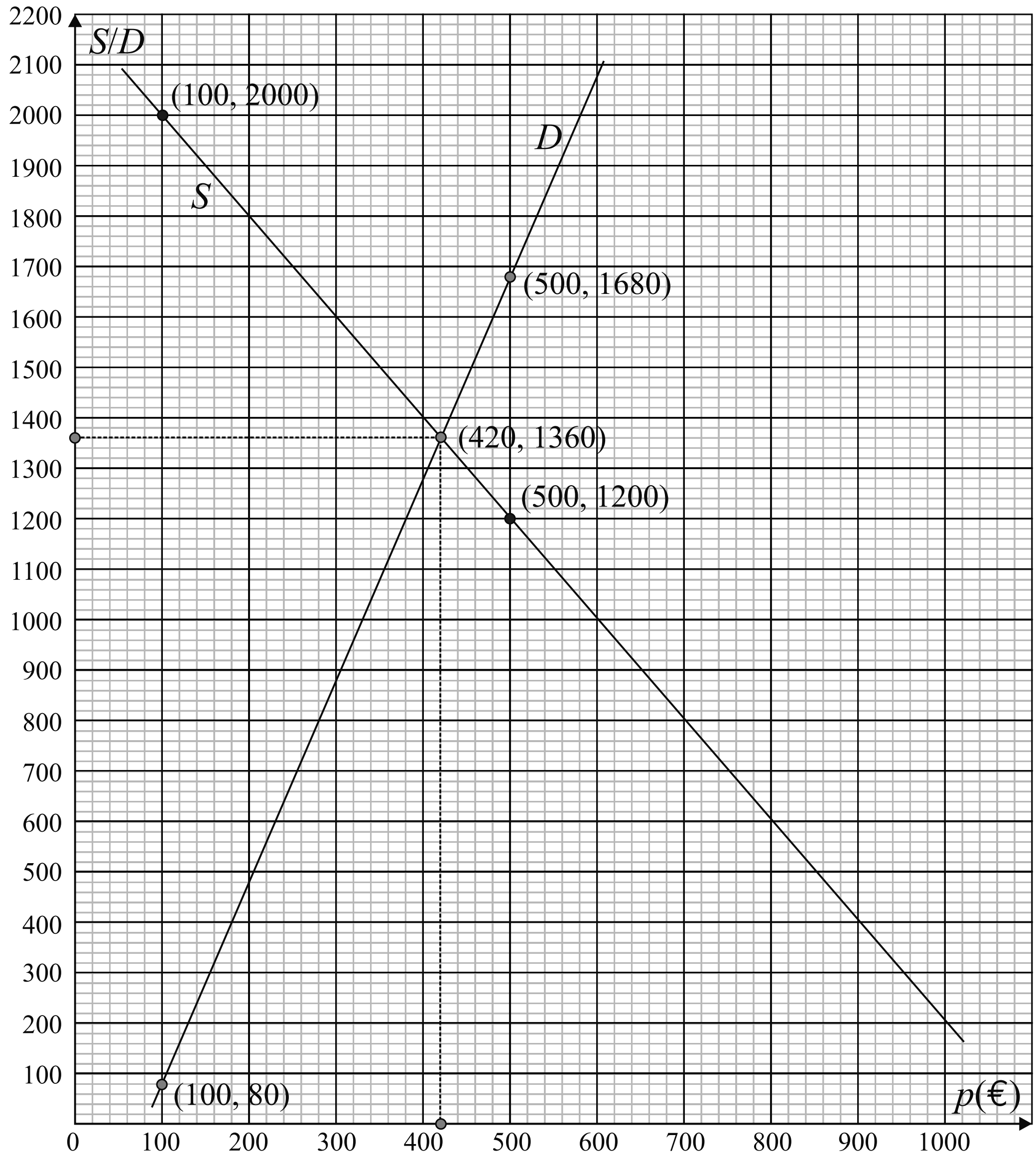
$1100 \geq p$ $\qquad$ $p \geq 80$

$\therefore p \leq 1100$

Price range: $€80 \leq p \leq €1100$

Question 9 (b)

p (€)	100	500
S	2000	1200
D	80	1680



Question 9 (c)

ALGEBRA

$S = D$

$2200 - 2p = 4p - 320$

$2200 + 320 = 6p$

$2520 = 6p$

$\therefore p = €420$

GRAPH

The supply and demand lines on the graph intersect at p = €420. This is the equilibrium point.

Question 9 (d)

ALGEBRA

$p = €420 : S = 2200 - 2p = 2200 - 2(420) = 1360$

$p = €420 : D = 4p - 320 = 4(420) - 320 = 1360$

GRAPH

According to the graphs, the number of items supplied and demanded at p = €420 is 1,360 items.

Question 9 (e)

$S > D$: Manufacturers will have warehouses filled with items that consumers won't buy.
$S < D$: Stores will have empty shelves and unhappy customers asking for the product.

Question 9 (f)

$S = 40 - 2p^2$

$D = 8p - 2$

$S = D \Rightarrow 40 - 2p^2 = 8p - 2$

$0 = 2p^2 + 8p - 42$

$0 = p^2 + 4p - 21$

$0 = (p-3)(p+7)$

$\therefore p = €3$

Sample Paper 4: Paper 2

Question 1 (25 marks)

Question 1 (a)

Centre (1, 2), $r = 5$ $\boxed{(x-h)^2+(y-k)^2=r^2}$

$(x-1)^2+(y-2)^2=5^2$

$(x-1)^2+(y-2)^2=25$

Question 1 (b)

$y=0:(x-1)^2+(0-2)^2=25$ $\boxed{\text{Cuts the } x\text{-axis: Put } y=0}$

$(x-1)^2+4=25$

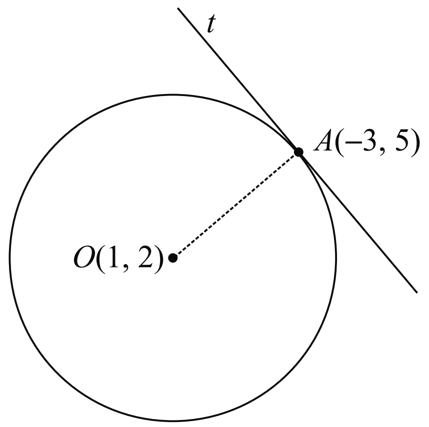
$(x-1)^2=21$

$(x-1)=\pm\sqrt{21}$

$x=1\pm\sqrt{21}$

$\therefore P(1+\sqrt{21},\ 0),\ Q(1-\sqrt{21},\ 0)$

Question 1 (c)



Is $A(-3,\ 5)\in(x-1)^2+(0-2)^2=25$?

$(-3-1)^2+(5-2)^2=(-4)^2+(3)^2=16+9=25$

Find the slope m_1 of OA:

$m_1=\frac{5-2}{-3-1}=\frac{3}{-4}=-\frac{3}{4}$ $\boxed{m=\frac{y_2-y_1}{x_2-x_1}}$

Slope of tangent t: $m_2=\frac{4}{3}$ $\boxed{\text{Perpendicular lines: } m_1\times m_2=-1}$

$y-5=\tfrac{4}{3}(x-(-3))$ $\boxed{y-y_1=m(x-x_1)}$

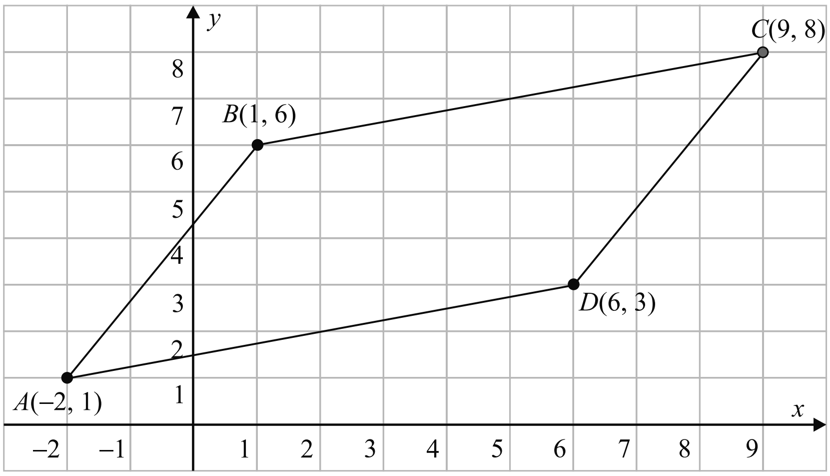
$3(y-5)=4(x+3)$

$3y-15=4x+12$

$0=4x-3y+27$

Question 2 (25 marks)

Question 2 (a)



$A(-2, 1) \to B(1, 6)$ [Add 3, Add 5]

$D(6, 3) \to C(9, 8)$

Question 2 (b)

$A(-2, 1)$, $D(6, 3)$

$m = \dfrac{3-1}{6-(-2)} = \dfrac{2}{8} = \dfrac{1}{4}$ $\boxed{m = \dfrac{y_2 - y_1}{x_2 - x_1}}$

Slope of BE: $m = -4$ $\boxed{\text{Perpendicular lines: } m_1 \times m_2 = -1}$

Equation of BE: $B(1, 6)$, $m = -4$ $\boxed{y - y_1 = m(x - x_1)}$

$y - 6 = -4(x - 1)$

$y - 6 = -4x + 4$

$4x + y - 10 = 0$

Question 2 (c)

Equation of AD: $A(-2, 1)$, $m = \frac{1}{4}$

$y - 1 = \frac{1}{4}(x - (-2))$

$4(y - 1) = 1(x + 2)$

$4y - 4 = x + 2$

$0 = x - 4y + 6$

Solve simultaneously the equations for AD and BE.

$4x + y = 10 ......(\mathbf{1})(\times 4)$

$x - 4y = -6 .....(\mathbf{2})$

$16x + 4y = 40$

$x - 4y = -6$

$17x = 34 \Rightarrow x = 2$

$x = 2 : 4(2) + y = 10 \Rightarrow y = 2$

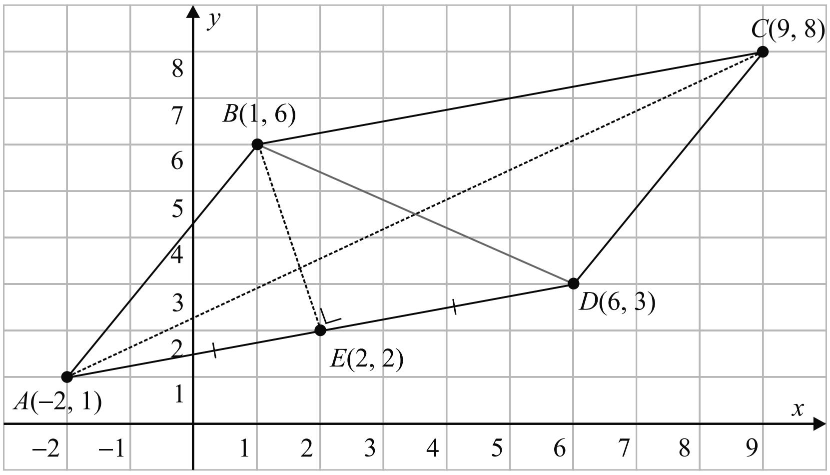
Point of intersection of BE and AD: $E(2, 2)$

Midpoint of $[AD]$: $A(-2, 1)$, $D(6, 3)$

$$\text{MP} = \left(\frac{-2+6}{2}, \frac{1+3}{2}\right) = (2, 2) \quad \boxed{\text{MP} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)}$$

Sample 4 Paper 2

Question 2 (d)



Method 1:

Area of a parallelogram: $\boxed{\text{Area} = b \times h}$

$$\text{Area} = b \times h = |AD| \times |BE|$$

$$|AD| = \sqrt{(6-(-2))^2 + (3-1)^2} = \sqrt{8^2 + 2^2} = \sqrt{64+4} = \sqrt{68} = \sqrt{4 \times 17} = 2\sqrt{17}$$

$$|BE| = \sqrt{(1-2)^2 + (6-2)^2} = \sqrt{(-1)^2 + 4^2} = \sqrt{1+16} = \sqrt{17}$$

$$\therefore \text{Area} = |AD| \times |BE| = 2\sqrt{17}\sqrt{17} = 2(17) = 34$$

Method 2:

The diagonal $[BD]$ bisects the area of the parallelogram.

$$\text{Area} = 2 \times |\Delta ABD|$$

$A(-2, 1) \rightarrow (0, 0)$ [Add 2, Subtract 1] $\boxed{A = \tfrac{1}{2}|x_1 y_2 - x_2 y_1|}$

$B(1, 6) \rightarrow (3, 5)$

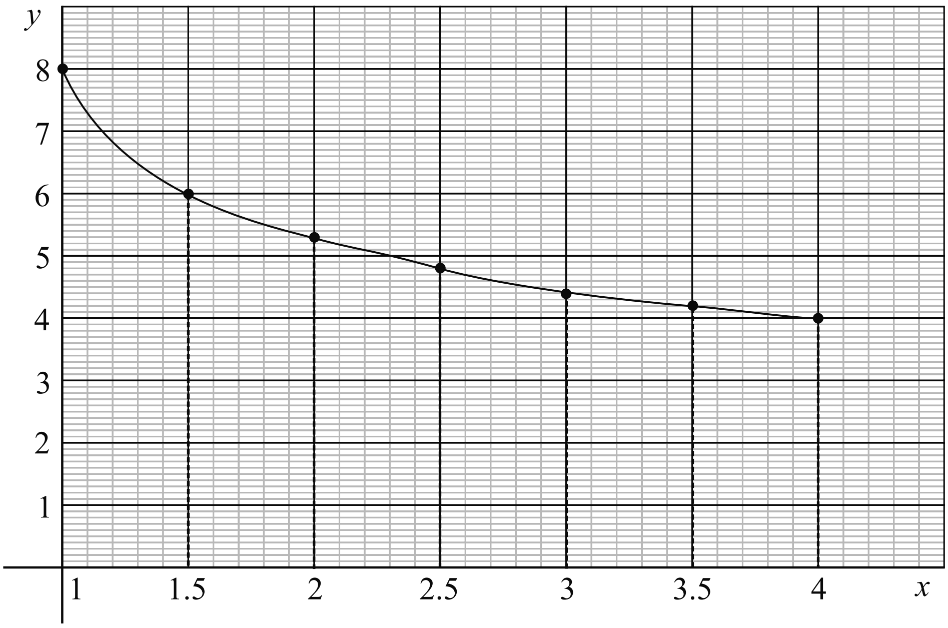
$D(6, 3) \rightarrow (8, 2)$

$$\text{Area} = 2 \times \tfrac{1}{2}|(3)(2) - (5)(8)| = |6 - 40| = |-34| = 34 \text{ square units}$$

QUESTION 3 (25 MARKS)

Question 3 (a) & (b)

x	1.0	1.5	2.0	2.5	3.0	3.5	4.0
y	8.0	6.0	5.3	4.8	4.4	4.2	4.0



Question 3 (c)

$$A \approx \frac{h}{2}[y_1 + y_n + 2(y_2 + y_3 + y_4 + ... + y_{n-1})]$$

$h = 0.5$

$$A \approx \frac{0.5}{2}[8 + 4 + 2(6 + 5.3 + 4.8 + 4.4 + 4.2)] = 15.35 \text{ square units}$$

Sample 4 Paper 2

Question 4 (25 marks)

Question 4 (a) (i)

If there are a ways of doing one operation, and b ways of doing another operation, then there are ab ways of doing both operations.

Question 4 (a) (ii)

$5 \times 6 \times 4 = 120$

Starter	Main	Dessert

There are five ways to fill the first box.
Once this box has been filled, there are four ways to fill the second box. Once the first two boxes have been filled, there are three ways to fill the third box.

Number of possible three-course meals = 120

Question 4 (b)

Digits can be repeated:

There are nine ways to fill the first box (you cannot have a zero here as it would not be a five-digit number).
There are ten ways to fill the second box (you can use all digits).
There are ten ways to fill the third box, and so on.

$9 \times 10 \times 10 \times 10 \times 10 = 90\,000$

Number of five-digit numbers = 90 000

Digits cannot be repeated:

There are nine ways to fill the first box (you cannot have a zero here as it would not be a five-digit number).
There are nine ways to fill the second box (you cannot use what is in the first box but you can use a zero).
There are eight ways to fill the third box (you cannot have a number that is in the first two boxes).

$9 \times 9 \times 8 \times 7 \times 6 = 27\,216$

Number of five-digit numbers with no digit repeated = 27 216

Question 4 (c)

$6 \times 5 \times 4 \times 3 \times 2 \times 1 = 720$

Number of arrangements of six people = 720

Glue the boys together and glue the girls together. There are two ways to arrange the groups. There are six ways to arrange the three boys in their group and there are six ways to arrange the three girls in their group.

$3 \times 2 \times 1 \qquad 3 \times 2 \times 1$

B	**B**	**B**		**G**	**G**	**G**

Number of arrangements where boys and girls sit together = $2(3 \times 2 \times 1 \times 3 \times 2 \times 1) = 72$

Question 5 (25 marks)

Question 5 (a)

Gain in cent per dozen	20	12	8	4	0	–2
Probability	0.26	x	0.35	0.04	0.03	0.02

$P(x) = 0.26 + x + 0.35 + 0.04 + 0.03 + 0.02 = 1$ $\boxed{\sum P(x) = 1}$

$\therefore x = 0.3$

Question 5 (b)

Gain in cent per dozen	20	12	8	4	0	–2
Probability	0.26	0.3	0.35	0.04	0.03	0.02

Expected gain: $\boxed{E(x) = \sum xP(x)}$

$E(x) = \sum xP(x)$

$= 20\times0.26+12\times0.3+8\times0.35+4\times0.04+0\times0.3+(-2)\times0.02=11.72 \text{ c}$

Question 5 (c)

Total number of eggs = 500 000

Number of dozens of eggs $= \dfrac{6\,000\,000}{12} = 500\,000$

Expected gain on his total production $= 500\,000\times11.72\text{ c} = 5\,860\,000 \text{ c} = €58\,600$

Question 5 (d)

Gain in cent per dozen	20	12	8	4	0	–2
Probability	0.26	0.3	y	0.04	0.03	0.02

$E(x) = 12 \text{ c}$

$\therefore 20\times0.26+12\times0.3+8y+4\times0.04+0\times0.3+(-2)\times0.02=12$

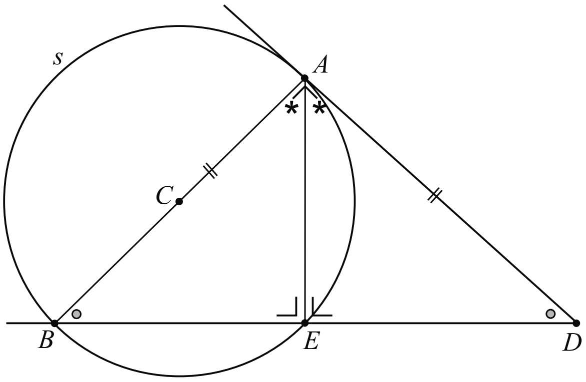
$8y+8.92=12$

$8y=12-8.92=3.08$

$y=0.385$

QUESTION 6 (25 MARKS)

Question 6 (a)



Consider triangles *ABE* and *AED*.

$|AB| = |AD|$ (Given)

$|\angle ABD| = |\angle ADB|$ (Isosceles triangle)

$|AE| = |AE|$ (Common side)

$|\angle AEB| = 90^{\circ}$ (Angle standing on diameter is a right angle)

$\therefore |\angle AED| = 90^{\circ}$ (Straight angle)

Therefore, triangles *ABE* and *AED* are congruent.

$\therefore |BE| = |ED|$

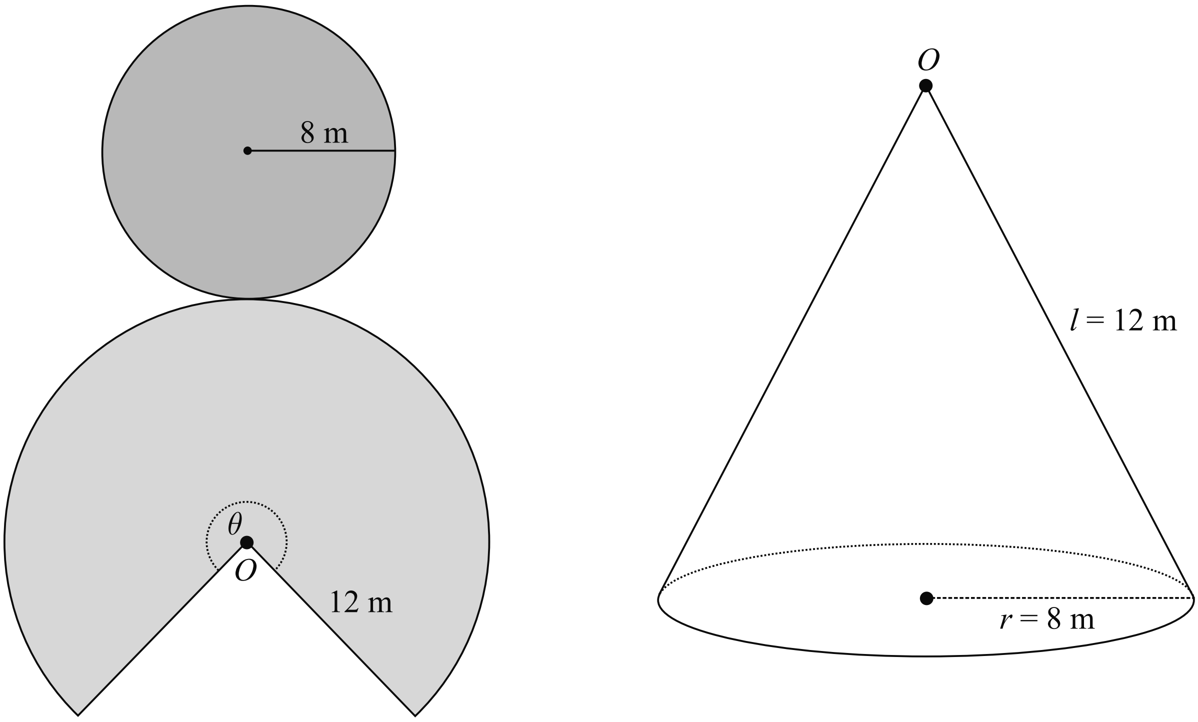
Question 6 (b)

$|\angle BAD| = 90^{\circ}$ (Tangent is perpendicular to the point of contact.)

$|\angle EBA| = |\angle EDA| = 45^{\circ}$ (Isosceles triangle)

QUESTION 7 (75 MARKS)

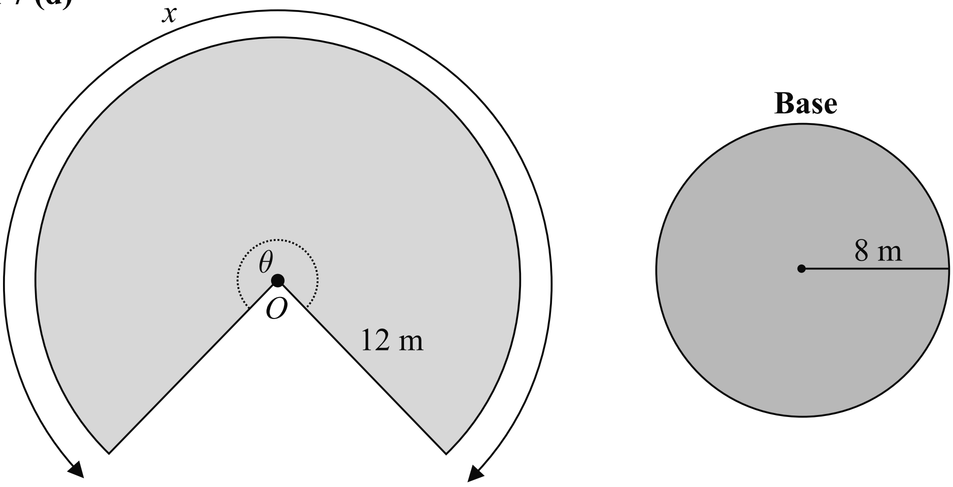
Question 7 (a)



Question 7 (b&c)

Radius of base of cone $r = 8$ m
Slant height of the cone $l = 12$ m

Question 7 (d)



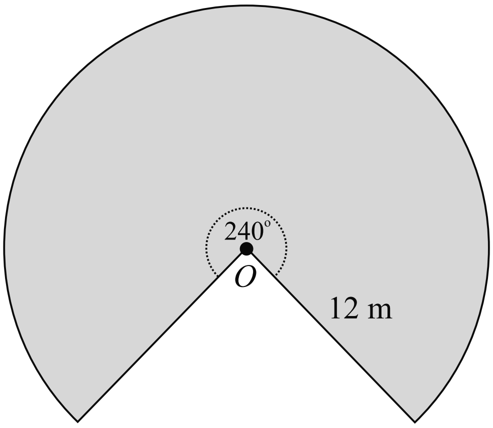
$\boxed{\text{Circumference of circle } l = 2\pi r}$

The length x is equal to the circumference of the circular base.
Length of circumference of base = $2\pi(8) = 16\pi$ m = x
Length of circumference of big circle = $2\pi(12) = 24\pi$ m

$$\frac{\text{Circumference of base}}{\text{Circumference of big circle}} = \frac{16\pi}{24\pi} = \frac{2}{3}$$

$$\theta = \tfrac{2}{3} \times 360^\circ = 240^\circ$$

Question 7 (e)



$\boxed{\text{Area of circle } A = \pi r^2}$

Curved surface area

$$= \frac{240^\circ}{360^\circ} \times \pi(12)^2$$

$$= 96\pi = 301.6 \text{ m}^2$$

Question 7 (f)

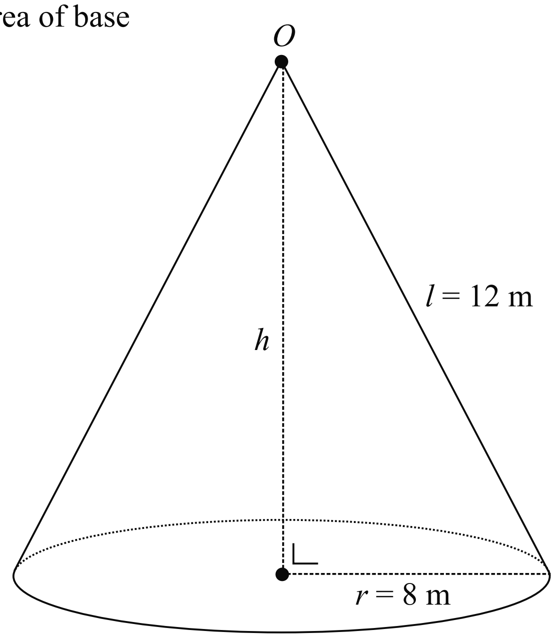
Area of base

$$= \pi(8)^2$$

$$= 64\pi = 201.1 \text{ m}^2$$

Total surface area = Curved surface area + Area of base
= 301.6 + 201.1 = 502.7 m^2

Question 7 (g)



$\boxed{\text{Volume of a cone } V = \tfrac{1}{3}\pi r^2 h}$

$$r^2 + h^2 = l^2$$

$$h^2 = l^2 - r^2$$

$$h = \sqrt{l^2 - r^2} = \sqrt{12^2 - 8^2} = 4\sqrt{5} \text{ m}$$

$$V = \tfrac{1}{3}\pi r^2 h = \tfrac{1}{3}\pi(8)^2(4\sqrt{5}) \approx 600 \text{ m}^3$$

Question 8 (35 marks)

Question 8 (a)

$n = 250$: Margin of error $= \pm\frac{1}{\sqrt{n}} = \pm\frac{1}{\sqrt{250}} = \pm 0.063 = \pm 6.3\%$

Question 8 (b)

$n = 500$: Margin of error $= \pm\frac{1}{\sqrt{n}} = \pm\frac{1}{\sqrt{500}} = \pm 0.045 = \pm 4.5\%$

As can be seen from the calculation, doubling the sample size to 500 does not cause the margin of error to be halved.

Question 8 (c)

Claim (H_o): 28% of second level students in Ireland are 180 cm or taller.

Proportion of those in the sample who are 178 cm or taller $= \frac{40}{250} = 0.16 = 16\%$

Test (using the sample proportion): 9.7% ← 16% → 22.3%
Since 28% is outside this interval, reject the null hypothesis where the teacher claims that 28% of second level students in Ireland are 180 cm or taller.

H_o ↓

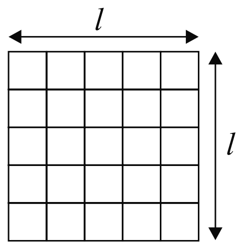
9.7% ← 16% → 22.3% 28%

Question 8 (d)

The population is the entire group under investigation.
The sample is a group chosen to test the claim.
Population: All second level students in Ireland.
Sample: 250 students from our school.

Question 9 (65 marks)

Question 9 (a)



Length l = 5 units = 5 × 20 m = 100 m
Area A = 5 × 5 = 25 units squared = 100 m × 100 m = 10 000 m^2

When converting units of length to metres, multiply by a factor of 20. (100 divided by 5)

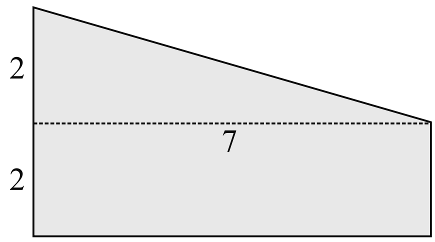
When converting units squared of area to m^2, multiply by a factor of 400. (10 000 divided by 25)

Question 9 (b)

Administration block

Area = Area of rectangle + Area of triangle

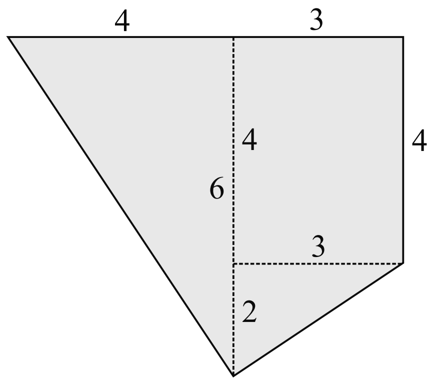
$$= 7\times 2+\tfrac{1}{2}\times 7\times 2$$
$$=14+7$$
$$= 21 \text{ units squared}$$
$$= 21\times 400 \text{ m}^2$$
$$= 8400 \text{ m}^2$$



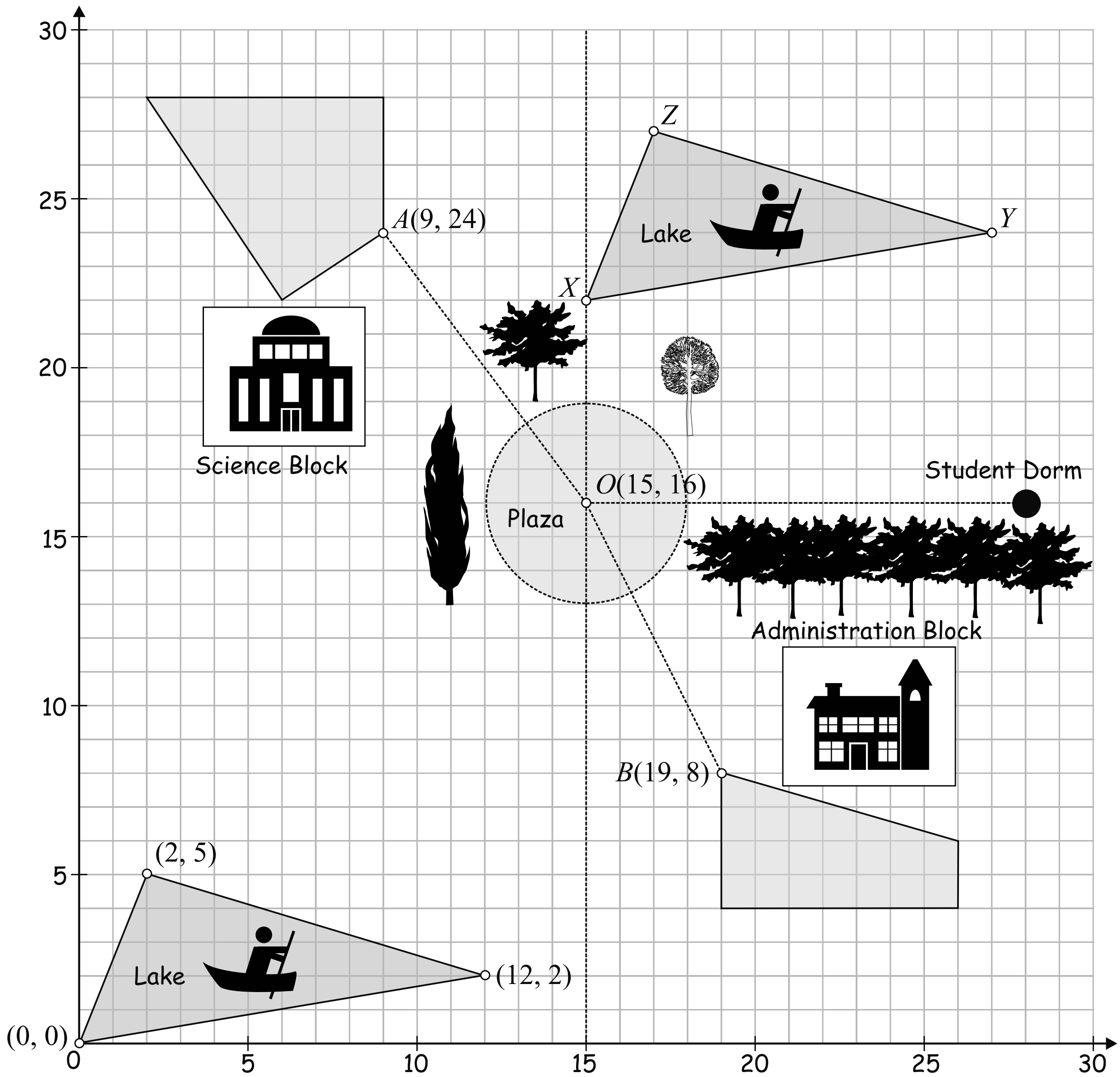
Science block

Area = Area of rectangle + Area of bigger triangle + Area of smaller triangle

$$= 4\times 3+\tfrac{1}{2}\times 4\times 6+\tfrac{1}{2}\times 3\times 2$$
$$=12+12+3$$
$$= 27 \text{ units squared}$$
$$= 27\times 400 \text{ m}^2$$
$$=10\,800 \text{ m}^2$$



Question 9 (c) (i)



Question 9 (c) (ii)

$(x_1, y_1) = (2, 5)$

$(x_2, y_2) = (12, 2)$

$A = \frac{1}{2}|x_1y_2 - x_2y_1| = \frac{1}{2}|(2)(2) - (12)(5)|$

$= \frac{1}{2}|4 - 60|$

$= \frac{1}{2}|-56|$

$= 28$ units squared

$= 28 \times 400 \text{ m}^2$

$= 11200 \text{ m}^2$

Question 9 (d) (i)

$A = (9, 24), O(15, 16), B(19, 8)$

Question 9 (d) (ii)

$$\boxed{|PQ| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}}$$

$A(9, 24), O(15, 16)$

$|AO| = \sqrt{(15-9)^2 + (16-24)^2} = 10$ units

$O(15, 16), B(19, 8)$

$|OB| = \sqrt{(19-15)^2 + (8-16)^2} = 8.94$ units

$|AO| + |OB| = 10 + 8.94 = 18.94$ units

$= 18.94 \times 20 \approx 379$ m

Sample 4 Paper 2

Question 9 (e) (i)

Radius of the circle r = 3 units = 60 m

Length of circumference: $l = 2\pi r = 2\pi(60) = 377$ m

Question 9 (e) (ii)

Distance walked $= 13 \times 20 + \frac{1}{4}(377) = 354.3$ m

Question 9 (e) (iii)

Distance $= 354.3$ m $= 0.3543$ km

Speed $= 2.5$ km/h

Time = ?

$$\boxed{\text{Speed} = \frac{\text{Distance}}{\text{Time}}}$$

$$\text{Time} = \frac{\text{Distance}}{\text{Speed}} = \frac{0.3543 \text{ km}}{2.5 \text{ km/h}} = 0.14172 \text{ hr} = 0.14172 \times 60 \text{s} \approx 9 \text{ mins}$$

Time of arrival: 10:09 am

Sample Paper 5: Paper 1

Question 1 (25 marks)

Question 1 (a)

$7-4x \geq 2x+1$

$-4x-2x \geq 1-7$

$-6x \geq -6$ [You reverse the inequality when you divide both sides by a negative number.]

$x \leq 1$

Answer: $x \leq 1$, $x \in \mathbb{R}$, All real numbers less than or equal to 1

Question 1 (b)

$\dfrac{1}{x^2+x-6}+1=\dfrac{3}{x+3}$ [Factorise the denominator.]

$\dfrac{1}{(x-2)(x+3)}+1=\dfrac{3}{x+3}$

$\dfrac{(x-2)(x+3)1}{(x-2)(x+3)}+(x-2)(x+3)1=\dfrac{(x-2)(x+3)3}{(x+3)}$ [Multiply each term by $(x-2)(x+3)$ to get rid of fractions.]

$1+(x-2)(x+3)=3(x-2)$

$1+x^2+3x-2x-6=3x-6$

$x^2-2x+1=0$

$(x-1)(x-1)=0$

$\therefore x=1$

Question 1 (c) (i)

$c: y=12-x-x^2$

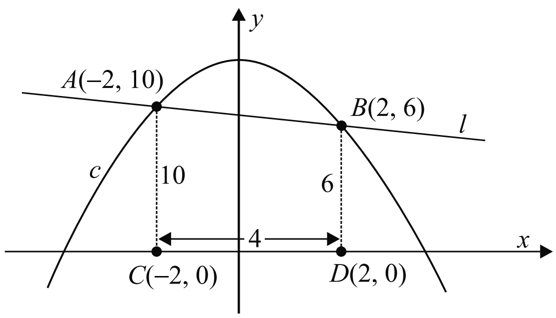
$l: x+y-8=0 \Rightarrow y=8-x$

$l \cap c \Rightarrow 8-x=12-x-x^2$

$\therefore x^2=4 \Rightarrow x=\pm\sqrt{4}=\pm 2$

$x=2: y=8-(2)=6 \Rightarrow B(2, 6)$

$x=-2: y=8-(-2)=10 \Rightarrow A(-2, 10)$

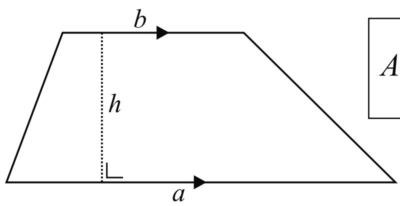


Coordinates of points: $A(-2, 10)$, $B(2, 6)$, $C(-2, 0)$, $D(2, 0)$

Question 1 (c) (ii)

$A=\left(\dfrac{10+6}{2}\right)4=8(4)=32$

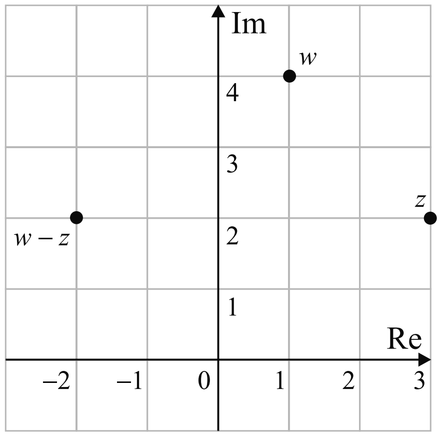
Area of a trapezium



$$A=\left(\frac{a+b}{2}\right)h$$

Question 2 (25 marks)

Question 2 (a)



(i) $z = 3 + 2i$

(ii) $w = 1 + 4i$

(iii) $w - z = 1 + 4i - 3 - 2i = -2 + 2i$

Question 2 (b)

$\frac{1}{5-12i}$ [Multiply above and below by the conjugate.]

$= \frac{1}{(5-12i)} \times \frac{(5+12i)}{(5+12i)}$ $\boxed{z = a + ib \Rightarrow \bar{z} = a - ib}$

$= \frac{5+12i}{25+60i-60i-144i^2}$

$= \frac{5+12i}{25+144} = \frac{5+12i}{169}$

$= \frac{5}{169} + \frac{12}{169}i$

Question 2 (c)

$|k + 6i| = 10$ $\boxed{z = a + ib \Rightarrow |z| = \sqrt{a^2 + b^2}}$

$\sqrt{k^2 + 6^2} = 10$

$k^2 + 36 = 100$

$k^2 = 64$

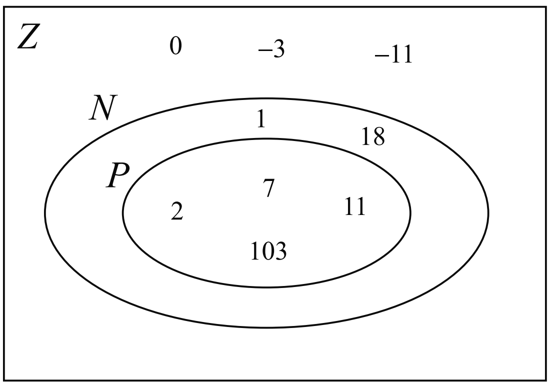
$k = \pm\sqrt{64} = \pm 8$

Question 3 (25 marks)

Question 3 (a)

Prime numbers: 2, 3, 5, 7, 11, 13, 17, 19, 23, 29

Question 3 (b)



Question 3 (c)

Common difference = 6

Possibility 1: **157**, 163, 169 [$169 = 13 \times 13$ so 169 is not a prime.]

Possibility 2: 151, **157**, 163

Possibility 3: 145, 151, **157** [145 is divisible by 5 so 145 is not a prime.]

Therefore, 151, 157 and 163 are the three consecutive prime numbers.

Question 4 (25 marks)

Question 4 (a) (i)

$a = 5,\ d = -2$

$T_n = 5 + (n-1)(-2)$ $\boxed{T_n = a + (n-1)d}$

$= 5 - 2n + 2$

$= 7 - 2n$

Question 4 (a) (ii)

$T_n = 7 - 2n$

$T_1 = 7 - 2(1) = 7 - 2 = 5$

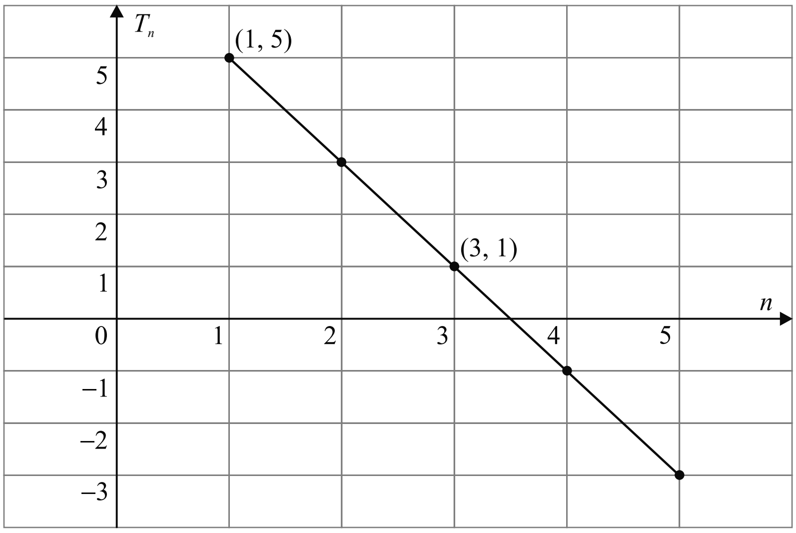
$T_2 = 7 - 2(2) = 7 - 4 = 3$

$T_3 = 7 - 2(3) = 7 - 6 = 1$

$T_4 = 7 - 2(4) = 7 - 8 = -1$

$T_5 = 7 - 2(5) = 7 - 10 = -3$

Question 4 (a) (iii)



Question 4 (a) (iv)

Points: (1, 5), (3, 1) $\boxed{m = \dfrac{y_2 - y_1}{x_2 - x_1}}$

$m = \dfrac{1-5}{3-1} = \dfrac{-4}{2} = -2$

The slope is equal to the common difference.

Question 4 (b)

$T_n = 4 + \frac{3}{2}n$

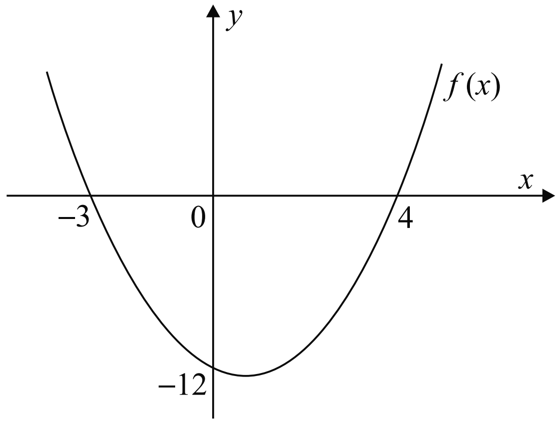
$T_1 = a = 4 + \frac{3}{2}(1) = \frac{11}{2}$

$d = \frac{3}{2}$ (slope of the line)

Sample 5 Paper 1

Question 5 (25 marks)

Question 5 (a)



$f(x)=k(x+a)(x-b)$

$f(x)=0 \Rightarrow k(x+a)(x-b)=0$

$(x+a)(x-b)=0$

$\therefore x=-a,\ b$

$x=-3,\ 4$

$\therefore a=3,\ b=4$

Question 5 (b)

$f(x)=k(x+3)(x-4)$

$f(0)=-12 \Rightarrow k(0+3)(0-4)=-12$

$-12k=-12$

$\therefore k=1$

Question 5 (c)

$f(x)=(x+3)(x-4)=x^2-x-12$

$f'(x)=2x-1$

$f'(x)=0 \Rightarrow 2x-1=0$

$\therefore x=\frac{1}{2}$

$f(\frac{1}{2})=(\frac{1}{2}+3)(\frac{1}{2}-4)=(\frac{7}{2})(-\frac{7}{2})=-\frac{49}{4}$

Local minimum: $(\frac{1}{2},\ -\frac{49}{4})$

Question 6 (25 marks)

Question 6 (a) (i)

$f(x)=x+3$

$g(x)=x^2+1$

$f(g(x))$

$=f(x^2+1)$

$=(x^2+1)+3$

$=x^2+4$

Question 6 (a) (ii)

$f(x)=x+3$

$g(x)=x^2+1$

$g(f(x))$

$=g(x+3)$

$=(x+3)^2+1$

$=x^2+6x+9+1$

$=x^2+6x+10$

Question 6 (a) (iii)

$f(g(x))=g(f(x))$

$x^2+4=x^2+6x+10$

$4-10=6x$

$-6=6x$

$\therefore x=-1$

$f(g(-1))=f(2)=5$

$g(f(-1))=g(2)=5$

Question 6 (b)

x	0.9	0.99	0.999	0.9999
$2x+1$	2.8	2.98	2.998	2.9998

$\lim_{x\to 1}(2x+1)=3$

Question 6 (c) (i)

$f(x)=x^2$

$f(x+h)=(x+h)^2=x^2+2hx+h^2$

Question 6 (c) (ii)

$f(x+h)-f(x)=x^2+2hx+h^2-x^2$

$=2hx+h^2$

$=h(2x+h)$

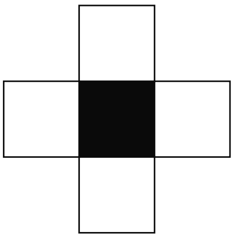
Question 6 (c) (iii)

$$\lim_{h\to 0}\left[\frac{f(x+h)-f(x)}{h}\right]=\lim_{h\to 0}\left[\frac{h(2x+h)}{h}\right]=\lim_{h\to 0}[2x+h]=2x$$

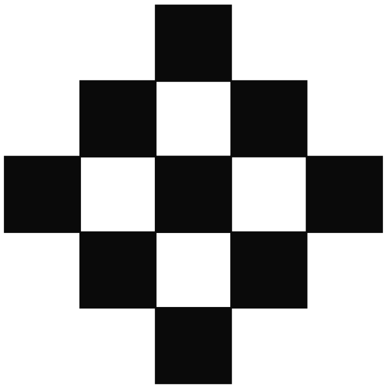
Question 7 (50 marks)

Question 7 (a)

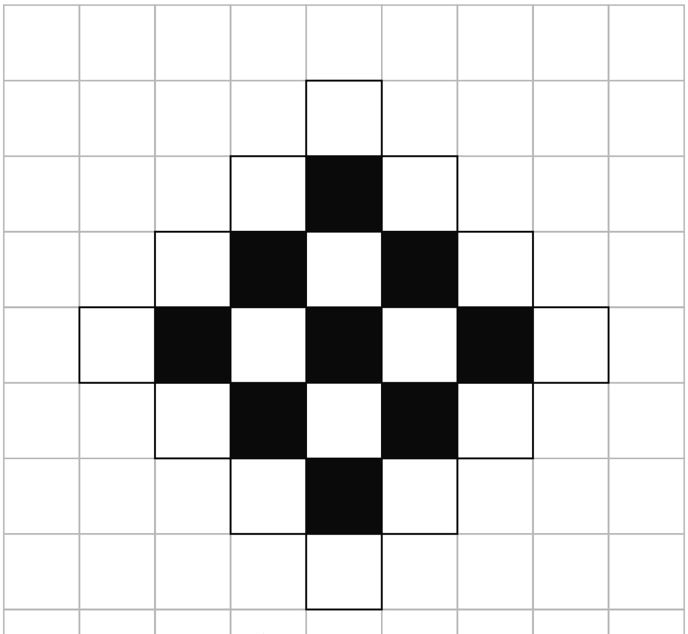
1st. arrangement



2nd. arrangement



3rd. arrangement



4th. arrangement

Question 7 (b)

	1st. arrangement	2nd. arrangement	3rd. arrangement	4th. arrangement
Row 1	1	1	1	1
Row 2		3	3	3
Row 3		1	5	5
Row 4			3	7
Row 5			1	5
Row 6				3
Row 7				1
Total	1	5	13	25

Question 7 (c)

Fifth arrangement: 1 + 3 + 5 + 7 + 9 + 7 + 5 + 3 + 1 = 41
Sixth arrangement: 1 + 3 + 5 + 7 + 9 + 11 + 9 + 7 + 5 + 3 + 1 = 61

Question 7 (d)

$T = 2n^2 + an + b$

1st. arrangement $(n = 1): T = 1$

$\therefore 1 = 2(1)^2 + a(1) + b$

$1 = 2 + a + b$

$a + b = -1......(\mathbf{1})$

$T = 2n^2 + an + b$

2nd. arrangement $(n = 2): T = 5$

$\therefore 5 = 2(2)^2 + a(2) + b$

$5 = 8 + 2a + b$

$2a + b = -3......(\mathbf{2})$

Solve equations (**1**) and (**2**) simultaneously to find *a* and *b*.

$a + b \quad = -1......(\mathbf{1})$

$2a + b = -3......(\mathbf{2})(\times -1)$

$$\begin{array}{l} a + b = -1 \\ \underline{-2a - b = 3} \\ -a \qquad = 2 \Rightarrow a = -2 \end{array}$$

Substitute into equation (**1**): $-2 + b = -1 \Rightarrow b = 1$

Answer: $a = -2$, $b = 1$

Question 7 (e)

$T = 2n^2 - 2n + 1$

10th. arrangement $(n = 10): T = ?$

$T = 2(10)^2 - 2(10) + 1$

$= 2(100) - 20 + 1$

$= 200 - 20 + 1$

$= 181$

Question 7 (f)

$T = 2n^2 - 2n + 1$

$T = 265,\ n = ?$

$265 = 2n^2 - 2n + 1$

$0 = 2n^2 - 2n - 264$

$0 = n^2 - n - 132$

$0 = (n - 12)(n + 11)$

$\therefore n = 12$

QUESTION 8 (50 MARKS)

Question 8 (a)

$2x + 2y = 160$

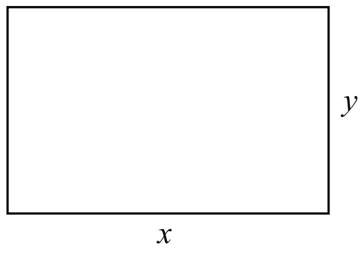
$x + y = 80$

$\therefore y = 80 - x$

Question 8 (b)

$A = xy = x(80 - x)$

$= 80x - x^2$



Question 8 (c)

$A = 80x - x^2$

$x = 0 : A = 80(0) - (0)^2 = 0$

$x = 10 : A = 80(10) - (10)^2 = 800 - 100 = 700$

$x = 20 : A = 80(20) - (20)^2 = 1600 - 400 = 1200$

$x = 30 : A = 80(30) - (30)^2 = 2400 - 900 = 1500$

$x = 40 : A = 80(40) - (40)^2 = 3200 - 1600 = 1600$

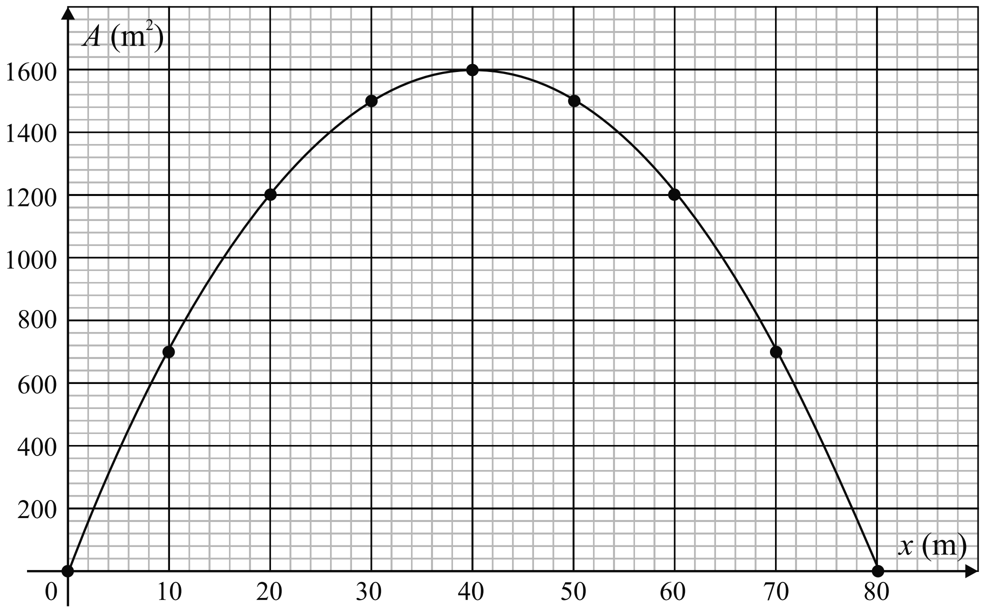
$x = 50 : A = 80(50) - (50)^2 = 4000 - 2500 = 1500$

$x = 60 : A = 80(60) - (60)^2 = 4800 - 3600 = 1200$

$x = 70 : A = 80(70) - (70)^2 = 5600 - 4900 = 700$

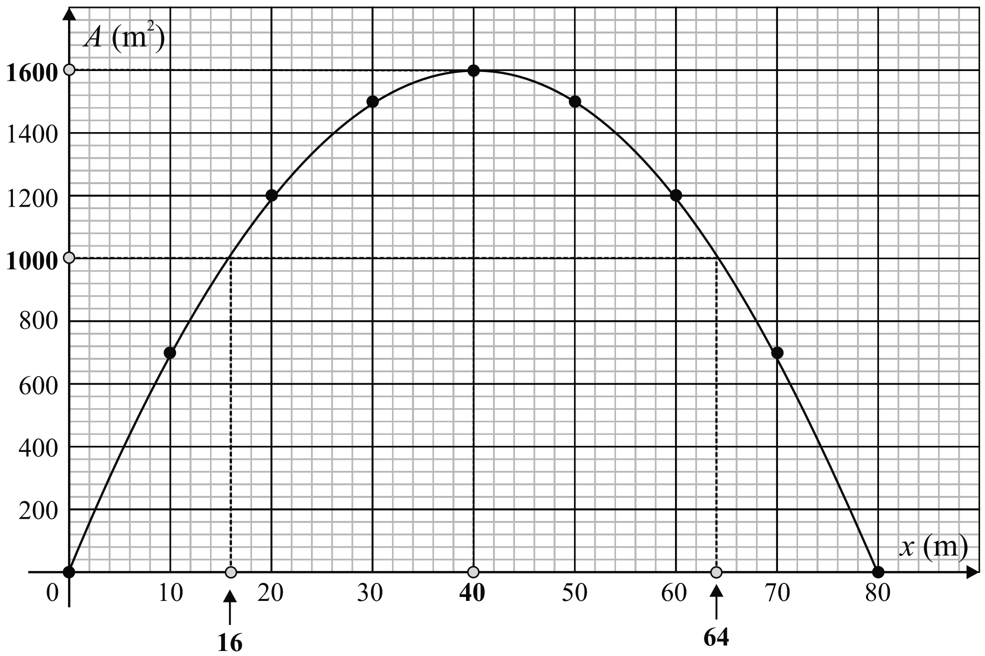
$x = 80 : A = 80(80) - (80)^2 = 6400 - 6400 = 0$

x (m)	0	10	20	30	40	50	60	70	80
A (m^2)	0	700	1200	1500	1600	1500	1200	700	0



Sample 5 Paper 1

Question 8 (d) (i)



Maximum area = 1600 m²
Go to this number on the vertical axis and read off the corresponding x value.
$\therefore x = 40$ m

$y = (80 - x) = (80 - 40) = 40$ m

Dimensions: 40 m by 40 m

Question 8 (d) (ii)
Go to 1000 m² on the vertical axis and read off the corresponding x values.
$\therefore x = 16$ m, 64 m

$x = 16$ m: $y = (80 - x) = (80 - 16) = 64$ m

$x = 64$ m: $y = (80 - x) = (80 - 64) = 16$ m

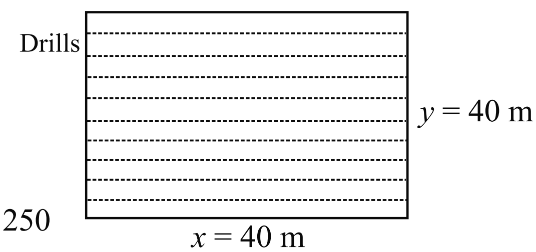
Dimensions: 16 m by 64 m

Question 8 (e)

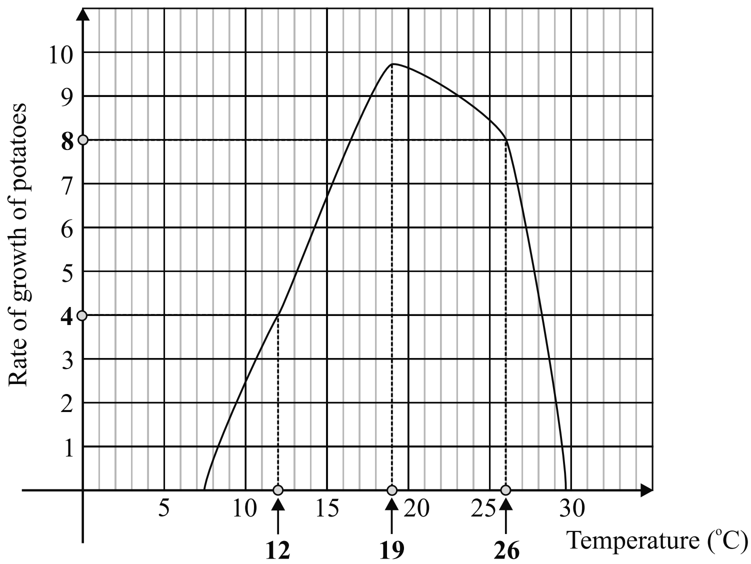
Maximum number of drills $= \dfrac{40 \text{ m}}{0.32 \text{ m}} = 125$

Number of potato plants per drill $= \dfrac{40 \text{ m}}{0.16 \text{ m}} = 250$

Maximum number of potato plants = 250 × 125 = 31 250



Question 8 (f) (i)



At 12°C the growth rate is 4.
At 26°C the growth rate is 8.
There is twice the growth rate between the maximum and minimum temperatures.

Question 8 (f) (ii)

19°C gives the maximum growth rate.

Sample 5 Paper 1

Question 9 (50 marks)

Question 9 (a) (i)

$y = \frac{1}{6}x(12-x)$

$y = 0 \Rightarrow \frac{1}{6}x(12-x) = 0$

$x(12-x) = 0$

$\therefore x = 0 \text{ m}, 12 \text{ m}$

Question 9 (a) (ii)

$|OA| = 12 - 0 = 12 \text{ m}$

Question 9 (a) (iii)

$y = \frac{1}{6}x(12-x)$

$\quad = 2x - \frac{1}{6}x^2$

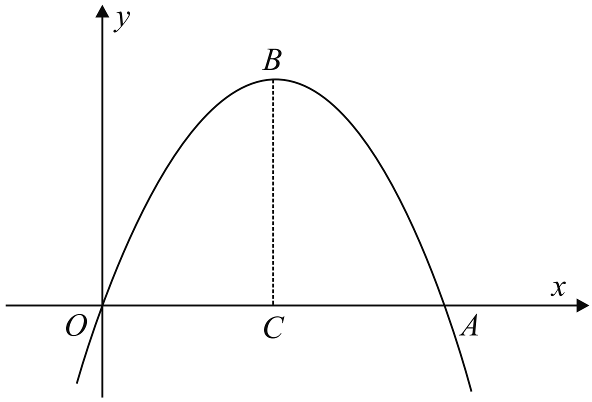
$\frac{dy}{dx} = 2 - \frac{1}{3}x$

$\frac{dy}{dx} = 0 \Rightarrow 2 - \frac{1}{3}x = 0$

$2 = \frac{1}{3}x$

$\therefore x = 6 \text{ m}$

$x = 6 : y = \frac{1}{6}(6)(12-6) = 6 \text{ m}$



Question 9 (b) (i)

$d = 6 - 1.9 = 4.1$ m

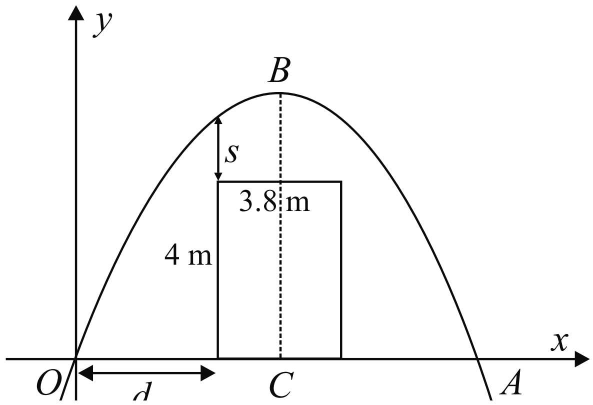
$x = 4.1 : y = \frac{1}{6}(4.1)(12-4.1) = 5.4 \text{ m}$

$s = 5.4 - 4 = 1.4 \text{ m}$

Question 9 (b) (ii)

$d = 6 - 3.8 = 2.2$ m

$x = 2.2 : y = \frac{1}{6}(2.2)(12-2.2) = 3.6 \text{ m} < 4 \text{ m}$



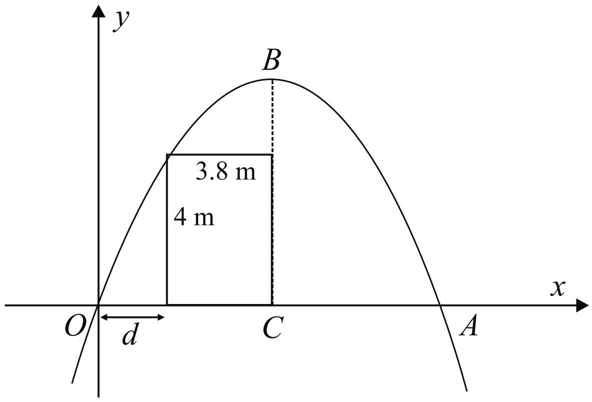
Question 9 (b) (iii)

$y = 4 : 4 = \frac{1}{6}x(12-x)$

$24 = 12x - x^2$

$x^2 - 12x + 24 = 0$

$$x = \frac{12 \pm \sqrt{(-12)^2 - 4(1)(24)}}{2} = 2.54 \text{ m}$$



Sample Paper 5: Paper 2

Question 1 (25 marks)

Question 1 (a)

$y=\sqrt{16-x^2}$

$x=0: y=\sqrt{16-0^2}=\sqrt{16}=4$

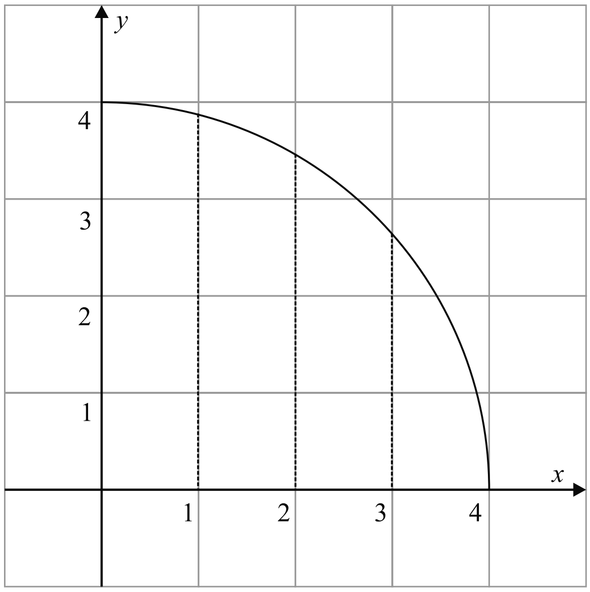
$x=1: y=\sqrt{16-1^2}=\sqrt{15}=3.87$

$x=2: y=\sqrt{16-2^2}=\sqrt{12}=3.46$

$x=3: y=\sqrt{16-3^2}=\sqrt{7}=2.65$

$x=4: y=\sqrt{16-4^2}=\sqrt{0}=0$

x	y
0	4
1	3.87
2	3.46
3	2.65
4	0



Question 1 (b)

$$\boxed{A \approx \frac{h}{2}[y_1+y_n+2(y_2+y_3+y_4+....+y_{n-1})]}$$

$h=1$

$A \approx \frac{1}{2}[4+0+2(3.87+3.46+2.65)]=11.98$ square units

Question 1 (c)

$r=4$

$\boxed{\text{Area of a circle: } A=\pi r^2}$

$A=\frac{1}{4}\times\pi(4)^2=4\pi$

Question 1 (d)

$4\pi=11.98$

$\therefore \pi=\frac{11.98}{4}\approx 3.00$

Question 2 (25 marks)

Question 2 (a)

	13-year-olds	5-year-olds	16-year-olds
Size N	150	2250	70
Mean μ	17	16.5	23
Standard Deviation σ	1.8	2	4.2

The biggest sample is 5-year-olds and the number sampled was 2,250.

Question 2 (b)

The data in the set of 16-year-olds is more spread out than the data in the other sets.

Question 2 (c)

$\mu = 16.5,\ N = 2250,\ \sum x = ?$

$$16.5 = \frac{\sum x}{2250} \Rightarrow \sum x = 16.5 \times 2250 = 37125$$

$$\boxed{\mu = \frac{\sum x}{n}}$$

Question 2 (d)

2250 5-year-olds: Sum of all results = 37,125

150 13-year-olds: Sum of all results $\sum x$ = 17 × 150 = 2550

70 16-year-olds: Sum of all results $\sum x$ = 23 × 70 = 1610

Total number of students = 2250 + 150 + 70 = 2,470

Sum of all results of all students = 37 125 + 2550 + 1610 = 41 285

Mean of all three sets $\mu = \dfrac{41285}{2470} = 16.7$

Question 2 (e)

> **Empirical Rule**
> 68% of all data points fall within 1 standard deviation of the mean.
> 95% of all data points fall within 2 standard deviations of the mean.
> 99.7% of all data points fall within 3 standard deviations of the mean.

$\mu = 23,\ \sigma = 4.2$

$\mu + 2\sigma = 23 + 2(4.2) = 31.4$

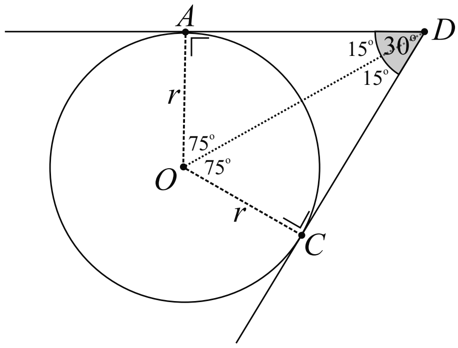
$\mu - 2\sigma = 23 - 2(4.2) = 14.6$

95% of all the data points in the 16-year-old set lie between 14.6 and 31.4.

Question 2 (f)

If all three sets are combined, the median results are most likely to be a value in the set of 5-year-olds.

Question 3 (25 marks)



Question 3 (a)

Draw radii from the centre of the circle O, to points of contact A and C. OA and OC are perpendicular to tangents AD and CD respectively.

Triangles ADO and DOC are congruent (RHS) because:

$|AO| = |OC| = r$

$|OD| = |OD|$ [Common side]

$|\angle OAD| = |\angle OCD|$ [Right angle]

0Therefore, the minor angle AOC at the centre of the circle is 150°.

Minor arc $|AC| = 2\pi(8)\left(\frac{150^\circ}{360^\circ}\right) = 20.9$ cm $\boxed{l = 2\pi r\left(\frac{\theta}{360^\circ}\right)}$

Question 3 (b) (i)

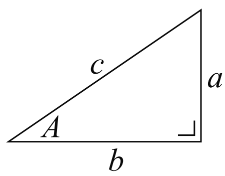
$|BC|^2 + 5^2 = 13^2$

$|BC|^2 + 25 = 169$

$|BC|^2 = 169 - 25 = 144$

$|BC| = \sqrt{144} = 12$

Pythagoras



$\boxed{\sin A = \frac{a}{c}}$

$\boxed{c^2 = a^2 + b^2}$

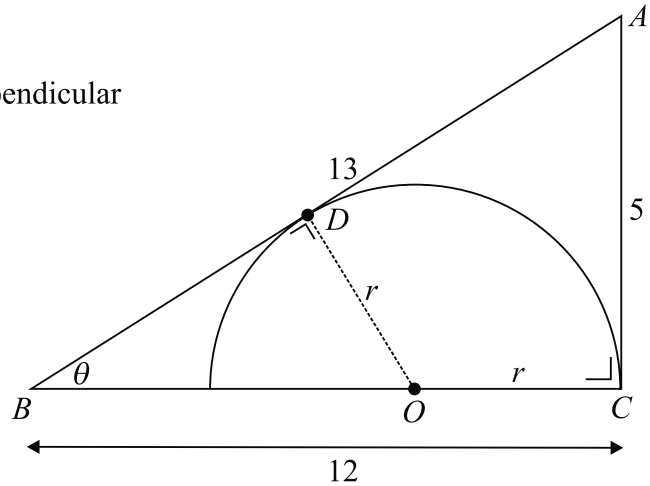
Question 3 (b) (ii)

Consider the right-angled triangle ABC:

$$\sin\theta = \frac{|AC|}{|AB|} = \frac{5}{13}$$

Consider the right-angled triangle BDO:

$|\angle BDO| = 90^\circ$ because the radius OD is perpendicular to tangent AB at the point of contact D.



$$\sin\theta = \frac{|DO|}{|BO|} = \frac{r}{12 - r} = \frac{5}{13}$$

$\therefore 13r = 5(12 - r)$

$13r = 60 - 5r$

$18r = 60$

$\therefore r = \frac{60}{18} = \frac{10}{3}$

Question 4 (25 marks)

Question 4 (a)

$l: 2x - 3y + k = 0$

$(1, 4) \in l \Rightarrow 2(1) - 3(4) + k = 0$

$2 - 12 + k = 0$

$-10 + k = 0$

$\therefore k = 10$

Question 4 (b)

$l: 2x - 3y + 10 = 0$

Slope of l: $m_1 = -\frac{2}{-3} = \frac{2}{3}$

Line: $ax + by + c = 0$

Slope $m = -\frac{a}{b}$

Slope of q: $m_2 = \frac{2}{3}$ [Parallel lines: $m_1 = m_2$]

Equation of q: Point $(5, 2) = (x_1, y_1)$, $m = \frac{2}{3}$ $\boxed{y - y_1 = m(x - x_1)}$

$y - 2 = \frac{2}{3}(x - 5)$

$3(y - 2) = 2(x - 5)$

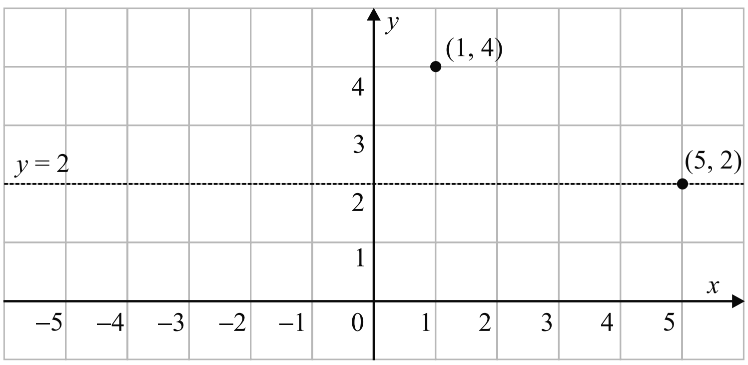
$3y - 6 = 2x - 10$

$0 = 2x - 3y - 4$

Question 4 (c)

Draw the line $y = 2$. [This is a horizontal line passing through the y-axis at 2.]
The x-axis is highlighted. The equation of the x-axis is $y = 0$.
These two lines are parallel and form opposite sides of the parallelogram.

The other two lines pass through (1, 4) and (5, 2) and are parallel to each other. You can see from the diagram that (5, 2) is one of the vertices. To calculate another vertex, find where q cuts the x-axis by letting $y = 0$.



q cuts x-axis:

$q: 2x - 3y - 4 = 0$

$y = 0: 2x - 3(0) - 4 = 0 \Rightarrow 2x = 4$

$\therefore x = 2$

Second vertex: (2, 0)

Find where l cuts the x-axis and the line $y = 2$:

$l: 2x - 3y + 10 = 0$

$y = 0$: $2x - 3(0) + 10 = 0 \Rightarrow 2x = -10$

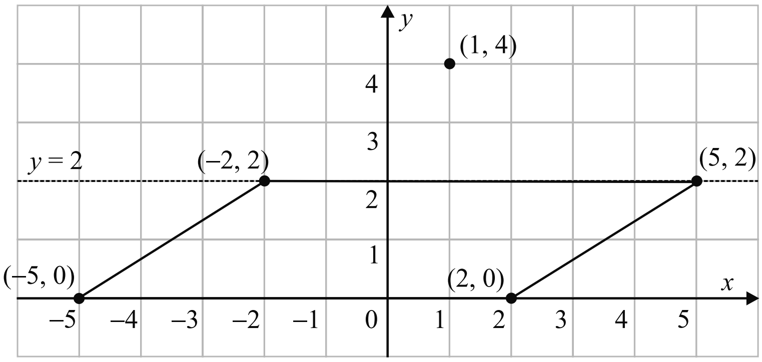
$\therefore x = -5$

Third vertex: $(-5, 0)$

$y = 2$: $2x - 3(2) + 10 = 0 \Rightarrow 2x = -4$

$\therefore x = -2$

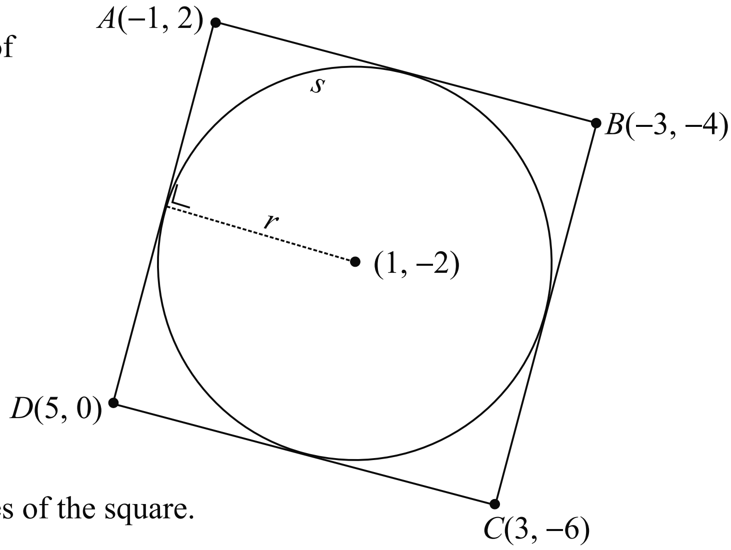
Fourth vertex: $(-2, 2)$



Vertices: $(-5, 0)$, $(2, 0)$, $(5, 2)$, $(-2, 2)$

Question 5 (25 marks)

Question 5 (a)



The centre of the circle is the midpoint of the diagonals $[AC]$ or $[BD]$.

$$\boxed{\text{Midpoint} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)}$$

$A(-1, 2)$, $C(3, -6)$

$$\text{Midpoint} = \left(\frac{-1+3}{2}, \frac{2-6}{2}\right) = (1, -2)$$

Question 5 (b)

The radius r is half the length of the sides of the square.

$A(-1, 2)$, $B(-3, -4)$

$$\boxed{|PQ| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}}$$

$$r = \tfrac{1}{2}|AB| = \tfrac{1}{2}\sqrt{(-3-(-1))^2 + (-4-2)^2}$$
$$= \tfrac{1}{2}\sqrt{(-2)^2 + (-6)^2}$$
$$= \tfrac{1}{2}\sqrt{4+36}$$
$$= \tfrac{1}{2}\sqrt{40} = \tfrac{1}{2}\sqrt{4\times 10}$$
$$= \sqrt{10}$$

Equation of s: Centre$(1, -2)$, $r = \sqrt{10}$ $\quad \boxed{(x-h)^2 + (y-k)^2 = r^2}$

$$(x-1)^2 + (y-(-2))^2 = (\sqrt{10})^2$$
$$(x-1)^2 + (y+2)^2 = 10$$

Question 5 (c)

$(x+4)^2 + y^2 = 10 \Rightarrow$ Centre$(-4, 0)$

$\therefore (1, -2) \to (-4, 0)$ [Subtract 5, Add 2]

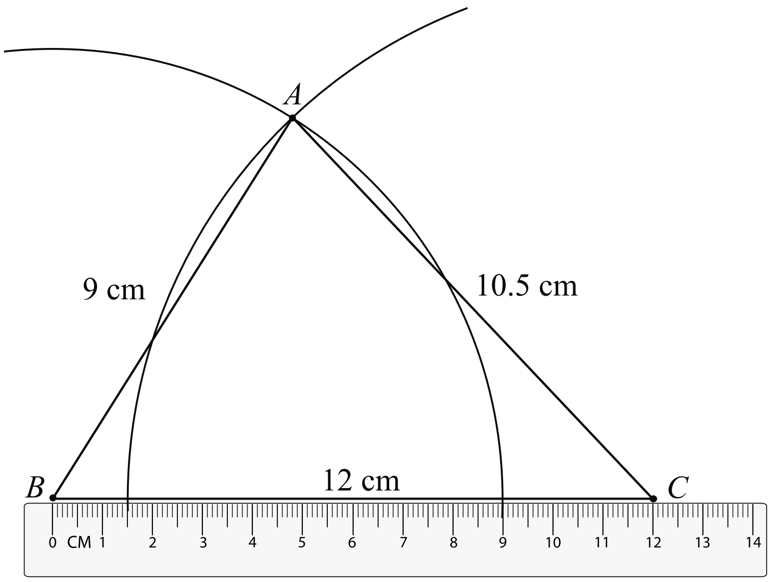
$\quad (p, q) \to (6, 5)$

$\therefore p - 5 = 6 \Rightarrow p = 11$

$\therefore q + 2 = 5 \Rightarrow q = 3$

Question 6 (25 marks)

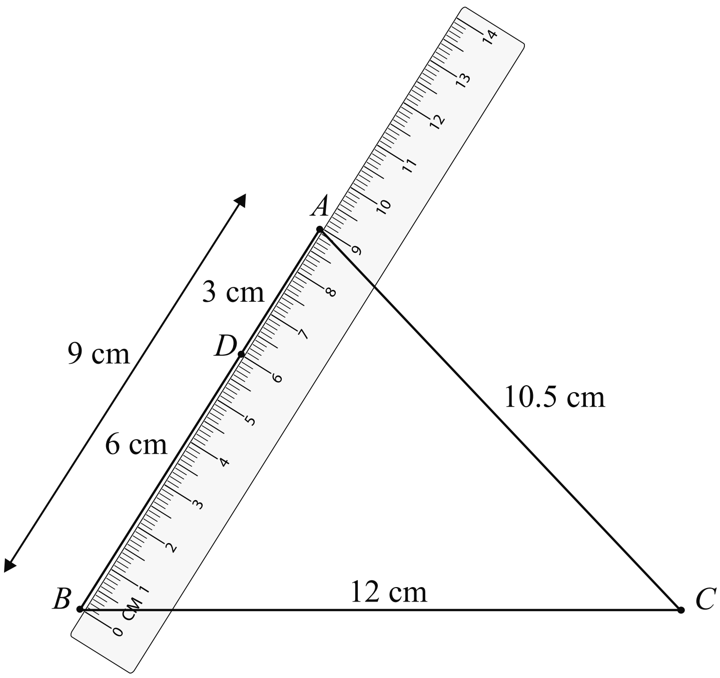
Question 6 (a)



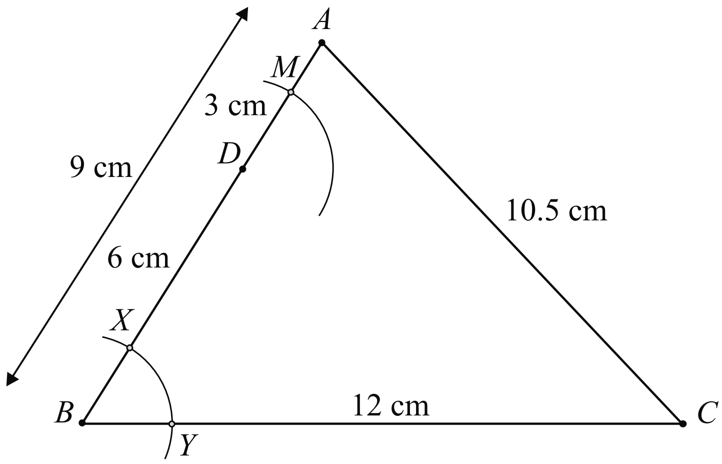
Rule out a line [*BC*] of length 12 cm.
Using a compass, draw an arc of length 9 cm with *B* as centre.
Using a compass, draw an arc of length 10.5 cm with *C* as centre.
A is the point of intersection of these arcs.
Join *B* to *A* and *C* to *A* to complete the construction.

Question 6 (b)

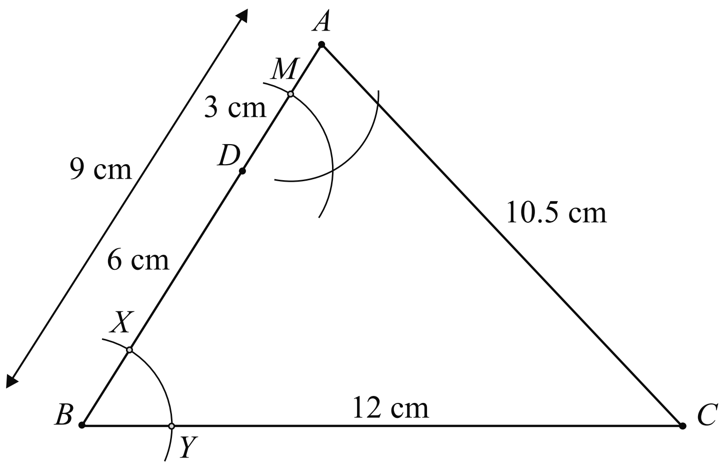
Step 1: Measure out a line segment [*BD*] of length 6 cm.



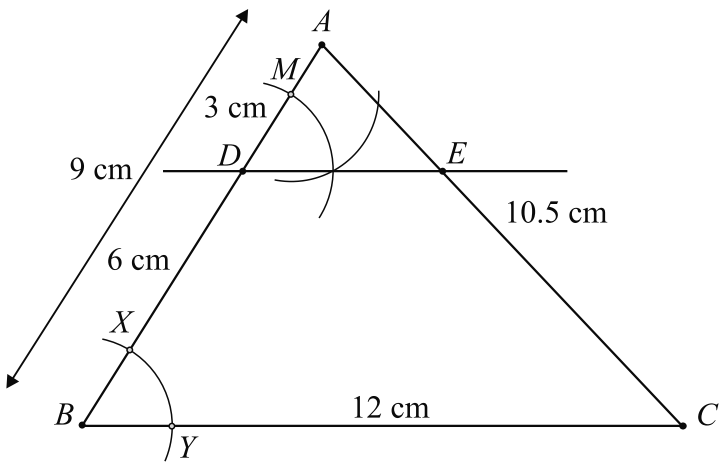
Step 2: Using a compass, draw an arc XY with B as centre.
Without changing the radius of the compass, draw another arc with D as centre.



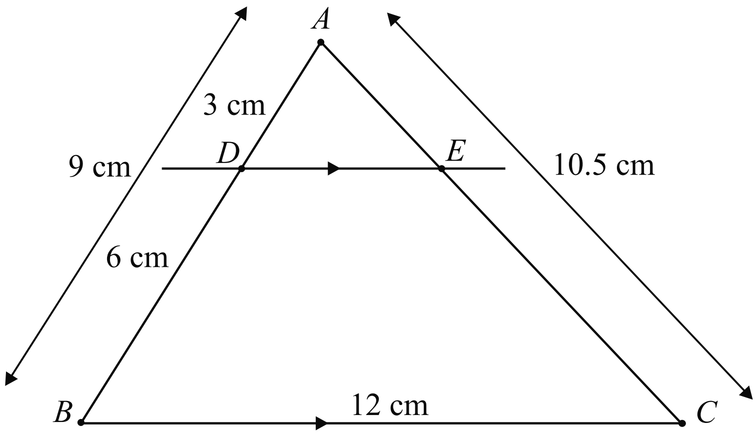
Step 3: Place the point of the compass on X and stretch it to Y, so that it has a radius of length $|XY|$. Without changing the radius of the compass, draw an arc with centre M.



Step 4: Draw a line through D and through the intersection of the arcs.
The line DE is parallel to BC.



Theorem 12. Let ΔABC be a triangle. If a line l is parallel to BC and cuts $[AB]$ in the ratio $s:t$, then it also cuts $[AC]$ in the same ratio.



$$\frac{|AB|}{|AD|}=\frac{|AC|}{|AE|}\Rightarrow \frac{9}{3}=\frac{10.5}{|AE|}$$

$$\therefore |AE|=\frac{3\times 10.5}{9}=3.5 \text{ cm}$$

$$|AE|+|EC|=10.5$$

$$\therefore |EC|=10.5-3.5=7 \text{ cm}$$

Question 6 (c)

Theorem 13. If two triangles ΔABC and $\Delta A'B'C'$ are similar, then their sides are proportional, in order:

$$\frac{|AB|}{|A'B'|}=\frac{|BC|}{|B'C'|}=\frac{|CA|}{|C'A'|}.$$

Triangles ADE and ABC are equiangular because:

$|\angle DAE|=|\angle BAC|$ [Common angle]

$|\angle ADE|=|\angle ABC|$ [Corresponding angles]

$|\angle AED|=|\angle ACB|$ [Corresponding angles]

$$\therefore \frac{|AD|}{|AB|}=\frac{|DE|}{|BC|}\Rightarrow \frac{3}{9}=\frac{|DE|}{12}$$

$$|DE|=\frac{3\times 12}{9}=4 \text{ cm}$$

Sample 5 Paper 2

Question 7 (70 marks)

Question 7 (a)

(i) Total number of deaths = 3324

(ii) Total number injured = 89 026

(iii) Mean number of deaths per year:

$$\mu = \frac{3324}{10} = 332.4$$

$$\mu = \frac{\sum x}{n}$$

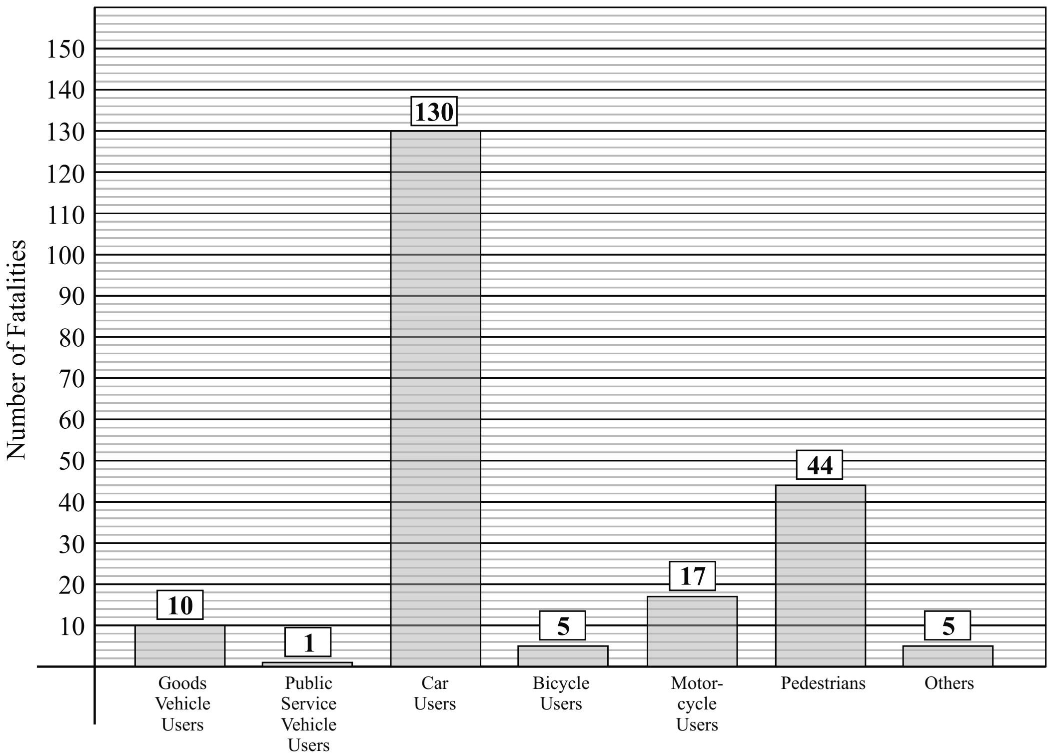
Year	Killed	Injured
2001	411	10 222
2002	376	9206
2003	335	8262
2004	374	7867
2005	396	9318
2006	365	8575
2007	338	7806
2008	279	9758
2009	238	9742
2010	212	8270
	3324	89 026

Question 7 (b)

(i) $\% \text{ decrease} = \frac{640 - 212}{640} \times 100\% = 66.875\%$

(ii) Reason 1: Better roads
Reason 2: Less drink-driving
Reason 3: Safer cars

Question 7 (c) (i)



Question 7 (c) (ii)

Number of car fatalities = 130
Total number of fatalities = 212

$$\% \text{ of car fatalities} = \frac{130}{212} \times 100\% = 61.3\%$$

Question 7 (c) (iii)

Fatalities highest among car users, number of pedestrian casualties is of concern, should there be compulsory wearing of bicycle helmets?

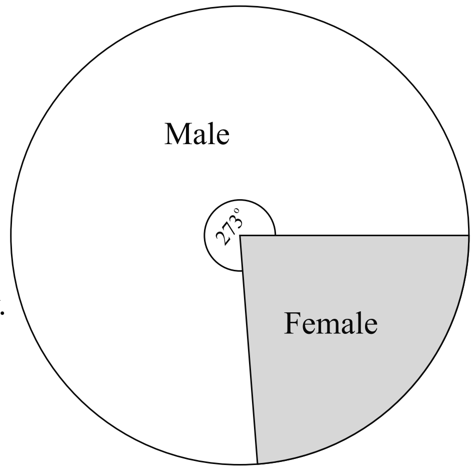
Question 7 (d) (i)

$$P(\text{Male Fatality}) = \frac{273}{360} = 0.7583$$

$$P(\text{Female Fatality}) = 1 - 0.76 = 0.2417$$

$$\frac{P(\text{Male Fatality})}{P(\text{Female Fatality})} = \frac{0.7583}{0.2417} = 3.1$$

A male fatality is 3.1 times more likely than a female fatality.



Question 7 (d) (ii)

Yes, lower level of women involved in car accidents should be reflected in insurance premiums.

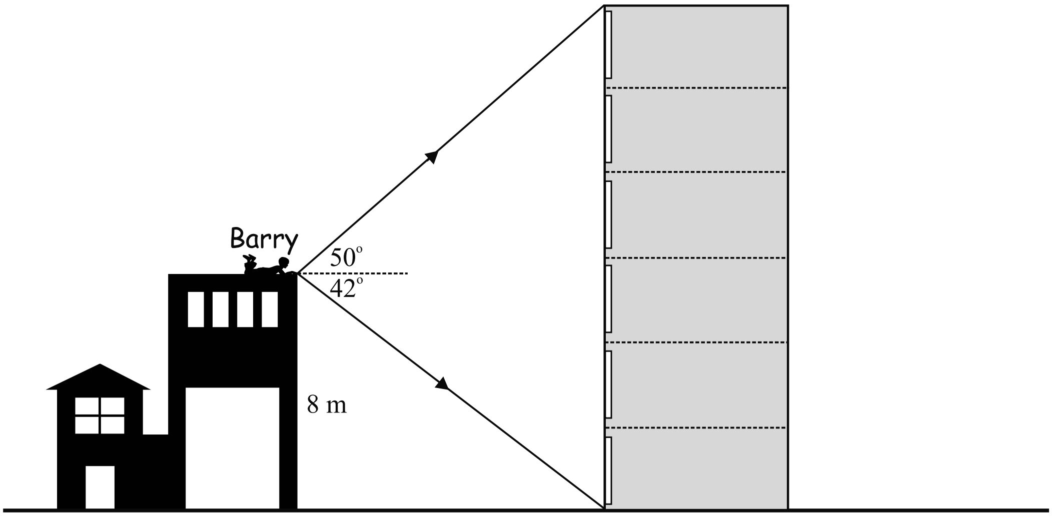
Question 7 (e) (i)

Type	Number of collisions	Cost per collision (euro)	Total cost (euro)
Fatal	185	**2 583 311**	477 912 535
Serious	409	345 121	**141 154 489**
Minor	5186	33 991	**176 277 326**
Material damage	**21 305**	2719	57 928 295
Total	**27 085**		**853 272 645**

Question 7 (e) (ii)

$$\text{Average cost per collision } = \frac{853\,272\,645}{27\,085} = €31503.51$$

Question 8 (35 marks)

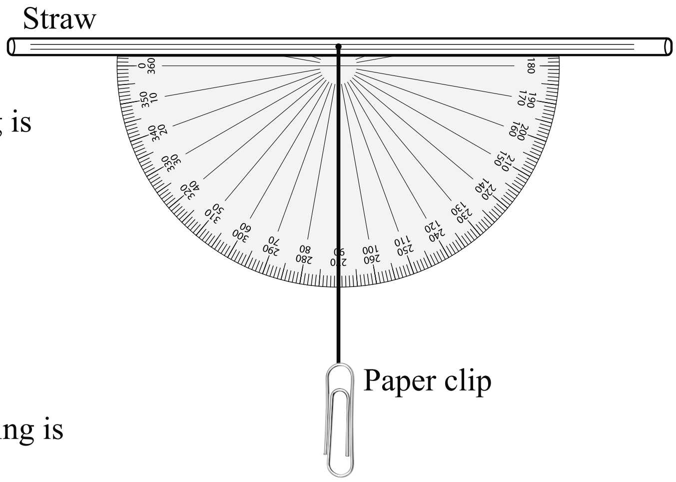


Question 8 (a)

Hold the clinometer so that the string is vertical.

Angle of elevation:

- Tilt the clinometer looking through the drinking straw so that the highest point on the top of the building is visible.
- Read the angle of elevation of this highest point to the nearest degree.



Angle of depression:

- Tilt the clinometer looking through the drinking straw so that the lowest point at the bottom of the building is visible.
- Read the angle of depression of this lowest point to the nearest degree.

Question 8 (b) (i)

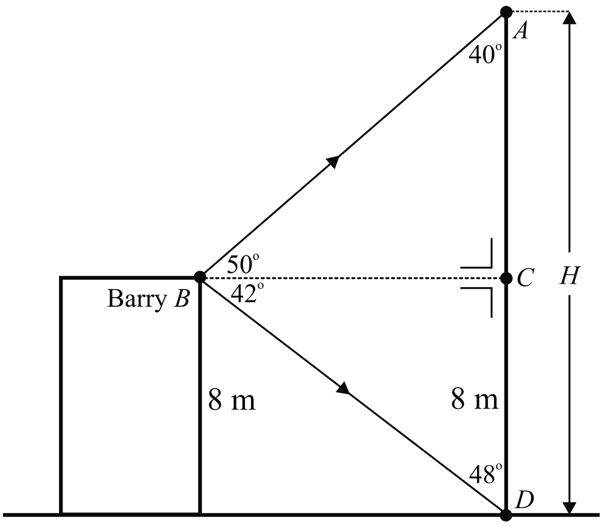
Angle of elevation: The angle the line of sight to an object makes with the horizontal when the object is above the level of the observer.

Angle of depression: The angle the line of sight to an object makes with the horizontal when the object is below the level of the observer.

Question 8 (b) (ii)

An angle of elevation of 0° means the height of the building opposite would be the same as the height of his building, i.e. 8 m.

Question 8 (c) (i)



Question 8 (c) (ii)

$$\boxed{\tan A = \frac{\text{Opposite}}{\text{Adjacent}}}$$

Consider triangle *BCD*:

$$\tan 42^\circ = \frac{8}{|BC|} \Rightarrow |BC| = \frac{8}{\tan 42^\circ} = 8.9 \text{ m}$$

Question 8 (c) (iii)

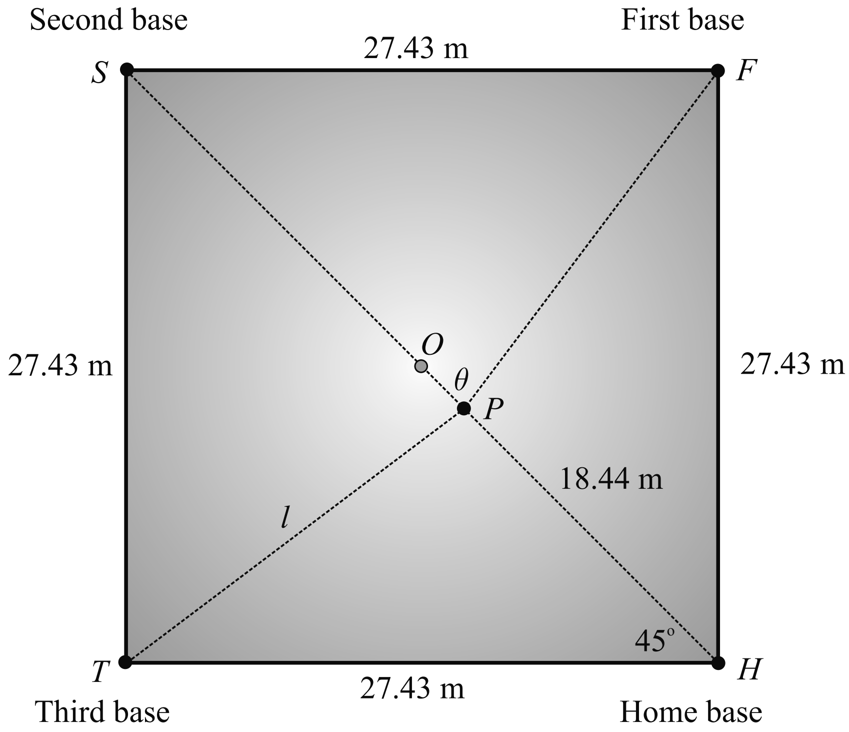
Consider triangle *BCA*:

$$\tan 50^\circ = \frac{|AC|}{8.9} \Rightarrow |AC| = 8.9 \tan 50^\circ = 10.6 \text{ m}$$

$$\therefore H = |DC| + |AC| = 8 + 10.6 = 18.6 \text{ m}$$

Sample 5 Paper 2

QUESTION 9 (45 MARKS)



Question 9 (a)

Length of diagonal $|HS| = \sqrt{27.43^2 + 27.43^2} = 38.79$ m

$|PS| = 38.79 - 18.44 = 20.35$ m

Question 9 (b)

Let O be the point where the diagonals of the square bisect each other.

$$|PO| = |HO| - |PH| = \frac{38.79}{2} - 18.44 = 0.96 \text{ m}$$

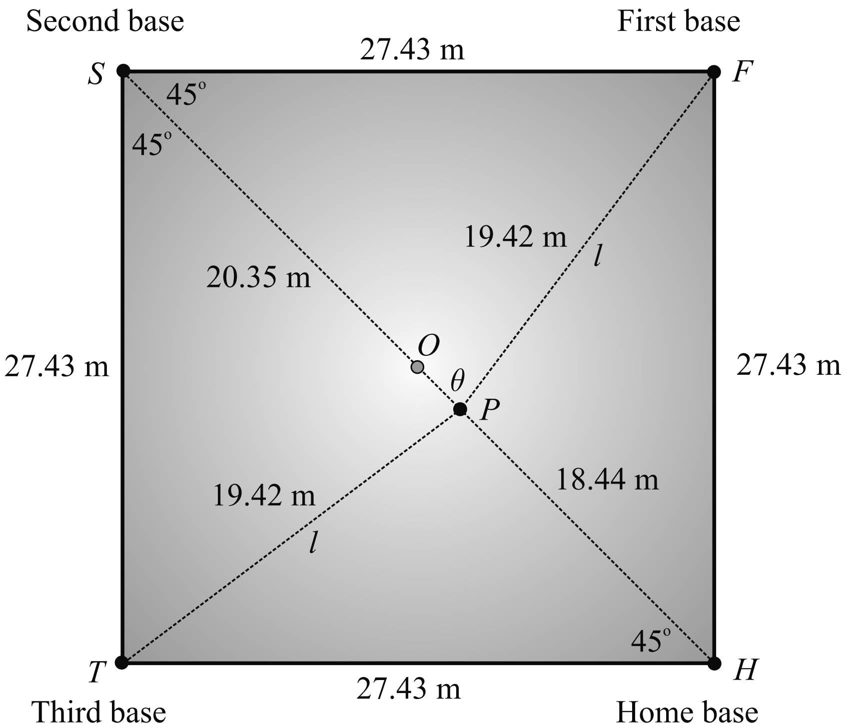
Question 9 (c)

$$\boxed{a^2 = b^2 + c^2 - 2bc\cos A}$$

$$l^2 = 27.43^2 + 18.44^2 - 2(27.43)(18.44)\cos 45^\circ$$

$$|PT| = l = \sqrt{27.43^2 + 18.44^2 - 2(27.43)(18.44)\cos 45^\circ} = 19.42 \text{ m}$$

Question 9 (d)
ΔSPF and ΔSPT are congruent (SAS) because:
$|SF| = |ST| = 27.43$ m
$|SP| = |SP|$ [Common]
$|\angle FSP| = |\angle TSP| = 45^\circ$
Therefore, $|FP| = |TP| = l = 19.42$ m



$$27.43^2 = 20.35^2 + 19.42^2 - 2(20.35)(19.42)\cos\theta \qquad \boxed{a^2 = b^2 + c^2 - 2bc\cos A}$$

$$2(20.35)(19.42)\cos\theta = 20.35^2 + 19.42^2 - 27.43^2$$

$$\cos\theta = \frac{20.35^2 + 19.42^2 - 27.43^2}{2(20.35)(19.42)}$$

$$\theta = \cos^{-1}\left(\frac{20.35^2 + 19.42^2 - 27.43^2}{2(20.35)(19.42)}\right) \approx 87^\circ$$

Question 9 (e)

$$24 \text{ km h}^{-1} = \frac{24 \text{ km}}{1 \text{ h}} = \frac{24\ 000 \text{ m}}{3600 \text{ s}} = \frac{20}{3} \text{ s}$$

$s = 4 \times 27.43 \text{ m} = 109.72 \text{ m}$

$v = \frac{20}{3} \text{ m s}^{-1}$

$t = ?$

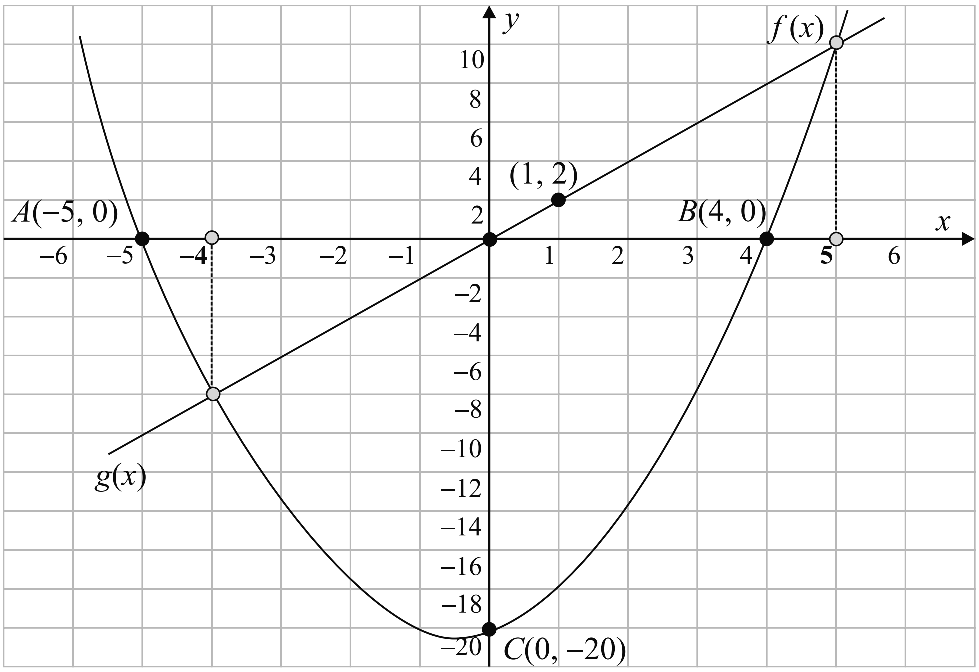
$$t = \frac{s}{v} = \frac{109.72}{\frac{20}{3}} = 16.5 \text{ s} \qquad \boxed{v = \frac{s}{t}}$$

Sample 5 Paper 2

Sample Paper 6: Paper 1

Question 1 (25 marks)

Question 1 (a)



$f(x)=0 \Rightarrow x^2+x-20=0$

$(x-4)(x+5)=0$

$\therefore x=4,\ -5$

$A(-5,\ 0),\ B(4,\ 0)$

To find out where the curve cuts the x-axis, put $f(x) = 0$.
To find out where the curve cuts the y-axis, put $x = 0$.

$x=0 \Rightarrow (0)^2+(0)-20=-20$

$\therefore C(0,\ -20)$

Question 1 (b)

(0, 0) and (1, 2) are two points on $g(x)$.
Draw a graph of $g(x)$ through these two points.
This straight line intersects the graph of $f(x)$ at $x = -4$ and $x = 5$.

Question 1 (c)

Points: $(x_1,\ y_1)=(0,\ 0),\ (x_2,\ y_2)=(1,\ 2)$

$$\boxed{m=\frac{y_2-y_1}{x_2-x_1}}$$

$$\boxed{y-y_1=m(x-x_1)}$$

$m=\dfrac{2-0}{1-0}=2$

$y-0=2(x-0)$

$\therefore y=2x$ or $g(x)=2x$

Question 2 (25 marks)

Question 2 (a)

$f(x) = 3\sqrt{x}$

$f(12) = 3\sqrt{12} = 3\sqrt{4\times 3} = 3\times 2\sqrt{3} = 6\sqrt{3}$

Question 2 (b)

Let x = Number of apples
Let y = Number of oranges

$3x + 4y = 130....(\mathbf{1})$

$4x + 2y = 120....(\mathbf{2})(\times -2)$

$$\begin{array}{rl} 3x + 4y &= 130 \\ -8x - 4y &= -240 \\ \hline -5x \quad\quad &= -110 \Rightarrow x = 22 \text{ c} \end{array}$$

Substitute this value of x into equation (**1**).

$3(22) + 4y = 130$

$66 + 4y = 130$

$4y = 130 - 66 = 64$

$\therefore y = 16 \text{ c}$

Cost of an apple = 22 cent, cost of an orange = 16 cent

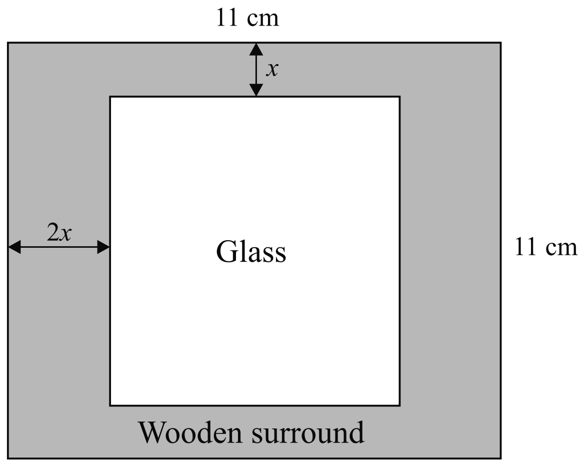
Cost of six apples and six oranges $= 6(22) + 6(16) = 228 \text{ c} = €2.28$

Question 2 (c) (i)

Area of glass $= (11 - 2x)(11 - 4x)$

$= 121 - 44x - 22x + 8x^2$

$= 121 - 66x + 8x^2$



Question 2 (c) (ii)

$121 - 66x + 8x^2 = 40$

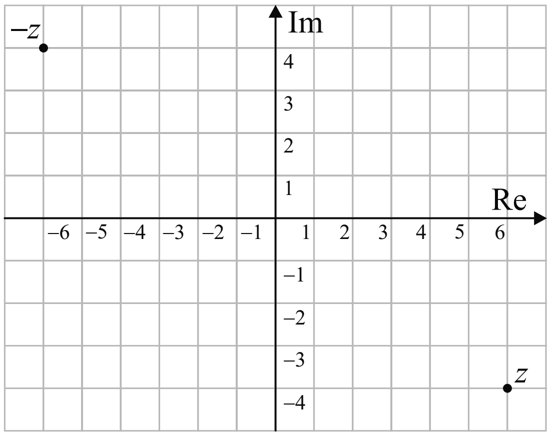
$8x^2 - 66x + 81 = 0$

$(2x - 3)(4x - 27) = 0$

$\therefore x = \frac{3}{2}, \frac{27}{4} = 1.5 \text{ cm}, 6.75 \text{ cm}$

Answer: x = 1.5 cm [Ignore 6.75 cm as $2x$ would be longer than 11 cm.]

Question 3 (25 marks)



Question 3 (a) (i)

$z = 6 - 4i$

$-z = -6 + 4i$

Question 3 (a) (ii)

$$\boxed{z = a + ib \Rightarrow |z| = \sqrt{a^2 + b^2}}$$

$$|z| = |6 - 4i| = \sqrt{6^2 + (-4)^2} = \sqrt{36 + 16}$$
$$= \sqrt{52} = 2\sqrt{13}$$

$$|-z| = |-6 + 4i| = \sqrt{(-6)^2 + 4^2} = \sqrt{36 + 16}$$
$$= \sqrt{52} = 2\sqrt{13}$$

They are both the same distance from the origin.

Question 3 (b) (i)

$z = 3 - 2i \Rightarrow \bar{z} = 3 + 2i$ $\boxed{z = a + bi \Rightarrow \bar{z} = a - bi}$

$$z\bar{z} = (3 - 2i)(3 + 2i) = 9 + 6i - 6i - 4i^2$$
$$= 9 + 4 = 13$$

$\dfrac{z}{\bar{z}} = \dfrac{3 - 2i}{3 + 2i}$ [Multiply above and below by the conjugate of the denominator.]

$$= \frac{(3 - 2i)}{(3 + 2i)} \times \frac{(3 - 2i)}{(3 - 2i)}$$

$$= \frac{9 - 6i - 6i + 4i^2}{13}$$

$$= \frac{5 - 12i}{13} = \frac{5}{13} - \frac{12}{13}i$$

Question 3 (b) (ii)

$$\left|\frac{z}{\bar{z}}\right| = \left|\frac{5}{13} - \frac{12}{13}i\right| = \sqrt{\left(\frac{5}{13}\right)^2 + \left(\frac{12}{13}\right)^2} = \sqrt{\frac{25}{169} + \frac{144}{169}} = \sqrt{\frac{169}{169}} = 1$$

QUESTION 4 (25 MARKS)

Question 4 (a)

$T_1 = a = 9$ $\boxed{T_n = a + (n-1)d}$

$T_{12} = a + 11d = -35$

$\therefore 9 + 11d = -35$

$11d = -44 \Rightarrow d = -4$

Sequence: 9, 5, 1, −3,.....

Question 4 (b)

Arithmetic sequence: 24, 21, 18,.....

$a = 24,\ d = 21 - 24 = -3$

Put $S_n = 0$ $\boxed{S_n = \frac{n}{2}[2a + (n-1)d]}$

$\therefore \frac{n}{2}[2(24) + (n-1)(-3)] = 0$

$[48 - 3n + 3] = 0$

$51 = 3n$

$\therefore n = 17$

ANSWER: At least 18 terms are needed to give a negative sum.

Question 4 (c)

Arithmetic sequence: 1000, 1500, 2000,.....
a = 1000, d = 500

$T_n = 12\ 500$ $\boxed{T_n = a + (n-1)d}$

$\therefore 1000 + (n-1)500 = 12\ 500$

$(n-1)500 = 11\ 500$

$n - 1 = \frac{11\ 500}{500} = 23$

$\therefore n = 24$

ANSWER: He covers 12 500 m on the 24th. day.

Sample 6 Paper 1

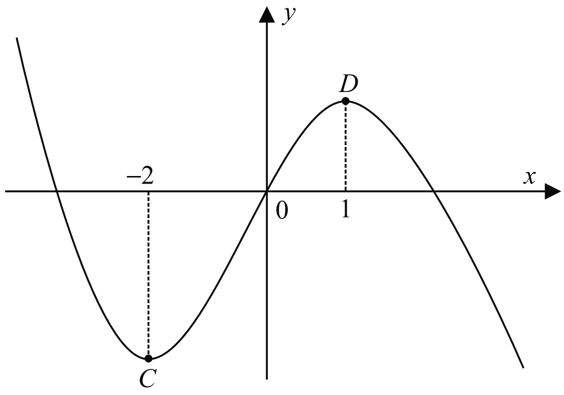
Question 5 (25 marks)

Question 5 (a)

$y = -x^3 + ax^2 + bx$

$x = 0: y = -(0)^3 + a(0)^2 + b(0) = 0$

$\therefore (0,\ 0) \in y = -x^3 + ax^2 + bx$



Question 5 (b)

$y = -x^3 + ax^2 + bx$

$\dfrac{dy}{dx} = -3x^2 + 2ax + b$

$\left(\dfrac{dy}{dx}\right)_{x=1} = 0 \Rightarrow -3(1)^2 + 2a(1) + b = 0$

$-3 + 2a + b = 0$

$2a + b = 3 \ldots\ldots(\mathbf{1})$

$\left(\dfrac{dy}{dx}\right)_{x=-2} = 0 \Rightarrow -3(-2)^2 + 2a(-2) + b = 0$

$-12 - 4a + b = 0$

$-4a + b = 12 \ldots\ldots(\mathbf{2})$

Solve the simultaneous equation to find a and b:

$2a + b = 3 \ldots\ldots(\mathbf{1})$

$\underline{-4a + b = 12 \ldots\ldots(\mathbf{2})}$

$6a \quad = -9 \Rightarrow a = -\frac{3}{2}$

$2a + b = 3 \ldots\ldots(\mathbf{1}) \Rightarrow 2(-\frac{3}{2}) + b = 3$

$-3 + b = 3$

$\therefore b = 6$

Question 5 (c)

$y = -x^3 - \frac{3}{2}x^2 + 6x$

$x = -2: y = -(-2)^3 - \frac{3}{2}(-2)^2 + 6(-2)$

$= -(-8) - \frac{3}{2}(4) - 12$

$= 8 - 6 - 12 = -10$

Local minimum: $C(-2,\ -10)$

$y = -x^3 - \frac{3}{2}x^2 + 6x$

$x = 1: y = -(1)^3 - \frac{3}{2}(1)^2 + 6(1)$

$= -1 - \frac{3}{2} + 6 = \frac{7}{2}$

Local maximum: $D(1,\ \frac{7}{2})$

Question 5 (d)

$y = -x^3 - \frac{3}{2}x^2 + 6x$

$\dfrac{dy}{dx} = -3x^2 - 3x + 6$

$\left(\dfrac{dy}{dx}\right)_{x=0} = -3(0)^2 - 3(0) + 6 = 6 = m$

Equation of t: $m = 6$, point $(x_1, y_1) = (0, 0)$

$y - 0 = 6(x - 0)$ $\boxed{y - y_1 = m(x - x_1)}$

$y = 6x$

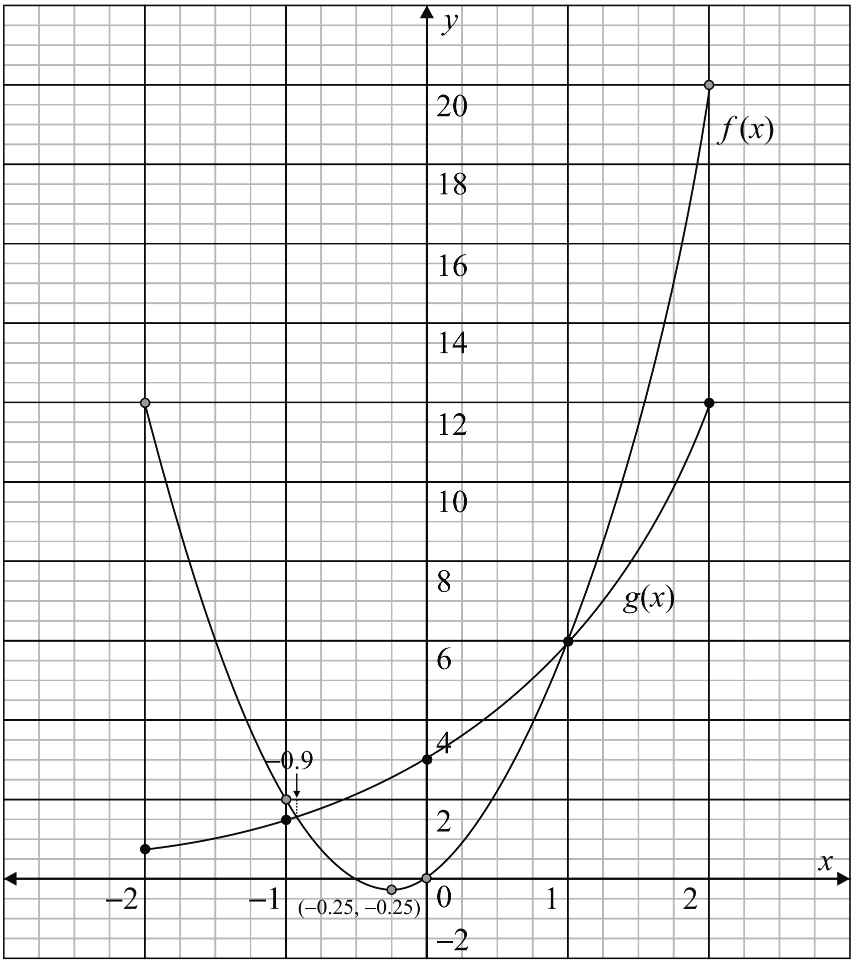
$6x - y = 0$

Question 6 (25 marks)

Question 6 (a)

x	-2	-1	0	1	2
$f(x)$	12	2	0	6	20
$g(x)$	0.75	1.5	3	6	12

Question 6 (b)



Question 6 (c)

$f(x) = 4x^2 + 2x$

$f'(x) = 8x + 2$

$f'(x) = 0 \Rightarrow 8x + 2 = 0$

$8x = -2$

$\therefore x = -\frac{1}{4}$

$f(-\frac{1}{4}) = 4(-\frac{1}{4})^2 + 2(-\frac{1}{4}) = \frac{1}{4} - \frac{1}{2} = -\frac{1}{4}$

Local minimum $(-\frac{1}{4}, -\frac{1}{4})$

Graph: Local minimum $(-\frac{1}{4}, -\frac{1}{4})$

Question 6 (d)

$f(x) = g(x): x = -0.9, 1$

Sample 6 Paper 1

QUESTION 7 (50 MARKS)

	GANDIA	MAJA	RIO	BRAVA
Number of bedrooms	1	2	3	4
Cost per person per week (€)	400	380	420	350
Based on number sharing	2	4	6	8
Under-occupancy charge for every person per week (€)	40	50	60	70
Reduction for over occupancy per extra person (€)	20 (Max. 2 extra)	30 (Max. 2 extra)	45 (Max. 4 extra)	50 (Max. 4 extra)

Question 7 (a)

The cost of staying in Gandia is €400 per person per week, based on two people sharing. One extra person gets a reduction of €20.

Total cost of 3 people = $3 \times 400 - 1 \times 20 =$ €1180

Question 7 (b) (i)

The cost of staying in Brava is €350 per person per week, based on eight people sharing.

Total cost of 8 people = $8 \times 350 =$ €2800

Question 7 (b) (ii)

The cost of staying in Brava is €350 per person per week, based on eight people sharing. Under-occupancy means each person must pay an extra €70 per week.

Total cost of 6 people = $6 \times 350 + 6 \times 70 =$ €2520

Question 7 (b) (iii)

$$\%\text{ loss} = \frac{€2800 - €2520}{€2800} \times 100\% = \frac{€280}{€2800} \times 100\% = 10\%$$

Question 7 (c)

The cost of staying in Rio is €420 per person per week, based on 6 people sharing.

Total cost of 10 people = $10 \times 420 - 4 \times 45 =$ €4020

The cost of staying in Brava is €350 per person per week, based on 8 people sharing.

Total cost of 10 people = $10 \times 350 - 2 \times 50 =$ €3400

Brava is cheaper by €620.

Question 7 (d)

The cost of staying in Maja is €380 per person per week, based on four people sharing. Under-occupancy means each person must pay an extra €50 per week.
Total cost of three people = 3 × 380 + 3 × 50 = €1290

The cost of staying in Gandia is €400 per person per week, based on two people sharing. An extra adult gets a reduction of €20.
Total cost of three people = 3 × 400 − 1 × 20 = €1180

$$\% \text{ saving} = \frac{1290-1180}{1290} \times 100\% = 8.5\%$$

Question 7 (e) (i)

The cost of staying in Brava is €350 per person per week, based on eight people sharing. An extra person gets a reduction of €50, costing that person €300 per week.

Occupancy per week x	Income I (€)
8	2800
9	3100
10	3400
11	3700
12	4000

Cost of 8 people $= 8 \times €350 = €2800$

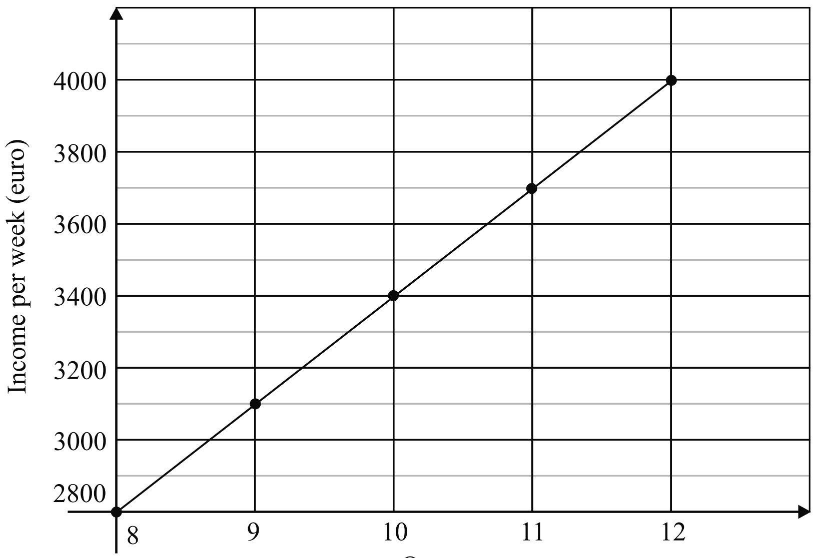
Cost of 9 people $= 8 \times €350 + 1 \times €300 = €3100$

Cost of 10 people $= 8 \times €350 + 2 \times €300 = €3400$

Cost of 11 people $= 8 \times €350 + 3 \times €300 = €3700$

Cost of 12 people $= 8 \times €350 + 4 \times €300 = €4000$

Question 7 (e) (ii)



Question 7 (e) (iii)

Two points on the graph: (8, 2800) and (12, 4000)

$$m = \frac{4000-2800}{12-8} = 300 \qquad \boxed{m = \frac{y_2 - y_1}{x_2 - x_1}}$$

$$y - 2800 = 300(x-8) \qquad \boxed{y - y_1 = m(x - x_1)}$$

$$y - 2800 = 300x - 2400$$

$$y = 300x + 400$$

or $I = 300x + 400$

Question 7 (e) (iv)

Occupancy per week x	Income I (€)
10	3500
11	3800
12	4100
⋮	⋮
16	5300

Cost of 10 people $= 10 \times €350 = €3500$

Cost of 11 people $= 10 \times €350 + 1 \times €300 = €3800$

Cost of 12 people $= 10 \times €350 + 2 \times €300 = €4100$

⋮

Cost of 16 people $= 10 \times €350 + 6 \times €300 = €5300$

Two points on the graph: (10, 3500) and (16, 5300)

$$m = \frac{5300 - 3500}{16 - 10} = \frac{1800}{6} = 300$$

$y - 3500 = 300(x - 10)$

$y = 300x - 3000 + 3500$

$y = 300x + 500$

or $I = 300x + 500$

Question 8 (50 marks)

Question 8 (a)

Arithmetic sequence: 1, 3, 5, 7,......

$a = 1,\ d = 2$

$T_n = 1 + (n-1)(2)$ $\boxed{T_n = a + (n-1)d}$

$= 1 + 2n - 2$

$= 2n - 1$

Question 8 (b)

$1 = 1^2 - 0^2$

$3 = 2^2 - 1^2$

$5 = 3^2 - 2^2$

$7 = 4^2 - 3^2$ (= 16 – 9 = 7)

Question 8 (c)

$T_n = 21$

$\therefore 2n - 1 = 21$

$2n = 22$

$\therefore n = 11$

$T_{11} = 21 = 11^2 - 10^2$

Question 8 (d)

$T_n = n^2 - (n-1)^2$

$= n^2 - (n^2 - 2n + 1)$

$= n^2 - n^2 + 2n - 1$

$= 2n - 1$

Question 8 (e)

Two consecutive odd numbers: $2n - 1$, $2n + 1$

$(2n-1)(2n+1)$

$= 4n^2 + 2n - 2n - 1$

$= 4n^2 - 1$

$4n^2 =$ Even number for all values of n

$\therefore 4n^2 - 1 =$ Odd number

Question 8 (f)

$\boxed{S_n = \frac{n}{2}[2a + (n-1)d]}$

Arithmetic sequence: 1, 3, 5, 7,......

$a = 1,\ d = 2,\ n = 100$

$S_{100} = \frac{100}{2}[2(1) + (100-1)(2)]$

$= 50[2 + (99)(2)]$

$= 50[200]$

$= 10\ 000$

Sample 6 Paper 1

QUESTION 9 (50 MARKS)

Question 9 (a)

$$\tan A = \frac{1}{50} \Rightarrow A = \tan^{-1}\left(\frac{1}{50}\right) = 1.146^{\circ}$$

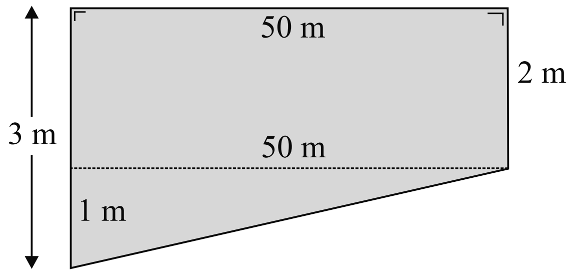
Question 9 (b)

Surface area: Area $= 25 \times 50 = 1250\ \text{m}^2$

Question 9 (c)

Volume = Area of the side of pool × Width of pool

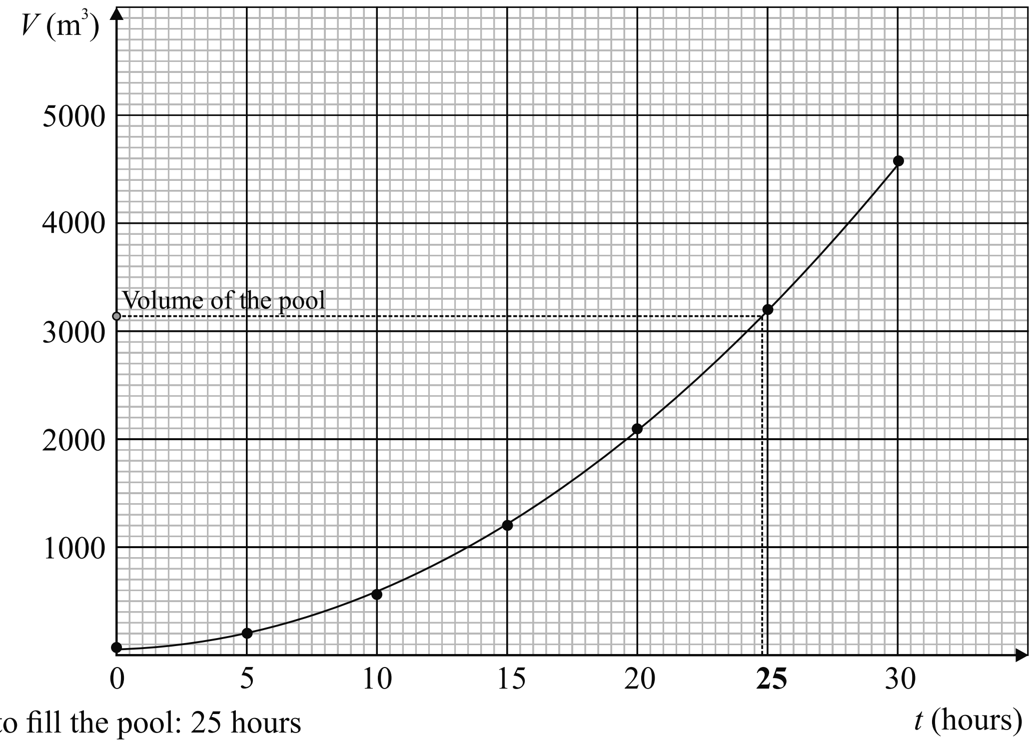
$= (50 \times 2 + \frac{1}{2} \times 50 \times 1) \times 25 = 3125\ \text{m}^3$



Question 9 (d)

t (h)	0	5	10	15	20	25	30
V (m^3)	80	205	580	1205	2080	3205	4580

Question 9 (e)



Time to fill the pool: 25 hours

Question 9 (f)

$V = 80 + 5t^2$

$\dfrac{dV}{dt} = 10t$

(i) $\left(\dfrac{dV}{dt}\right)_{t=6} = 10(6) = 60\ \text{m}^3\ \text{h}^{-1}$

(ii) $\left(\dfrac{dV}{dt}\right)_{t=16} = 10(16) = 160\ \text{m}^3\ \text{h}^{-1}$

Question 9 (g)

$t = 12 : V = 80 + 5(12)^2 = 800\ \text{m}^3$

$\%\ \text{filled} = \dfrac{800}{3125} \times 100\% = 25.6\%$

Volume to be filled $= 3125 - 800 = 2325\ \text{m}^3$

$\text{Time} = \dfrac{2325}{85} = 1641$ minutes

Sample Paper 6: Paper 2

Question 1 (25 marks)

Question 1 (a)

40, 48, 50, 52, 56, 58, 60, 65, 68, 70

(i) Mean $\mu = 56.7$

(ii) Standard deviation $\sigma = 8.99$

Question 1 (b)

48, 48, 52, 57, **59**, **61**, 63, 63, 98, 100

(i) There are two middle values when the data is laid out in order: 59 and 61. Get the mean of these two values.

Median (middle value) $= \dfrac{59+61}{2} = 60$

(ii) 48, 48, **52**, 57, **59**, **61**, 63, **63**, 98, 100

$Q_1 = 52,\ Q_3 = 63$

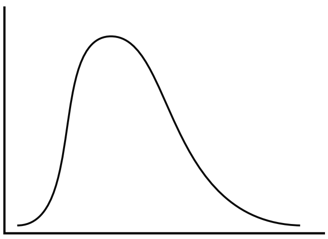
Interquartile range $= Q_3 - Q_1 = 63 - 52 = 11$

> Casio *fx*-85*GT PLUS*
> **Finding mean and standard deviation**
> Press Mode
> Press 2: Stat
> Press 1:1-Var
> Input values pressing = after each value
> Press AC
> Press Shift followed by 1
> Press 4: Var
> Press 2: $\bar{x}$ (mean)
> Press equals
> Press Shift followed by 1
> Press 4: Var
> Press 3: σx (standard deviation)
> Press equals

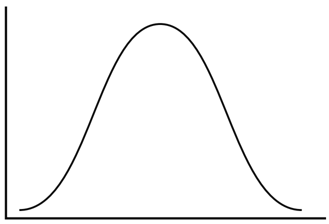
Question 1 (c)

In **part (a)**, there are no extreme values or outliers so values are normally distributed. Therefore, the mean and standard deviation take account of all values.
In **part (b)**, there are extreme values or outliers (10, 98, 100). Therefore, the median and interquartile range give a better picture of central tendency and the spread. The mean and standard deviation are sensitive to outliers.

Question 1 (d)



(i) Median and interquartile range



(ii) Mean and standard deviation

Question 2 (25 marks)

Question 2 (a)

A standard pack of playing cards has four suits (diamonds (red), hearts (red), spades (black), clubs (black)). Each suit has 13 cards, including an ace, three face cards (king, queen, jack) and cards numbers 2 to 10, giving 52 cards in total.

(i) $P(\text{Black card}) = \dfrac{\text{Number of black cards}}{\text{Total number of cards}} = \dfrac{26}{52} = \dfrac{1}{2}$

(ii) $P(\text{Jack}) = \dfrac{\text{Number of jacks}}{\text{Total number of cards}} = \dfrac{4}{52} = \dfrac{1}{13}$

(iii) $P(\text{Jack of diamonds}) = \dfrac{\text{Number of jack of diamonds}}{\text{Total number of cards}} = \dfrac{1}{52}$

Question 2 (b)

(i) $P(\text{Either a face card } or \text{ a diamond})$

$= P(\text{Face card}) + P(\text{Diamond}) - P(\text{Face card and a diamond})$

$= \dfrac{12}{52} + \dfrac{13}{52} - \dfrac{3}{52} = \dfrac{22}{52} = \dfrac{11}{26}$

(ii) $P(\text{4 of spades})$ **and then** $P(\text{4 of spades})$ [with replacement]

$= \dfrac{1}{52} \times \dfrac{1}{52} = \dfrac{1}{2704}$

Question 2 (c)

Write out all the possibilities for tossing a coin three times:
HHH, HHT, HTH, THH, HTT, THT, TTH, TTT (Eight outcomes)

(i) HHH, **HHT**, **HTH**, **THH**, HTT, THT, TTH, TTT (Eight outcomes)

$P(\text{2 heads and 1 tail}) = \dfrac{3}{8}$

(ii) **HHH**, HHT, HTH, THH, HTT, THT, TTH, TTT (Eight outcomes)

$P(\text{3 heads}) = \dfrac{1}{8}$

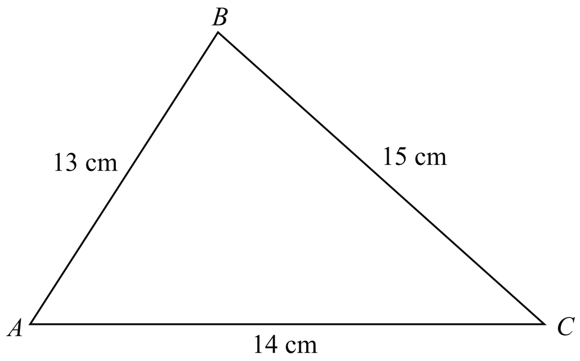
(iii) HHH, HHT, **HTH**, **THH**, HTT, THT, TTH, TTT (Six outcomes)

$P(\text{2 heads and 1 tail with the second head on the third roll}) = \dfrac{2}{8} = \dfrac{1}{4}$

Question 3 (25 marks)

Question 3 (a)

Biggest angle: $\angle BAC$ [The biggest angle is opposite the longest side.]



Question 3 (b)

$$\boxed{a^2 = b^2 + c^2 - 2bc\cos A}$$

$$15^2 = 13^2 + 14^2 - 2(13)(14)\cos(\angle BAC)$$

$$2(13)(14)\cos(\angle BAC) = 13^2 + 14^2 - 15^2$$

$$\cos(\angle BAC) = \frac{13^2 + 14^2 - 15^2}{2(13)(14)}$$

$$\angle BAC = \cos^{-1}\left(\frac{13^2 + 14^2 - 15^2}{2(13)(14)}\right) = 67.4^\circ$$

Question 3 (c)

$$\text{Area} = \tfrac{1}{2}(13)(14)\sin 67.4^\circ = 84 \text{ cm}^2 \qquad \boxed{\text{Area} = \tfrac{1}{2}ab\sin C}$$

Question 3 (d)

$$\boxed{k^2 = \frac{|\text{Image area}|}{|\text{Object area}|}}$$

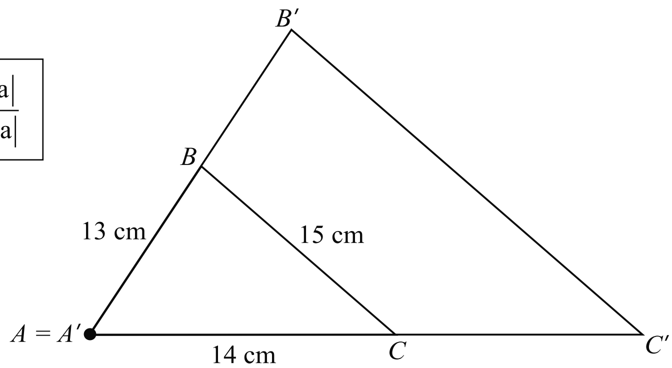
$k = 1.5$

$|\text{Object area}| = 84 \text{ cm}^2$

$|\text{Image area}| = ?$

$|\text{Image area}| = k^2\,|\text{Object area}|$

$\qquad = (1.5)^2(84) = 189 \text{ cm}^2$



$$\boxed{\text{Scale factor } k = \frac{|\text{Image length}|}{|\text{Object length}|}}$$

$k = 1.5$

$|\text{Object length}| = |BC| = 15 \text{ cm}$

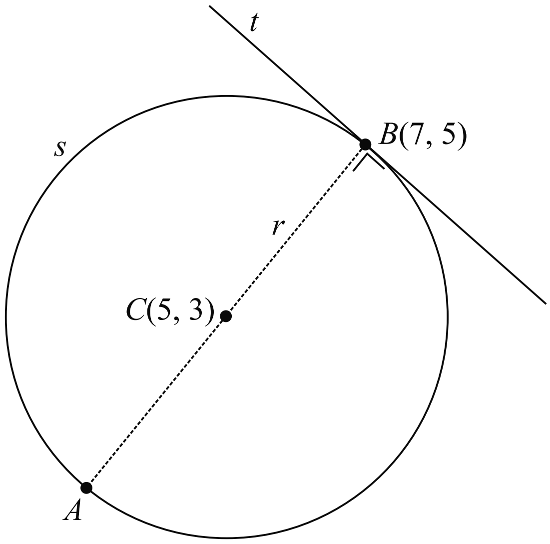
$|\text{Image area}| = |B'C'| = ?$

$|\text{BC}| = k\,|B'C'|$

$\qquad = (1.5)(15) = 22.5 \text{ cm}$

Sample 6 Paper 2

QUESTION 4 (25 MARKS)



Question 4 (a)

$B \rightarrow C \rightarrow A$

$B(7, 5) \rightarrow C(5, 3) \rightarrow A(3, 1)$

$A(3, 1),\ B(7, 5)$

$m = \dfrac{5-1}{7-3} = \dfrac{4}{4} = 1$ $\boxed{m = \dfrac{y_2 - y_1}{x_2 - x_1}}$

Question 4 (b)

$\boxed{|PQ| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}}$

$r = |BC|$

$B(7, 5),\ C(5, 3)$

$r = \sqrt{(5-7)^2 + (3-5)^2} = \sqrt{4+4} = \sqrt{8}$

Centre $C(5, 3)$, $r = \sqrt{8}$

$s: (x-5)^2 + (y-3)^2 = (\sqrt{8})^2$

$(x-5)^2 + (y-3)^2 = 8$

Equation of circle: Centre (h, k), Radius r
$(x-h)^2 + (y-k)^2 = r^2$

Question 4 (c) (i)

Equation of t: Point of contact: $B(7, 5)$, slope $m = -1$ [The tangent t is perpendicular to the diameter at the point of contact.]

$t: y - 5 = -1(x - 7)$ $\boxed{y - y_1 = m(x - x_1)}$

$y - 5 = -x + 7$

$x + y - 12 = 0$

Question 4 (c) (ii)

$x + y - 12 = 0$

$x = 0: 0 + y - 12 = 0 \Rightarrow y = 12$

$y = 0: x + 0 - 12 = 0 \Rightarrow x = 12$

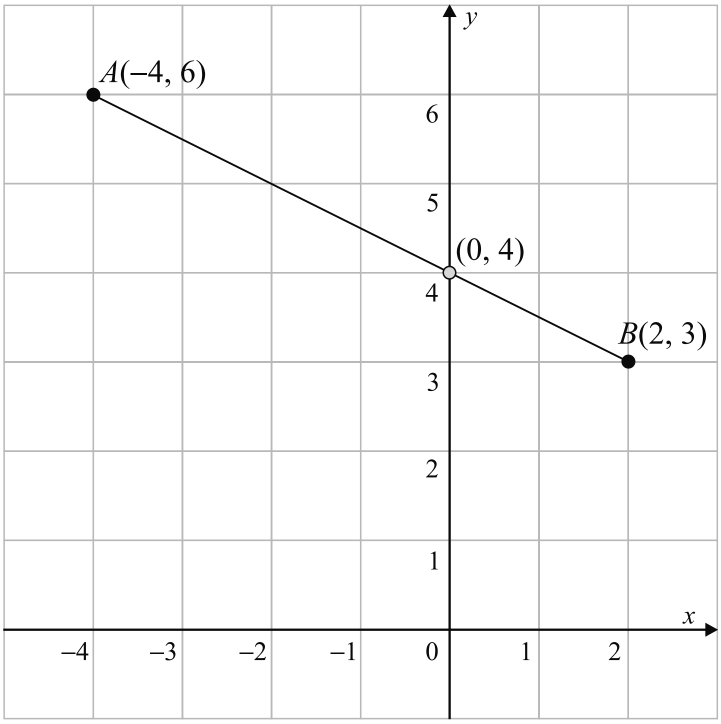
Cuts y-axis: Put $x = 0$
Cuts x-axis: Put $y = 0$

x-intercept: (12, 0); y-intercept: (0, 12)

Question 5 (25 marks)

Question 5 (a) & (b)

y-intercept: $y = 4$, Point $(0, 4)$



Question 5 (c)

Equation of AB: $A(-4, 6)$, $B(2, 3)$

$$m = \frac{3-6}{2-(-4)} = \frac{-3}{6} = -\frac{1}{2} \qquad \boxed{m = \frac{y_2 - y_1}{x_2 - x_1}}$$

$$\boxed{y - y_1 = m(x - x_1)}$$

$$y - 3 = -\tfrac{1}{2}(x - 2)$$
$$2y - 6 = -x + 2$$
$$x + 2y - 8 = 0$$

Cuts y-axis: Put $x = 0$

$$x = 0: 0 + 2y - 8 = 0$$
$$2y = 8$$
$$\therefore y = 4$$

Both answers agree.

Question 5 (d)

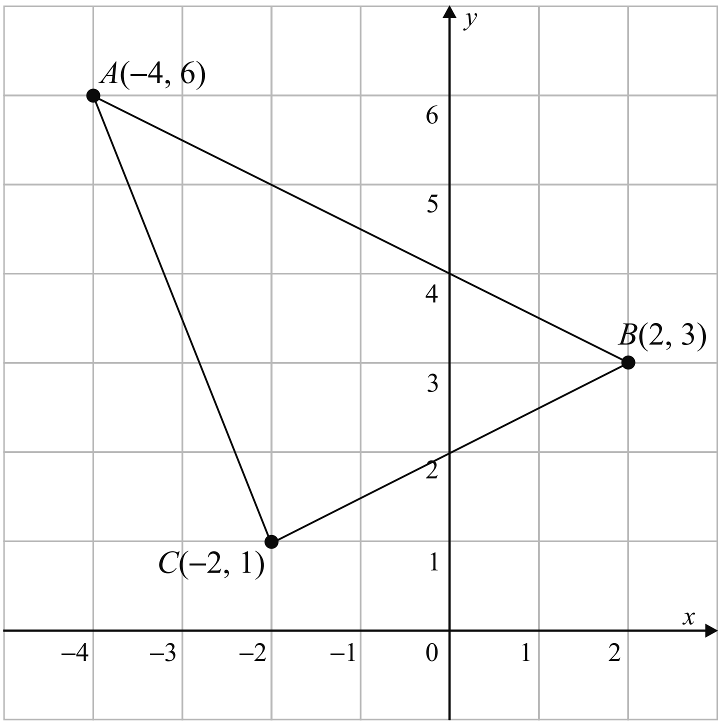
$$\boxed{\text{Area} = \tfrac{1}{2}\left|x_1y_2 - x_2y_1\right|}$$

$C(-2, 1) \rightarrow (0, 0)$ [Add 2, Subtract 1]

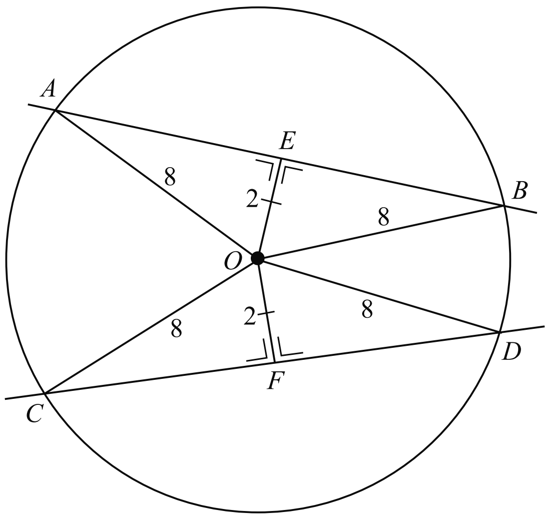
$A(-4, 6) \rightarrow (-2, 5)$

$B(2, 3) \rightarrow (4, 2)$

$$\begin{aligned} \text{Area} &= \tfrac{1}{2}\left|(-2)(2) - (4)(5)\right| \\ &= \tfrac{1}{2}\left|-4 - 20\right| \\ &= \tfrac{1}{2}\left|-24\right| \\ &= 12 \end{aligned}$$



QUESTION 6 (25 MARKS)



Draw lines from the centre O to points A, B, C and D. These lines are radii of length 8 cm. Apply Pythagoras to triangle BOE.

$$|BE|^2 + 2^2 = 8^2$$

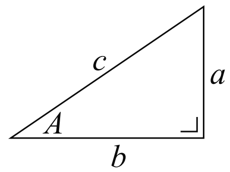
$$|BE|^2 = 64 - 4 = 60$$

$$|BE| = \sqrt{60} = \sqrt{4 \times 15} = 2\sqrt{15} \text{ cm}$$

Similarly, $|AE| = 2\sqrt{15}$ cm

$$\therefore |AB| = 2\sqrt{15} + 2\sqrt{15} = 4\sqrt{15} \text{ cm}$$

PYTHAGORAS



$$\boxed{c^2 = a^2 + b^2}$$

Applying Pythagoras to the triangles COF and FOD will give:

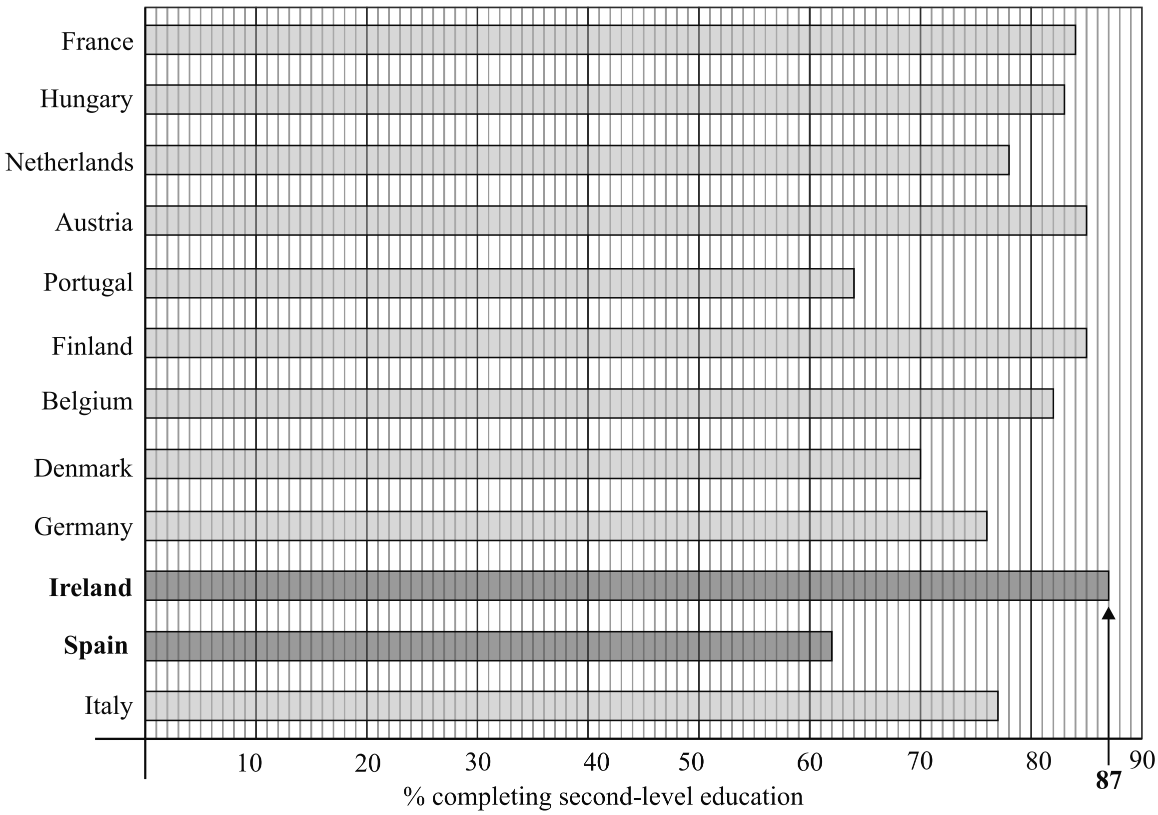
$$|CF| = |FD| = 2\sqrt{15} \text{ cm}$$

$$\therefore |CD| = 4\sqrt{15} \text{ cm}$$

$$\therefore |AB| = |CD|$$

Question 7 (75 marks)

Question 7 (a)



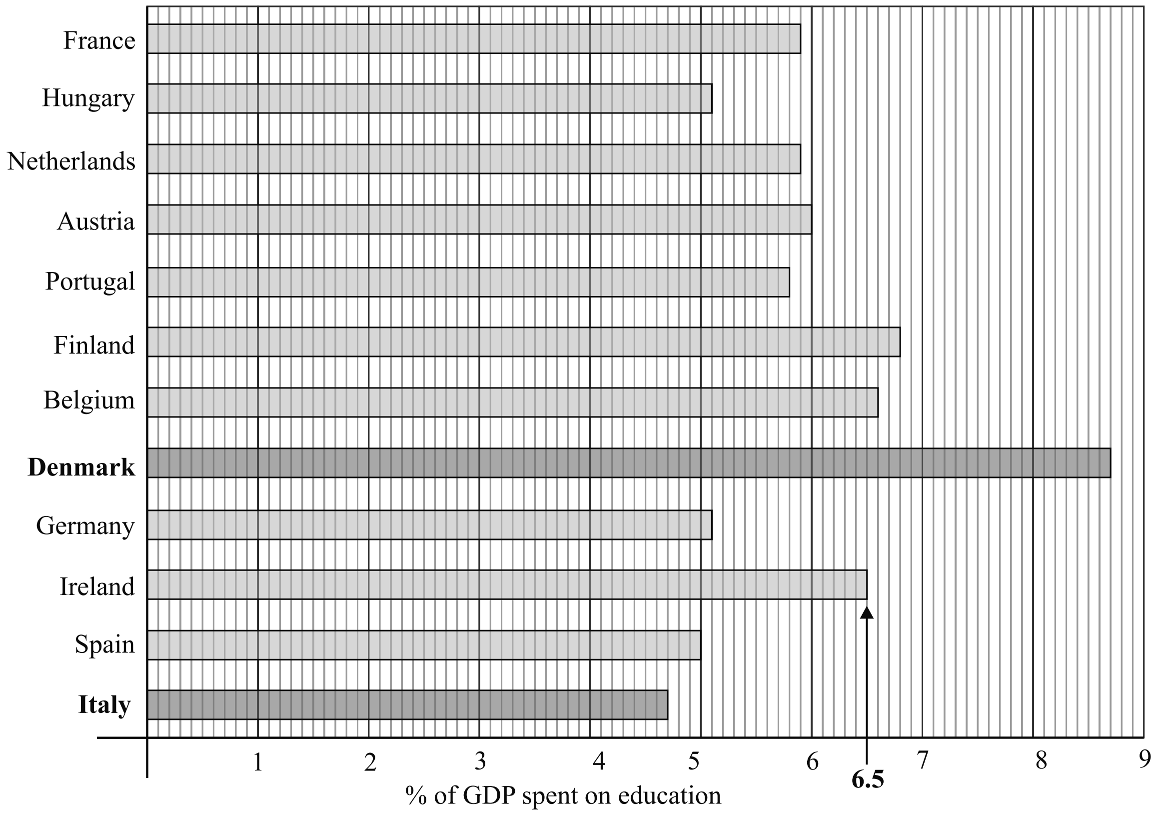
(i) Ireland has the highest percentage of students who completed second-level education.

(ii) Spain has the lowest percentage of students who completed second-level education.

(iii) Ireland's completion rate is 87%.

Sample 6
Paper 2

Question 7 (b)



(i) Denmark has the highest spending as a percentage of GDP.

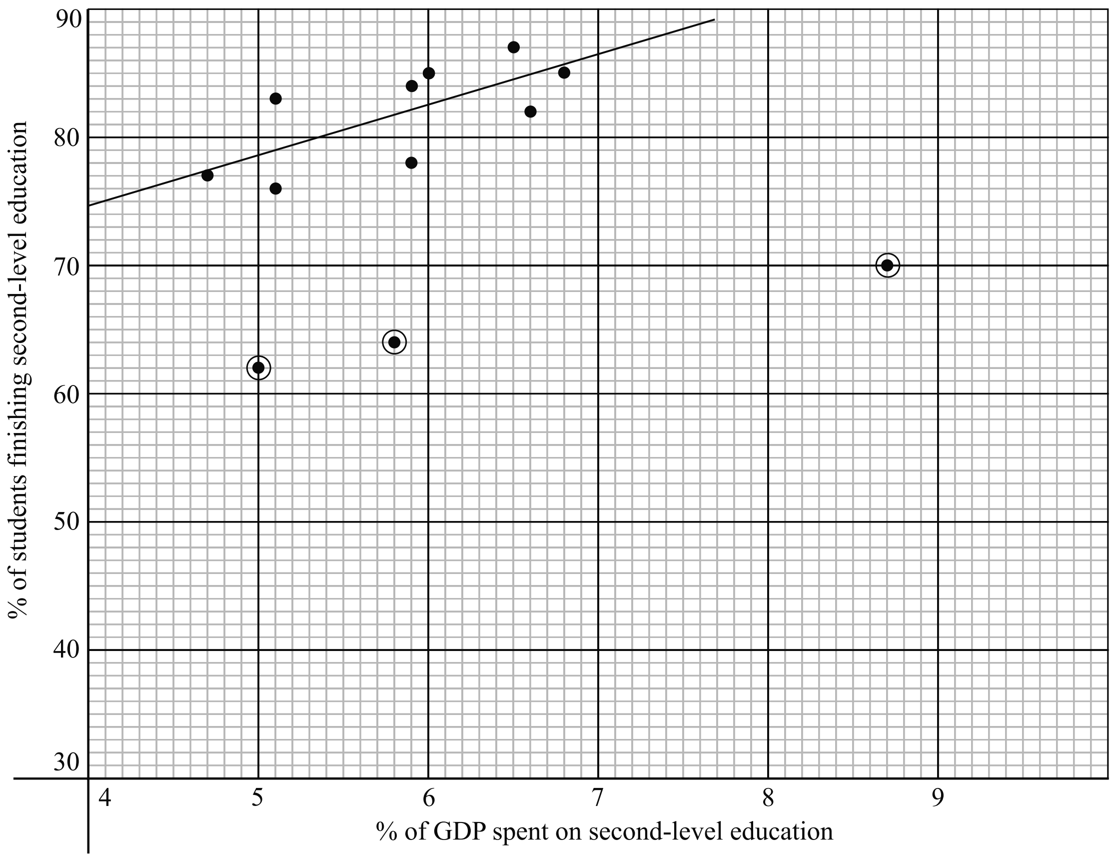
(ii) Italy has the lowest spending as a percentage of GDP.

(iii) Ireland's spending as a percentage of GDP is 6.5%

Question 7 (c)

Country	% of GDP spent on second-level education	% who finished second-level education
Austria	6.0	85
Belgium	6.6	82
Denmark	**8.7**	**70**
France	5.9	84
Finland	6.8	85
Germany	5.1	76
Hungary	5.1	83
Ireland	6.5	87
Italy	4.7	77
Netherlands	5.9	78
Portugal	**5.8**	**64**
Spain	**5.0**	**62**

Question 7 (d)



Question 7 (e)
Outlier countries: Spain, Portugal, Denmark.

Question 7 (f)
Draw a line of best fit through the points (excluding the outliers) to see how closely the points cluster about this line. You can see that there is a moderate positive correlation. You can estimate r to be anywhere between 0.6 and 0.8. [The actual value is $r = +0.68$].

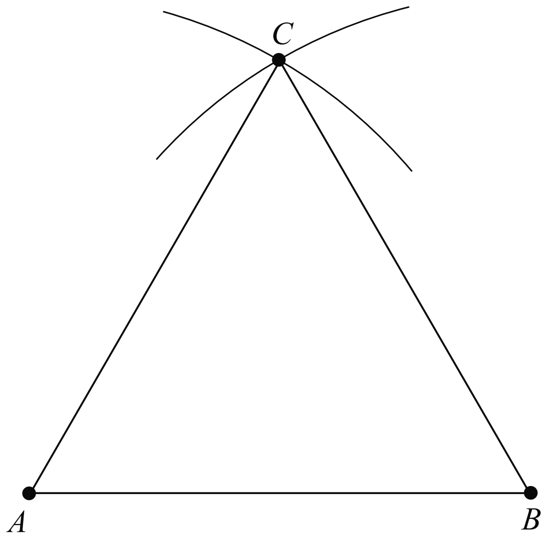
Question 7 (g)
No, it is clearly a factor, but many other variables exist like levels of poverty, the affordability of second-level education and unemployment.

Sample 6
Paper 2

Question 8 (50 marks)

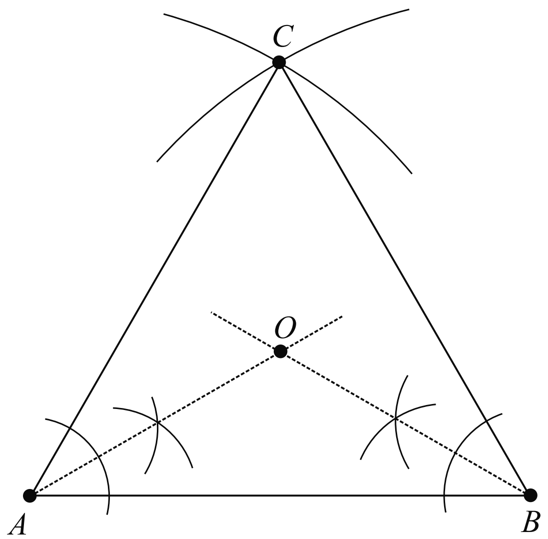
Question 8 (a) (i)

Stretch your compass so it is equal to length $|AB|$.
With the compass at A, draw an arc above AB.
With the compass at B (leaving the radius of the compass unchanged), draw another arc above AB.
The intersection of these arcs is point C.
Join A to C and B to C to construct triangle ABC.

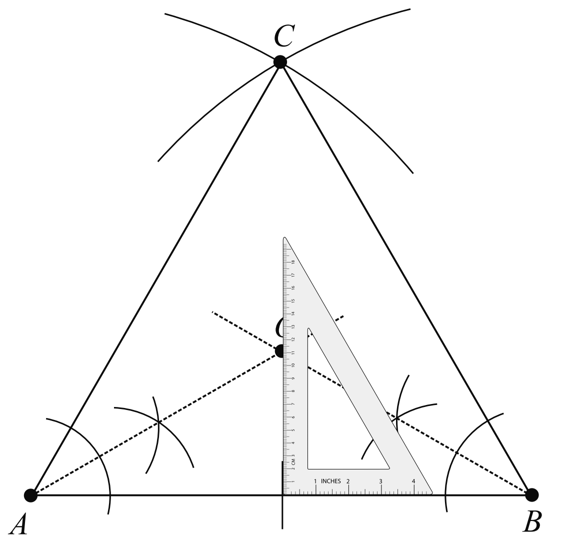


Question 8 (a) (ii)

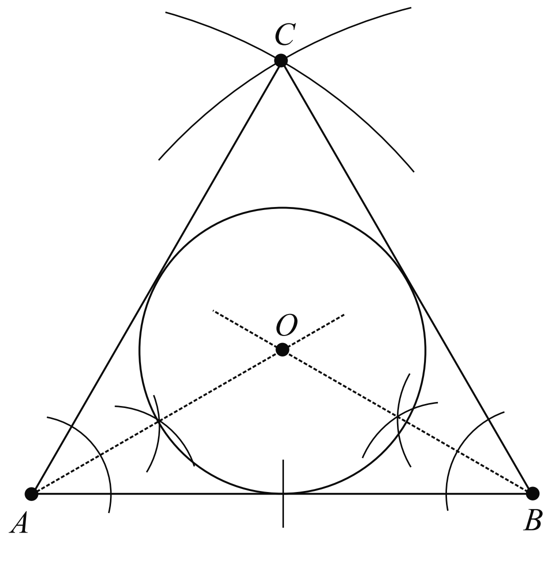
Step 1: Use your compass to find the angle bisectors of each of the base angles. The intersection of these angle bisectors gives you the incentre O.



Step 2: Using a set square, mark the perpendicular line through O that cuts side AB.

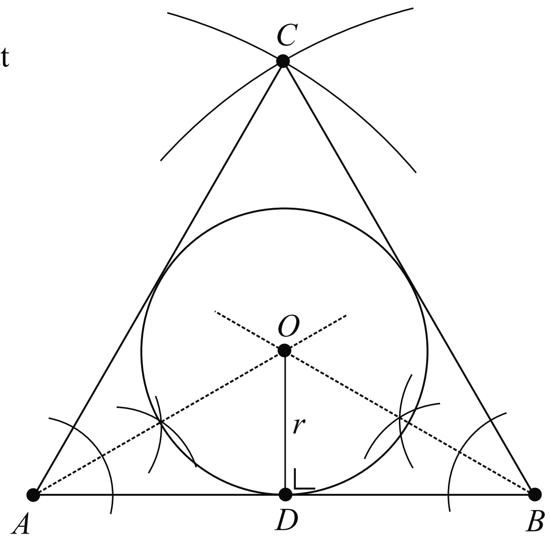


Step 3: Stretch your compass from O to the point of intersection of this perpendicular line and side AB, and draw the incircle.



Question 8 (a) (iii)
A line which touches a circle at one point only is called a **tangent**.

Question 8 (a) (iv)
The radius $r = |OD|$ makes an angle of 90° with side AB because a tangent is perpendicular to the radius at the point of contact.



Question 8 (a) (v)
Consider triangles AOD and DOB.

$|OD| = |OD|$ [Common side]

$|\angle OAD| = |\angle OBD|$ [30°]

$|\angle DOA| = |\angle DOB|$ [60°]

$|\angle ODA| = |\angle BDO|$ [90°]

Therefore, triangles AOD and DOB are congruent (ASA).
$\therefore |AD| = |DB|$

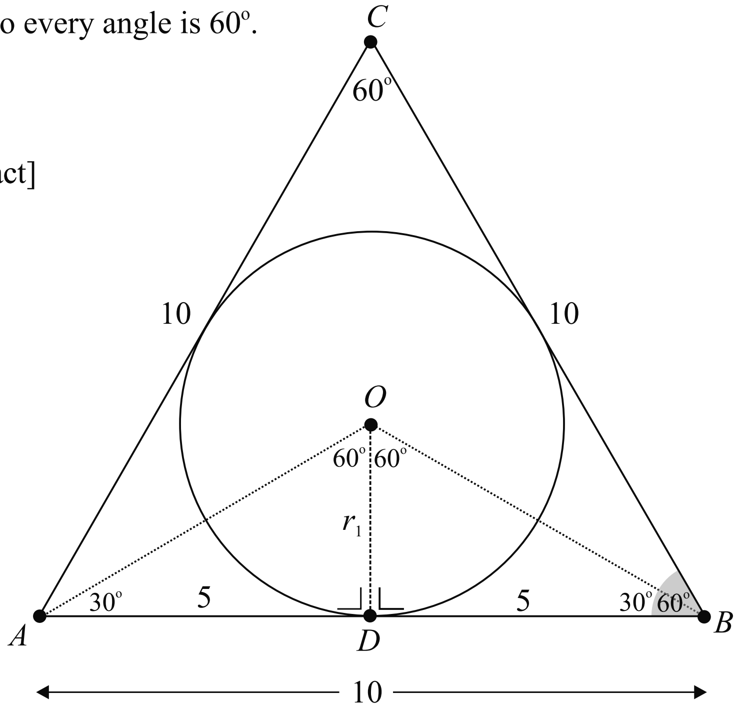
Question 8 (b) (i)
Triangle ABC is an equilateral triangle so every angle is 60°.
$|AD| = |DB| = 5$ [Already proved].

$|\angle OBD| = 30^\circ$ [Angle bisector]

$|\angle ODB| = 90^\circ$ [Tangent at point of contact]

$$\boxed{\tan A = \frac{\text{Opposite}}{\text{Adjacent}}}$$

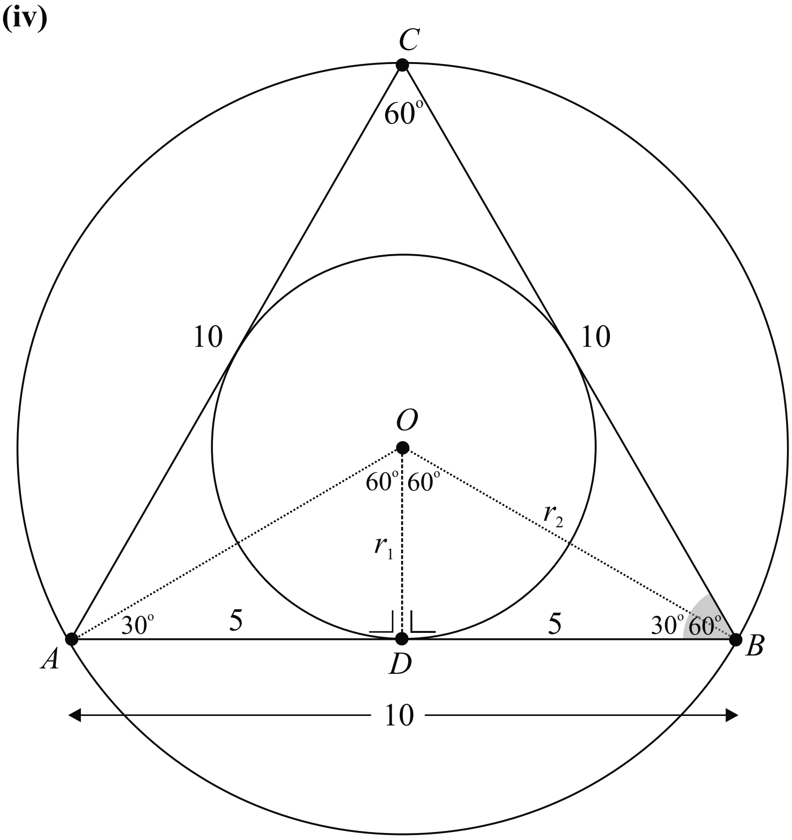
$$\tan 30^\circ = \frac{r_1}{5} \Rightarrow r_1 = 5 \tan 30^\circ = \frac{5\sqrt{3}}{3}$$



Question 8 (b) (ii)
The circumcentre is the intersection of the perpendicular bisectors of the sides.

Question 8 (b) (iii)
The perpendicular bisectors of the sides intersect at O, as do the angle bisectors.

Question 8 (b) (iv)



Question 8 (b) (v)

$$\cos 30^\circ = \frac{5}{r_2} \Rightarrow r_2 = \frac{5}{\cos 30^\circ} = \frac{10\sqrt{3}}{3}$$

$$\boxed{\cos A = \frac{\text{Adjacent}}{\text{Hypotenuse}}}$$

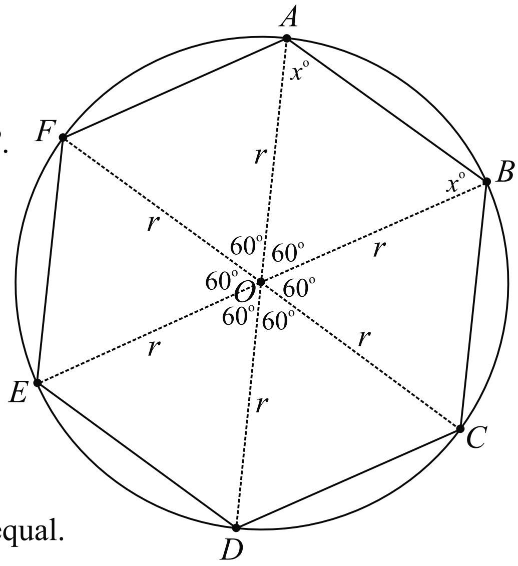
Question 8 (b) (vi)

$$\frac{r_1}{r_2} = \frac{\left(\frac{5\sqrt{3}}{3}\right)}{\left(\frac{10\sqrt{3}}{3}\right)} = \frac{1}{2} \Rightarrow r_1 : r_2 = 1:2$$

Question 9 (25 marks)

Question 9 (a)

Draw radii from O to each of the other points on the circle. Each of the angles at the centre of the circle is 60°.
$|\angle AOB| = 60^\circ$



Triangle AOB is an isosceles triangle as $|OA| = |OB| = r$.
$\therefore |\angle BAO| = |\angle ABO| = x^\circ$

$60^\circ + x^\circ + x^\circ = 180^\circ$ [Three angles add up to 180°]

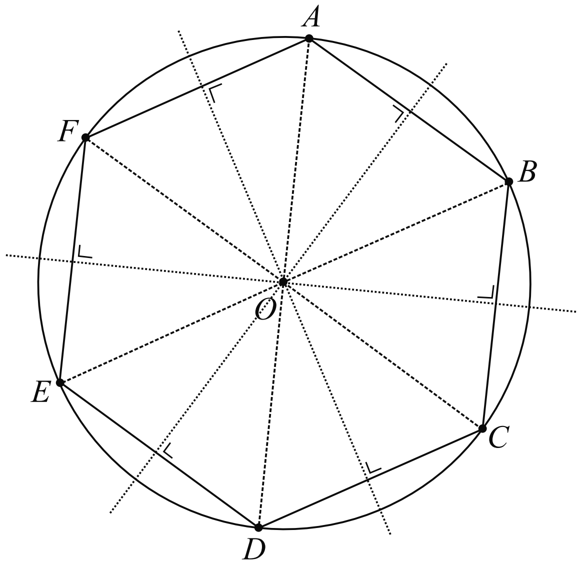
$2x^\circ = 120^\circ$

$\therefore x = 60^\circ$

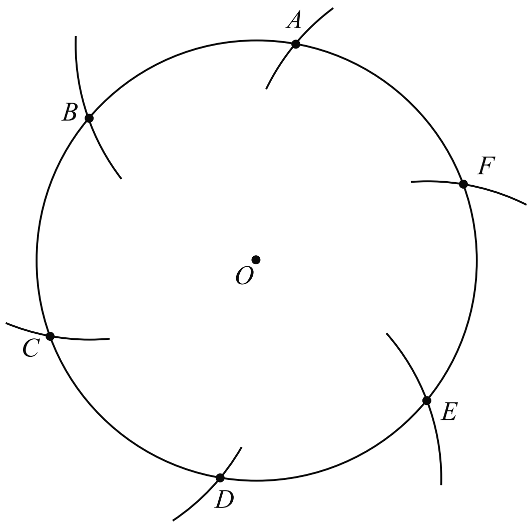
Triangle AOB is an equilateral triangle as all angles are equal.
$\therefore |AB| = r$

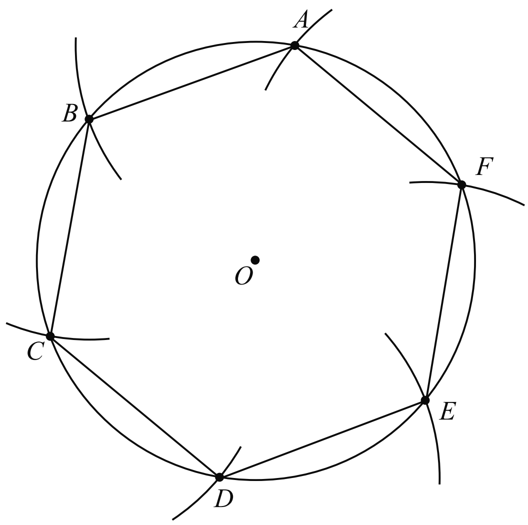
Question 9 (b)

Axes of symmetry: AD, BE, CF and the three perpendicular bisectors of the sides



Question 9 (c)





Mark a point called A anywhere on the circle.
Set the radius of the compass to the distance $|OA|$.
With the compass placed at A, draw arc AB. Then with the compass placed at B (do not change radius of compass), draw arc BC. Continue around the circle until you arrive back at A.
Join the points in straight lines to form the hexagon $ABCDEF$.

Sample Paper 7: Paper 1

Question 1 (25 marks)

Question 1 (a)

$$4(2-3i)+i(-5+2i)$$
$$=8-12i-5i+2i^2$$
$$=8-12i-5i-2$$
$$=6-17i$$

Question 1 (b)

$$\frac{2-3i}{1+5i}$$
$$=\frac{(2-3i)}{(1+5i)}\times\frac{(1-5i)}{(1-5i)}$$ [Multiply above and below by the conjugate of the denominator.]
$$=\frac{2-10i-3i+15i^2}{1-5i+5i-25i^2}$$
$$=\frac{2-10i-3i-15}{1+25}$$
$$=\frac{-13-13i}{26}$$
$$=-\frac{1}{2}-\frac{1}{2}i$$

$$\boxed{z=a+ib\Rightarrow |z|=\sqrt{a^2+b^2}}$$

$$|u|=\left|-\frac{1}{2}-\frac{1}{2}i\right|=\sqrt{\left(-\frac{1}{2}\right)^2+\left(-\frac{1}{2}\right)^2}$$
$$=\sqrt{\frac{1}{4}+\frac{1}{4}}=\sqrt{\frac{1}{2}}=\frac{1}{\sqrt{2}}$$

Question 1 (c)

$$z=2-3i,\ w=4-i$$
$$|z-w|=k$$
$$|2-3i-4+i|=k$$
$$|-2-2i|=k$$

$$\boxed{z=a+ib\Rightarrow |z|=\sqrt{a^2+b^2}}$$

$$\sqrt{(-2)^2+(-2)^2}=k$$
$$\sqrt{8}=k$$
$$\therefore k=2\sqrt{2}$$

$$4|u|=4\left(\frac{1}{\sqrt{2}}\right)=2\sqrt{2}=k$$

Question 2 (25 marks)

Question 2 (a)

$$\frac{x}{8}-y=-\frac{5}{2}\ (\times 8)$$

$$3x+\frac{y}{3}=13\ (\times 3)$$

$x-8y=-20....(\mathbf{1})(\times -9)$

$9x+y=39......(\mathbf{2})$

$$\begin{array}{r} -9x+72y=180 \\ 9x+y\quad =39 \\ \hline 73y=219\Rightarrow y=3 \end{array}$$

Answer: $x=4$, $y=3$

Substitute this value of y into equation (**1**) to find x:

$x-8(3)=-20....(\mathbf{1})$

$x-24=-20$

$x=-20+24=4$

Question 2 (b)

Roots: $x=-3,\ 4$

Factors: $(x+3),\ (x-4)$

$\therefore (x+3)(x-4)=0$

$x^2-4x+3x-12=0$

$x^2-x-12=0$

Question 2 (c)

$$\frac{2}{x-1}-\frac{1}{x+2}=\frac{1}{2}$$

$$\frac{2(x-1)(x+2)2}{(x-1)}-\frac{2(x-1)(x+2)1}{(x+2)}=\frac{2(x-1)(x+2)1}{2}$$ [Multiply across by the denominators.]

$4(x+2)-2(x-1)=(x-1)(x+2)$

$4x+8-2x+2=x^2+2x-x-2$

$0=x^2+2x-x-4x+2x-2-2-8$

$0=x^2-x-12$

$0=(x+3)(x-4)$

$\therefore x=-3,\ 4$

Sample 7 Paper 1

Question 3 (25 marks)

Question 3 (a)

$2\sqrt{x+1}=1$

$4(x+1)=1$

$4x+4=1$

$4x=-3$

$\therefore x=-\frac{3}{4}$

Check: $x=-\frac{3}{4}$

$2\sqrt{(-\frac{3}{4})+1}=2\sqrt{\frac{1}{4}}=2\times\frac{1}{2}=1$

Question 3 (b) (i)

$\sqrt{125}=\sqrt{5^3}=(5^3)^{\frac{1}{2}}=5^{\frac{3}{2}}$

Question 3 (b) (ii)

$$\frac{5^{2x+1}}{\sqrt{5}}=\left(\frac{1}{\sqrt{125}}\right)^3$$

$$\frac{5^{2x+1}}{5^{\frac{1}{2}}}=\left(\frac{1}{5^{\frac{3}{2}}}\right)^3$$

$$5^{2x+1-\frac{1}{2}}=(5^{-\frac{3}{2}})^3$$

$$5^{2x+\frac{1}{2}}=5^{-\frac{9}{2}}$$

$$\therefore 2x+\tfrac{1}{2}=-\tfrac{9}{2}$$

$$2x=-\tfrac{9}{2}-\tfrac{1}{2}=-\tfrac{10}{2}=-5$$

$$\therefore x=-\tfrac{5}{2}$$

Power Rules

$a^p a^q=a^{p+q}$

$\frac{a^p}{a^q}=a^{p-q}$

$(a^p)^q=a^{pq}$

$a^0=1$

$a^{-p}=\frac{1}{a^p}$

Question 4 (25 marks)

Question 4 (a)

Number of litres: 11 360 – 7160 = 4200 litres

Cost = $4200 \times 20.5\text{ cent} = 86100\text{ cent} = €861.00$

Amount of VAT paid = $€1041.81 - €861.00 = €180.81$

Rate of VAT $= \dfrac{€180.81}{€861.00} \times 100\% = 21\%$

Question 4 (b) (i)

True value = €3.85 + €7.45 + €8.40 + €11.55 = €31.25

Estimated value = €3 + €7 + €8 + €11 = €29

Absolute error $= |31.25 - 29| = 2.25$

Fractional error $= \dfrac{2.25}{31.25}$

% error $= \dfrac{2.25}{31.25} \times 100\% = 7.2\%$

Question 4 (b) (ii)

True value = €3.85 + €7.45 + €8.40 + €11.55 = €31.25

Estimated value = €4 + €7 + €8 + €12 = €31

Absolute error $= |31.25 - 31| = 0.25$

Fractional error $= \dfrac{0.25}{31.25}$

% error $= \dfrac{0.25}{31.25} \times 100\% = 0.8\%$

Question 5 (25 marks)

Question 5 (a)

Turning Points (TP): Put $\dfrac{dy}{dx} = 0$ and solve for x.

$\left(\dfrac{d^2y}{dx^2}\right)_{\text{TP}} < 0 \Rightarrow$ Local maximum

$\left(\dfrac{d^2y}{dx^2}\right)_{\text{TP}} > 0 \Rightarrow$ Local minimum

$y = f(x) = x^3 - 6x^2 + 9x$

$\dfrac{dy}{dx} = f'(x) = 3x^2 - 12x + 9$

$\dfrac{d^2y}{dx^2} = f''(x) = 6x - 12$

$\dfrac{dy}{dx} = f'(x) = 3x^2 - 12x + 9$

$\dfrac{dy}{dx} = 0 \Rightarrow 3x^2 - 12x + 9 = 0$

$x^2 - 4x + 3 = 0$

$(x-1)(x-3) = 0$

$\therefore x = 1, 3$

$f(x) = x^3 - 6x^2 + 9x$

$f(1) = (1)^3 - 6(1)^2 + 9(1) = 4 \Rightarrow (1, 4)$ is a TP.

$f(3) = (3)^3 - 6(3)^2 + 9(3) = 0 \Rightarrow (3, 0)$ is a TP.

Deciding which of the turning points is a local maximum or local minimum:

Method 1: Look at the y-coordinates in each turning point.
(1, 4) is a local maximum and (3, 0) is a local minimum, as 4 is greater than 0.

Method 2: Use calculus.

$$\frac{d^2y}{dx^2} = f''(x) = 6x - 12$$

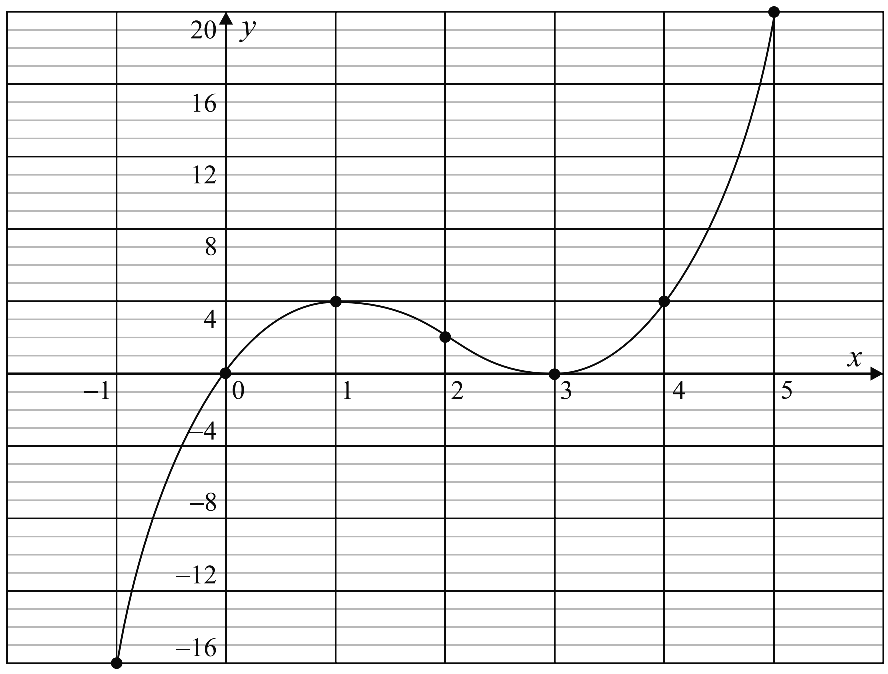
$f''(1) = 6(1) - 12 = -6 \Rightarrow (1, 4)$ is a local maximum.

$f''(3) = 6(3) - 12 = 6 \Rightarrow (3, 0)$ is a local minimum.

Question 5 (b)

x	-1	0	1	2	3	4	5
y	-16	0	4	2	0	4	20

Question 5 (c)



Question 5 (d)

$$\boxed{ax + by + c = 0 \Rightarrow \text{Slope: } m = -\frac{a}{b}}$$

$t: 9x - y + 3 = 0$

$m = 9$

$f(x) = x^3 - 6x^2 + 9x$

$f'(x) = \frac{dy}{dx} = 3x^2 - 12x + 9$

$\frac{dy}{dx} = 9 \Rightarrow 3x^2 - 12x + 9 = 9$

$3x^2 - 12x = 0$

$x(x-4) = 0$

$\therefore x = 0, 4$

$f(x) = x^3 - 6x^2 + 9x$

$f(0) = (0)^3 - 6(0)^2 + 9(0) = 0$

$f(4) = (4)^3 - 6(4)^2 + 9(4) = 64 - 96 + 36 = 4$

(0, 0) and (4, 4) are the points of contact of these two tangents.

$t_1: m = 9$, point$(x_1, y_1) = (0, 0)$

$y - 0 = 9(x - 0)$

$y = 9x$

$9x - y = 0$

$t_2: m = 9$, point$(x_1, y_1) = (4, 4)$

$y - 4 = 9(x - 4)$

$y - 4 = 9x - 36$

$9x - y - 32 = 0$

Question 6 (25 marks)

Question 6 (a)

$h = 8t^2 - 80t + 250$

$\frac{dh}{dt} = 16t - 80$

$\frac{dh}{dt} = 0 \Rightarrow 16t - 80 = 0$

$16t = 80$

$\therefore t = 5$ s

Question 6 (b)

$h = 8t^2 - 80t + 250$

$h_{\text{Min}} = 8(5)^2 - 80(5) + 250$

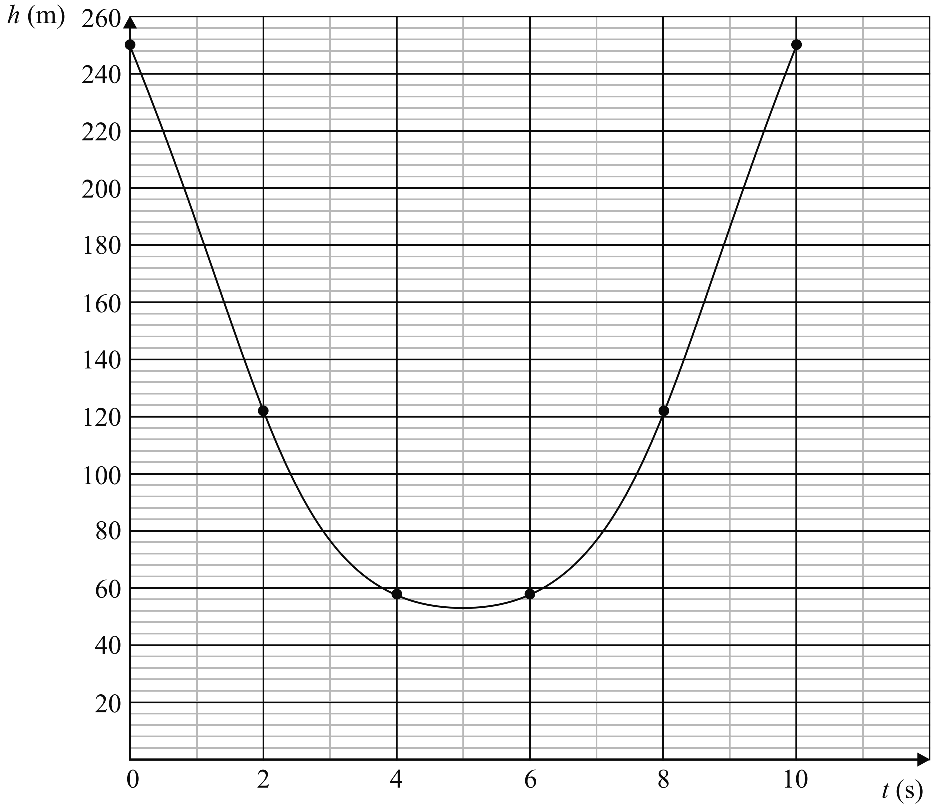
$= 200 - 400 + 250$

$= 50$ m

Question 6 (c)

t (s)	0	2	4	6	8	10
h (m)	250	122	58	58	122	250

Question 6 (d)

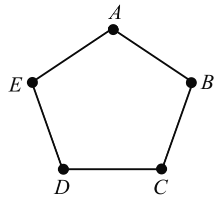


Sample 7 Paper 1

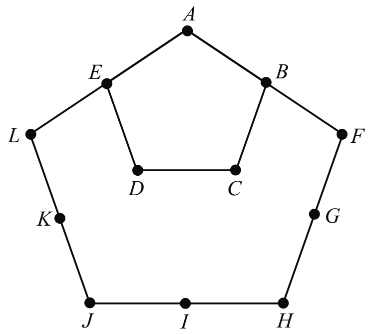
Question 7 (50 marks)

Question 7 (a)

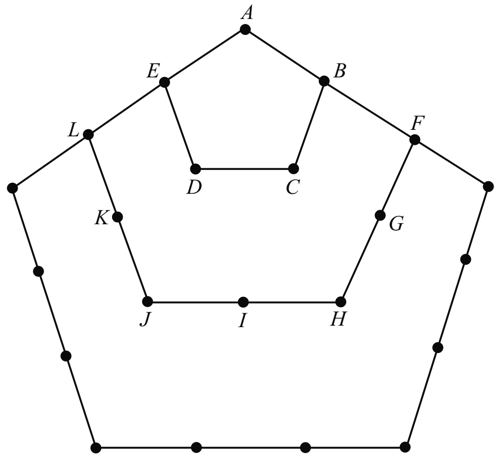
$1^{st.}$ pattern



$2^{nd.}$ pattern



$3^{rd.}$ pattern



$4^{th.}$ pattern

Question 7 (b)

	$1^{st.}$ pattern	$2^{nd.}$ pattern	$3^{rd.}$ pattern	$4^{th.}$ pattern
Branch 1	• 1	• 1	• 1	1
Branch 2		•••• 4	•••• 4	4
Branch 3			••••••• 7	7
Branch 4				10
Total	1	5	12	22

Question 7 (c)

Fifth: 1 + 4 + 7 + 10 +13 = 35
Sixth: 1 + 4 + 7 + 10 +13 + 16 = 51

Question 7 (d)

Arithmetic series: $1+4+7+....$

$a=1,\ d=3$

$$S_{50}=\tfrac{50}{2}[2(1)+(50-1)(3)] \quad \boxed{S_n=\tfrac{n}{2}[2a+(n-1)d]}$$
$$=25[2+49(3)]$$
$$=25[149]$$
$$=3725$$

Question 7 (e)

Arithmetic series: $1+4+7+....$

$a=1,\ d=3$

$$S_n = \tfrac{n}{2}[2(1)+(n-1)(3)]$$
$$= \tfrac{n}{2}[2+3n-3]$$
$$= \tfrac{n}{2}(3n-1)$$

Question 7 (f) (i)

$1335 = \tfrac{n}{2}(3n-1)$

$2670 = 3n^2 - n$

$0 = 3n^2 - n - 2670$

$0 = (3n+89)(n-30)$

$\therefore n = 30,\ \cancel{-\tfrac{89}{3}}$

1335 is in the sequence because 30 is a whole number.

Question 7 (f) (ii)

$98 = \tfrac{n}{2}(3n-1)$

$196 = 3n^2 - n$

$0 = 3n^2 - n - 196$

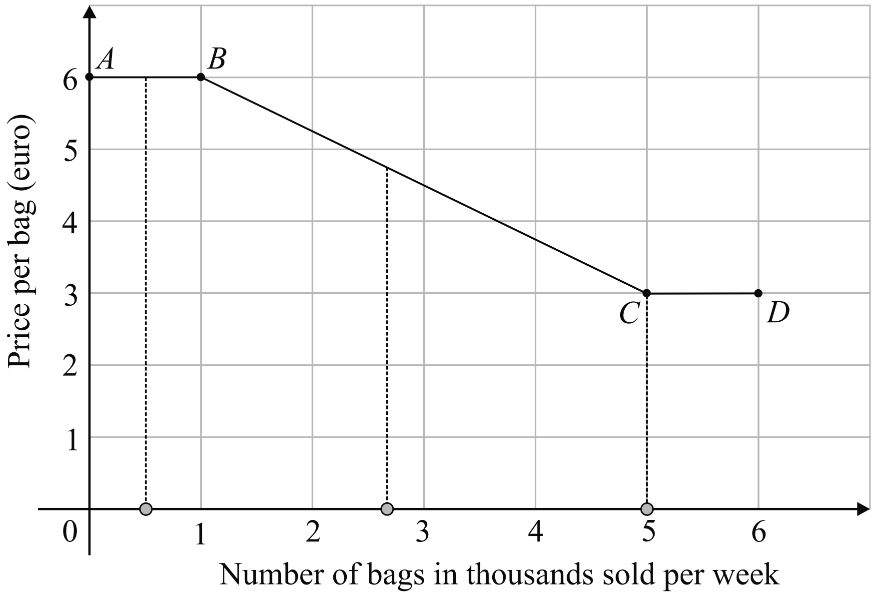
$$n = \frac{1 \pm \sqrt{1+4(3)(196)}}{6} = 8.25$$

98 is not in the sequence because 8.25 is not a whole number.

Sample 7 Paper 1

QUESTION 8 (50 MARKS)

Question 8 (a)



(i) *AB*: For 0–1000 bags per week, the price is €6 per bag.
(ii) *BC*: For 1000–5000 bags per week, the price decreases from €6 to €3.
(iii) *CD*: For 5000–6000 bags per week, the price is €3 per bag.

Question 8 (b)

$B(1, 6)$, $C(5, 3)$

Slope of BC: $m = \dfrac{3-6}{5-1} = \dfrac{-3}{4} = -\dfrac{3}{4}$ $\quad \boxed{m = \dfrac{y_2 - y_1}{x_2 - x_1}}$

Equation of BC: $y - 6 = -\tfrac{3}{4}(x-1)$ $\quad \boxed{y - y_1 = m(x - x_1)}$

$$4y - 24 = -3x + 3$$
$$3x + 4y - 27 = 0$$

$x = 2.8$: $3(2.8) + 4y - 27 = 0$
$$8.4 + 4y = 27$$
$$4y = 27 - 8.4 = 18.6$$
$$y = 4.65$$
$\therefore y = €4.65$

Question 8 (c)

First 10 weeks: 500 bags
500 bags costing €6 each for 10 weeks: Income = 500 × 10 × €6 = €30 000

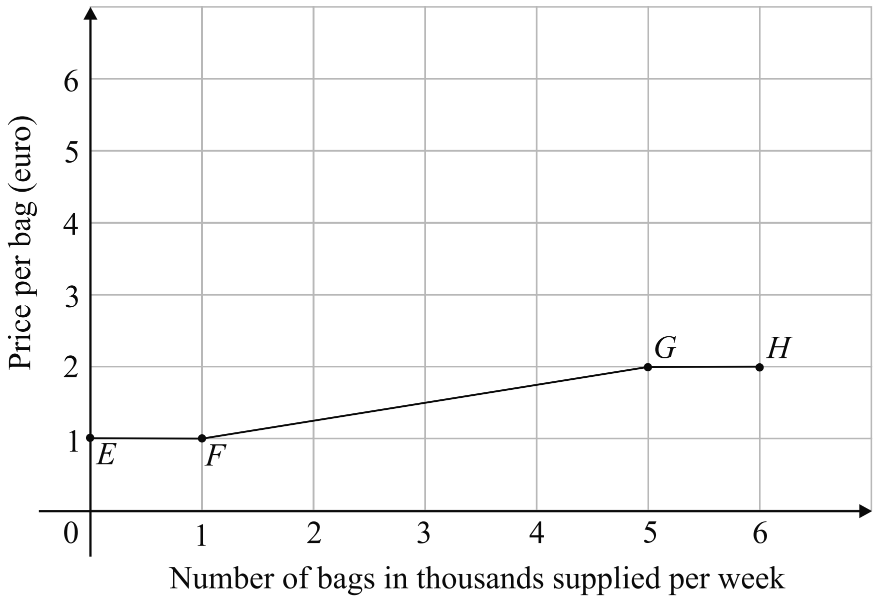
Next 30 weeks: 2800 bags
2800 bags costing €4.65 each for 30 weeks: Income = 2800 × 30 × €4.65 = €390 600

Final 12 weeks: 5000 bags
5000 bags costing €3 each for 12 weeks: Income = 5000 × 12 × €3 = €180 000

Number of weeks	Number of bags per week	Price per bag (€)	Total (€)
10	500	6	30 000
30	2800	4.65	390 600
12	5000	3	180 000
			600 600

Total income = €30 000 + €390 600 + €180 000 = €600 600

Question 8 (d)



(i) *EF*: For 0–1000 bags per week, the price is €1 per bag.
(ii) *FG*: For 1000–5000 bags per week, the price increases from €1 to €2.
(iii) *GH*: For 5000–6000 bags per week, the price is €2 per bag.

Question 8 (e)

$F(1, 1)$, $G(5, 2)$

Slope of FG: $m = \dfrac{2-1}{5-1} = \dfrac{1}{4}$

Equation of FG: $y-1 = \frac{1}{4}(x-1)$

$4y-4 = x-1$

$0 = x-4y+3$

$\boxed{m = \dfrac{y_2 - y_1}{x_2 - x_1}}$

$\boxed{y - y_1 = m(x - x_1)}$

$x = 2.8$: $(2.8)-4y+3=0$

$5.8 = 4y$

$y = 1.45$

$\therefore y = €1.45$

Question 8 (f) (i)

0–1000 bags:
Income per bag sold = €6
Cost per bag supplied = €1
Profits per bag = €6 – €1 = €5

Question 8 (f) (ii)

5000–6000 bags:
Income per bag sold = €3
Cost per bag supplied = €2
Profits per bag = €3 – €2 = €1

Sample 7 Paper 1

QUESTION 9 (50 MARKS)

Question 9 (a) (i)

$x + x + h = 18$

$\therefore h = 18 - 2x$

Question 9 (a) (ii)

$V = (x)(x)(h)$

$= x^2(18 - 2x)$

$= 18x^2 - 2x^3$

Question 9 (a) (iii)

$V = 18x^2 - 2x^3$

$\frac{dV}{dx} = 36x - 6x^2$

$\frac{dV}{dx} = 0 \Rightarrow 36x - 6x^2 = 0$

$6x - x^2 = 0$

$x(6 - x) = 0$

$\therefore x = 6 \text{ m}$

Question 9 (a) (iv)

$V = 18x^2 - 2x^3$

$V_{\text{Max}} = 18(6)^2 - 2(6)^3 = 216 \text{ m}^3$

Question 9 (b) (i)

$V = 195 - 2t - t^2$

$t = 0 : V = 195 - 2(0) - (0)^2 = 195 \text{ m}^3$

Question 9 (b) (ii)

$V = 195 - 2t - t^2$

$t = 10 : V = 195 - 2(10) - (10)^2 = 75 \text{ m}^3$

Question 9 (b) (iii)

$V = 195 - 2t - t^2$

$V = 0 : 0 = 195 - 2t - t^2$

$t^2 + 2t - 195 = 0$

$0 = (t - 13)(t + 15)$

$\therefore t = 13 \text{ s}$

Question 9 (b) (iv)

$V = 195 - 2t - t^2$

$\frac{dV}{dt} = -2 - 2t$

$\left(\frac{dV}{dt}\right)_{t=7} = -2 - 2(7) = -16 \text{ m}^3 \text{ min}^{-1}$

Question 9 (b) (v)

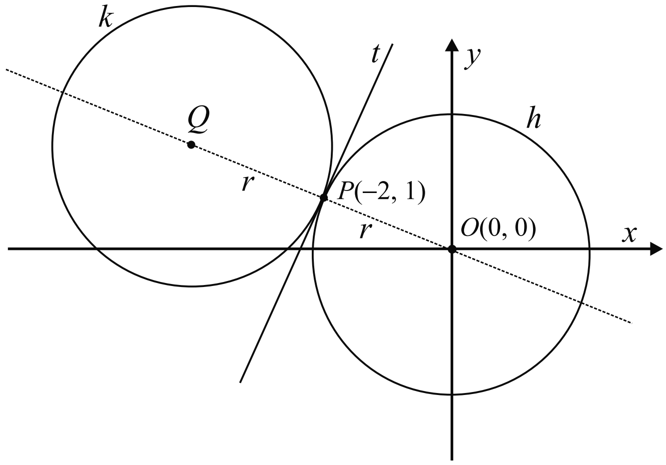
$\frac{dV}{dt} = -2 - 2t < 0$

The rate of change of volume V is negative for all values of t. Therefore, the volume is always decreasing.

Question 1 (25 marks)

Question 1 (a)

$$\boxed{|OP| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}}$$



$O(0, 0)$, $P(-2, 1)$

$|OP| = r = \sqrt{(-2-0)^2 + (1-0)^2}$

$= \sqrt{4+1} = \sqrt{5}$

$$\boxed{(x-h)^2 + (y-k)^2 = r^2}$$

Equation of h: $(h, k) = (0, 0)$, $r = \sqrt{5}$

$(x-0)^2 + (y-0)^2 = (\sqrt{5})^2$

$x^2 + y^2 = 5$

Question 1 (b)

Find the centre Q of k by a central symmetry of (0, 0) through $P(-2, 1)$.
$O(0, 0) \rightarrow P(-2, 1) \rightarrow Q(-4, 2)$ [Subtract 2, Add 1]

Equation of k: $(h, k) = (-4, 2)$, $r = \sqrt{5}$

$(x-(-4))^2 + (y-2)^2 = (\sqrt{5})^2$

$(x+4)^2 + (y-2)^2 = 5$

Question 1 (c)

t is perpendicular to OQ.

Slope of OQ: $O(0, 0)$, $Q(-4, 2)$

$m_1 = \dfrac{2-0}{-4-0} = \dfrac{2}{-4} = -\dfrac{1}{2}$ $\quad \boxed{m = \dfrac{y_2 - y_1}{x_2 - x_1}}$

$\perp$ slope of t: $m_2 = 2$

Equation of t: $(x_1, y_1) = P(-2, 1)$, $m_2 = 2$ $\quad \boxed{y - y_1 = m(x - x_1)}$

$y - 1 = 2(x-(-2))$

$y - 1 = 2x + 4$

$0 = 2x - y + 5$

QUESTION 2 (25 MARKS)

Question 2 (a) (i)

$$\boxed{m=\frac{y_2-y_1}{x_2-x_1}}$$

Slope of AC: $A(3, 3)$, $C(-1, 5)$

$$m_1=\frac{5-3}{-1-3}=\frac{2}{-4}=-\frac{1}{2}$$

Slope of BC: $B(-3, 1)$, $C(-1, 5)$

$$m_2=\frac{5-1}{-1-(-3)}=\frac{4}{2}=2$$

$$m_1\times m_2=-\tfrac{1}{2}\times 2=-1\Rightarrow AC\perp BC$$

Question 2 (a) (ii)

$$\boxed{|PQ|=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}}$$

$A(3, 3)$, $C(-1, 5)$

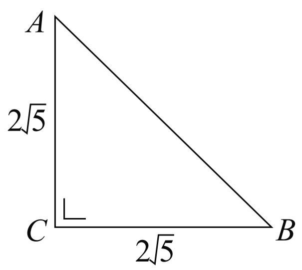
$$|AC|=\sqrt{(-1-3)^2+(3-5)^2}$$
$$=\sqrt{16+4}=\sqrt{20}=2\sqrt{5}$$

Slope of BC: $B(-3, 1)$, $C(-1, 5)$

$$|BC|=\sqrt{(-1-(-3))^2+(5-1)^2}$$
$$=\sqrt{4+16}=\sqrt{20}=2\sqrt{5}$$

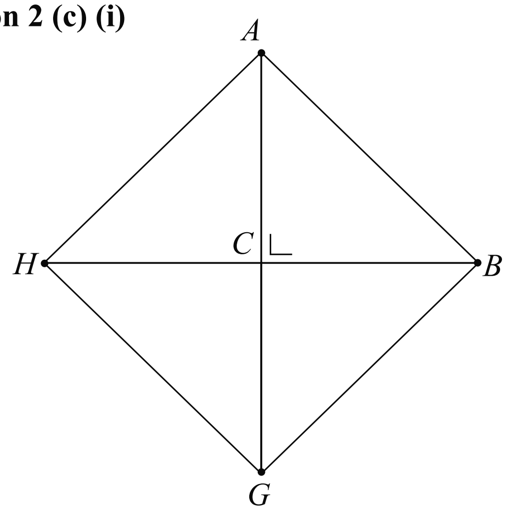
$$\therefore |AC|=|BC|$$

Question 2 (b)



$$\text{Area}=\tfrac{1}{2}bh=\tfrac{1}{2}(2\sqrt{5})(2\sqrt{5})=10$$

Question 2 (c) (i)



$B(-3, 1)\rightarrow C(-1, 5)\rightarrow H(1, 9)$ [Add 2, Add 4]

$A(3, 3)\rightarrow C(-1, 5)\rightarrow G(-5, 7)$ [Subtract 4, Add 2]

Question 2 (c) (ii)

Equation of BC: $(x_1,\ y_1)=B(-3, 1),\ m=2$ $\boxed{y-y_1=m(x-x_1)}$

$$y-1=2(x-(-3))$$
$$y-1=2(x+3)$$
$$y-1=2x+6$$
$$0=2x-y+7$$

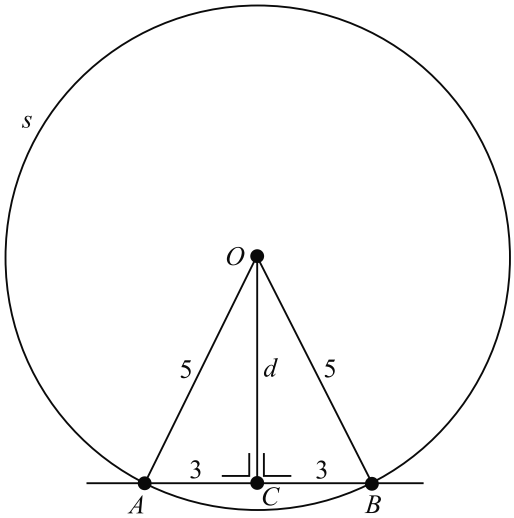
$H(1, 9)\in BC: 2x-y+7=0?$

$$2(1)-9+7$$
$$=2-9+7$$
$$=0\ (\text{Yes, } H(1, 9)\in BC)$$

Question 3 (25 marks)

Question 3 (a) (i)

Theorem
(1) The perpendicular from the centre to a chord bisects the chord.
(2) The perpendicular bisector of a chord passes through the centre.



$3^2 + d^2 = 5^2$

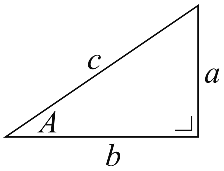
$9 + d^2 = 25$

$d^2 = 25 - 9$

$d^2 = 16$

$\therefore d = \sqrt{16} = 4$

Pythagoras



$\boxed{\sin A = \frac{a}{c}}$

$\boxed{c^2 = a^2 + b^2}$

Question 3 (a) (ii)

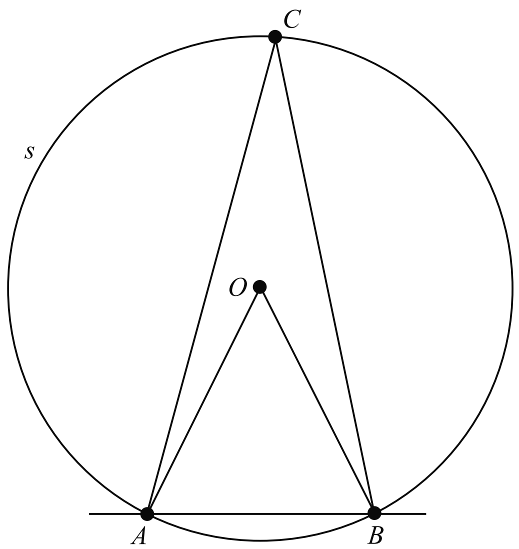
$$\sin|\angle AOC| = \frac{3}{5}$$

$$\therefore |\angle AOC| = \sin^{-1}\left(\frac{3}{5}\right) = 36.87^\circ$$

$$|\angle AOB| = 2|\angle AOC| = 2 \times 36.87^\circ = 73.74^\circ$$

Question 3 (b)

Theorem: The angle at the centre of a circle standing on a given arc is twice the angle at any point of the circle standing on the same arc.



$$|\angle AOB| = 2|\angle ACB|$$

$$\therefore |\angle ACB| = \tfrac{1}{2} \times 73.74^\circ = 36.87^\circ$$

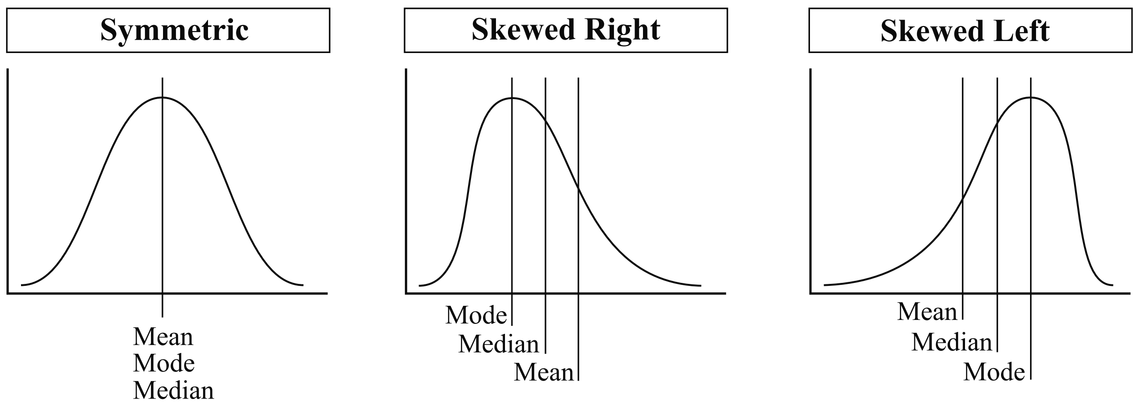
QUESTION 4 (25 MARKS)

Question 4 (a)

1. Frequency	A. Type of numerical data that could take on any real number in an interval, including decimals.
2. Pie Chart	B. The number of occurrences of an item or an event.
3. Line Plot	C. Shows data on a number line with x, or other marks, to show frequency.
4. Scatter Diagram	D. It is the range of the middle 50% of the data values.
5. Continuous Data	E. A chart that uses sectors of a disk to represent what fraction of the data falls into different categories.
6. Interquartile Range	F. $25^{th.}$ percentile
7. Univariate Data Set	G. A chart that displays whether there is a relationship between two random variables.
8. First Quartile	H. A data set in which one measurement has been made on each item.

1.	2.	3.	4.	5.	6.	7.	8.
B	E	C	G	A	D	H	F

Question 4 (b)



Question 4 (c)

(i) Heights of males aged between 18 and 30: Normal

(ii) Salaries of 20–65-year-olds working in a company: Positively skewed (skewed right)

(iii) The life expectancy of people in a number of countries: Negatively skewed (skewed left)

Question 5 (25 marks)

Question 5 (a)

24	28	27	26	27	20
30	25	31	33	28	29
24	32	30	40	36	28
27	28	24	42	20	32
33	26	31	35	44	22

2	0	0	2	4	4	4	5	6	6	7	7	7	8	8	8	8	9
3	0	0	1	1	2	2	3	3	5	6							
4	0	2	4														

Key: 2|4 = 24

Question 5 (b)

It is handy for putting numbers into numerical order without writing the whole numbers. It shows the range, minimum and maximum, gaps and clusters, and outliers easily.

Question 5 (c)

2	0	0	2	**4**	**4**	**4**	5	6	6	7	7	7	8	8	8	8	9
3	0	0	1	1	2	2	3	3	5	6							
4	0	2	4														

Three students received €24 per month.

Question 5 (d)

2	0	0	2	4	4	4	5	6	6	7	7	7	**8**	**8**	**8**	**8**	9
3	0	0	1	1	2	2	3	3	5	6							
4	0	2	4														

The modal amount is €28 per month.

Question 5 (e)

2	0	0	2	4	4	4	5	6	6	7	7	7	8	8	$\boxed{8}$	$\boxed{8}$	9
3	0	0	1	1	2	2	3	3	5	6							
4	0	2	4														

The middle (median) students are the $15^{\text{th.}}$ and $16^{\text{th.}}$ students.

$$\text{Median} = \frac{€28 + €28}{2} = €28$$

Question 6 (25 marks)

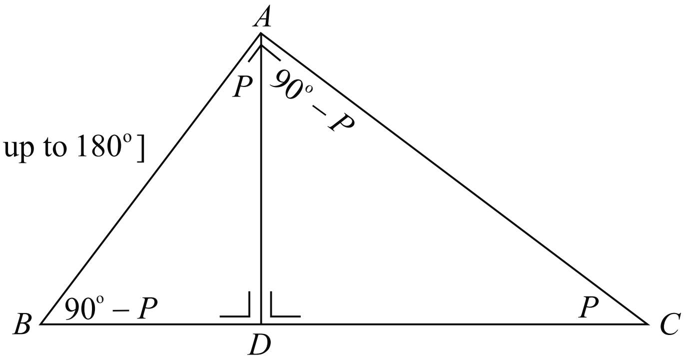
Question 6 (a)

Consider triangle ACD:

Let $P = |\angle ACD|$.

$|\angle CAD| + P + 90^\circ = 180^\circ$ [Three angles add up to 180°]

$\therefore |\angle CAD| = (90^\circ - P)$



Consider triangle ABD:

$|\angle BAD| = 90^\circ - (90^\circ - P) = 90^\circ - 90^\circ + P = P$

$P + |\angle DBA| + 90^\circ = 180^\circ$

$\therefore |\angle DBA| = 180^\circ - 90^\circ - P = (90^\circ - P)$

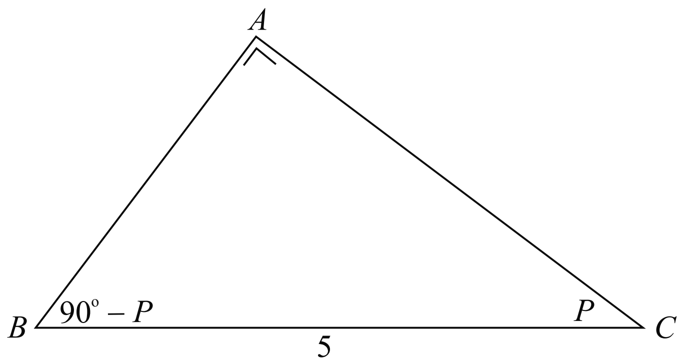
Therefore, triangles ABC and ADC are similar.

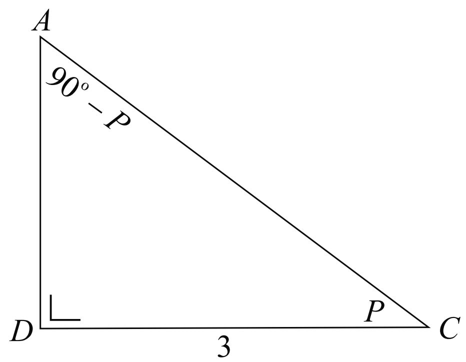
Question 6 (b)

Theorem

If two triangles ΔABC and $\Delta A'B'C'$ are similar, then their sides are proportional, in order:

$$\frac{|AB|}{|A'B'|} = \frac{|BC|}{|B'C'|} = \frac{|CA|}{|C'A'|}.$$





$\dfrac{|BC|}{|AC|} = \dfrac{|AC|}{|DC|} \Rightarrow \dfrac{5}{|AC|} = \dfrac{|AC|}{3}$

$\therefore 15 = |AC|^2$

$|AC| = \sqrt{15}$

$|AB|^2 + |AC|^2 = |BC|^2$

$|AB|^2 + (\sqrt{15})^2 = 5^2$

$|AB|^2 = 25 - 15 = 10$

$\therefore |AB| = \sqrt{10}$

$|AD|^2 + |DC|^2 = |AC|^2$

$|AD|^2 = (\sqrt{15})^2 - 3^2$

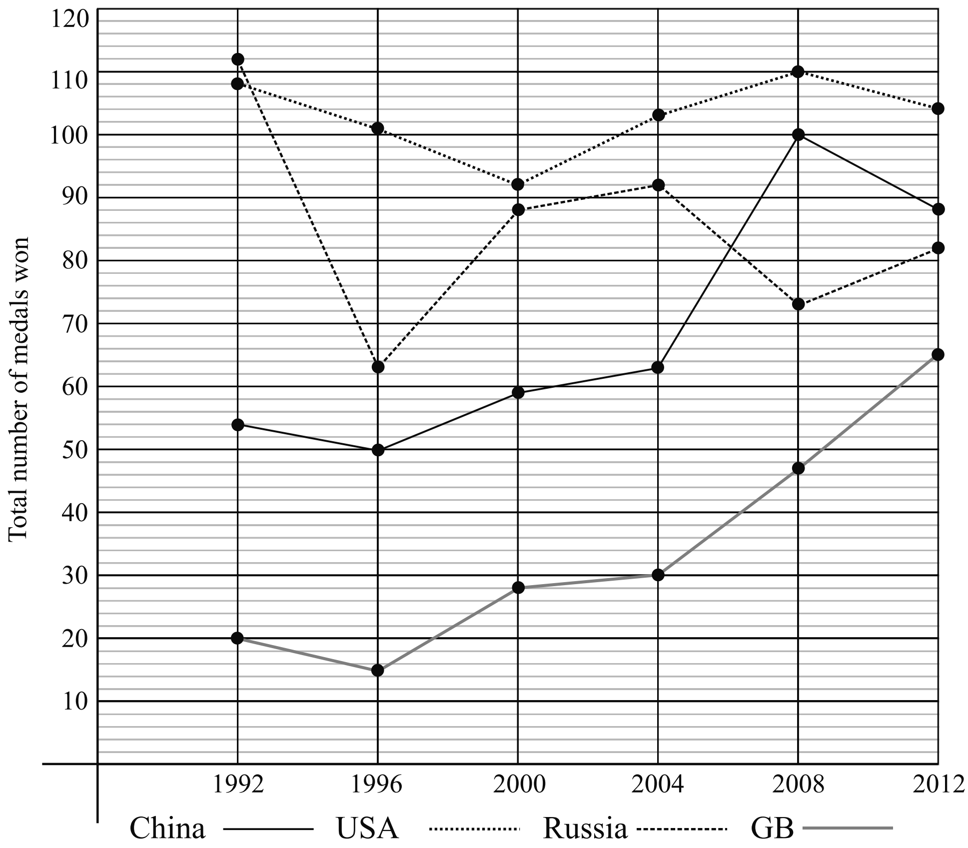
$|AD|^2 = 15 - 9 = 6$

$\therefore |AD| = \sqrt{6}$

Question 7 (75 marks)

Question 7 (a)

	China	USA	Russia	GB
London (2012)	88	104	82	65
Beijing (2008)	100	110	73	47
Athens (2004)	63	103	92	30
Sydney (2000)	59	92	88	28
Atlanta (1996)	50	101	63	15
Barcelona (1992)	54	108	112	20



China: There is a steady rise in the number of medals won as the economy grows.

USA: Relatively steady over the years.

Russia: Rapid fall in medals from 1992–1996 as the Soviet Union breaks up into individual states.

GB: Big improvement from Atlanta to the last Olympics in London can be attributed to hosting the Olympics in 2012, and a strategic investment in areas likely to bring success such as cycling.

Sample 7 Paper 2

Question 7 (b)

	China	USA	Russia	GB
London (2012)	88	104	82	65
Beijing (2008)	100	110	73	47
Athens (2004)	63	103	92	30
Sydney (2000)	59	92	88	28
Atlanta (1996)	50	101	63	15
Barcelona (1992)	54	108	112	20
				205

(i) Number of medals = 20 + 15 + 28 + 30 + 47 + 65 = 205

(ii) % increase $= \left(\dfrac{65-20}{20}\right) \times 100\% = 225\%$

(iii) Mean $= \dfrac{205}{6} \approx 34$

(iv) 1. Russia (112), 2. USA (108), 3. China (54)

(v) USA: $36 \times 3 + 38 \times 2 + 36 \times 1 = 220$ points

China: $51 \times 3 + 21 \times 2 + 28 \times 1 = 223$ points

No, China would be ahead by 223 points to 220 points.

Question 7 (c)

18 Gold 2 Silver 2 Bronze

22 Medals

(i) $P(\text{Gold}) = \dfrac{\text{Number of Gold medals}}{\text{Number of medals}} = \dfrac{18}{22} = \dfrac{9}{11}$

(ii) $P(\text{Silver or Bronze}) = \dfrac{\text{Number of Silver or Bronze medals}}{\text{Number of medals}} = \dfrac{4}{22} = \dfrac{2}{11}$

(iii) $P(\text{Gold})$ **and then** $P(\text{Silver})$ **and then** $P(\text{Bronze}) = \dfrac{9}{11} \times \dfrac{1}{11} \times \dfrac{1}{11} = \dfrac{9}{1331}$

(iv) GGG, GGS, GGB, GSS, GBB, GSB, SSB, SBB

Question 7 (d)

7.6	1	9					
7.7	6						
7.8	9						
7.9	0	3	8				
8.0							
8.1	3						
8.2	1	**4**	**8**				
8.3	1	1	4	5	5	9	9
8.4							
8.5							
8.6							
8.7							
8.8							
8.9	0	5					

Key: 8.2|1 = 8.21 m

(i) Range R = 8.95 m − 7.61 m = 1.34 m

(ii) Median $= \dfrac{8.24+8.28}{2} = 8.26$ m [The median is in the 10th. and 11th. positions.]

(iii) 13 world records are over 8 m.

Percentage of world records over 8 m $= \dfrac{13}{20} \times 100\% = 65\%$

(iv) Most records are under 8.4 m. There are two remarkable records at 8.9 m and 8.95 m.

Question 7 (e) (i)

Area of gold region:

$r = 12.2$ cm

$\text{Area} = \pi r^2 = \pi(12.2)^2 = 467.6 \text{ cm}^2$

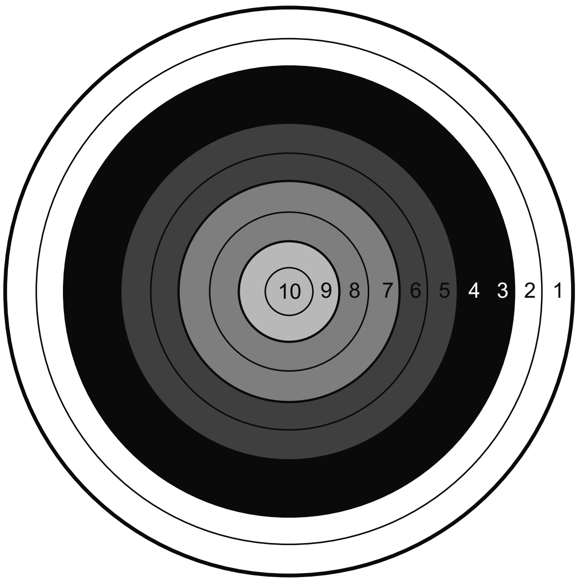
Area of black region:

Area of black circle − Area of blue circle

$\text{Area} = \pi(6.1\times 8)^2 - \pi(6.1\times 6)^2 = 3273.2 \text{ cm}^2$

$$\frac{\text{Area of black region}}{\text{Area of gold region}} = \frac{3273.2}{467.6} = 7$$

The arrow is seven times more likely to hit the black region.



Question 7 (e) (ii)

Score x	10	9	8	7	6	5	4	3	2	1
Probability $P(x)$	0.38	0.4	0.12	0.08	y	0.004	0.003	0.002	.0006	.0004
$xP(x)$										

Sample 7 Paper 2

The probabilities add to 1.

0.38 + 0.4 + 0.12 + 0.08 + y + 0.004 + 0.003 + 0.002 + 0.0006. + 0.0004 = 1

$y = 0.01$

Question 7 (e) (iii)

Score x	10	9	8	7	6	5	4	3	2	1
Probability $P(x)$	0.38	0.4	0.12	0.08	0.01	0.004	0.003	0.002	.0006	.0004
$xP(x)$	3.8	3.6	0.96	0.56	0.06	0.02	0.012	0.006	.0012	.0004

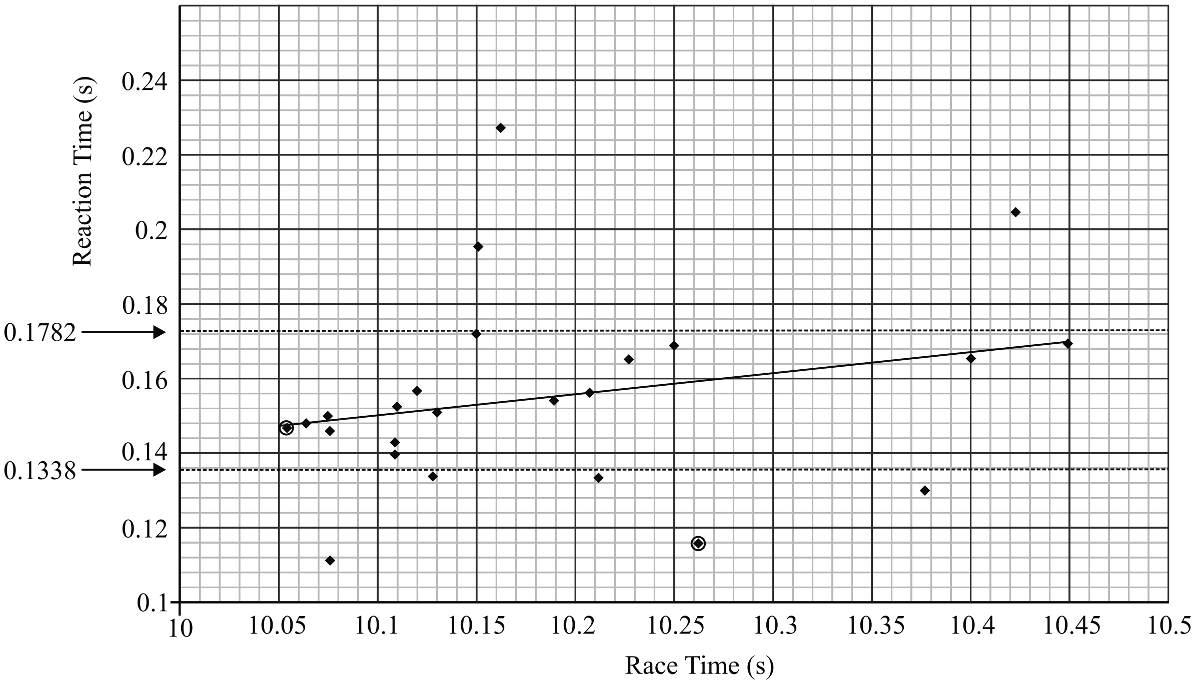
Question 7 (e) (iv)

Average score per arrow $\mu = \sum xP(x) = 9.0196$

Question 7 (e) (v)

Expected score (15 arrows) $= 15 \times 9.0196 = 135.294$

Question 7 (f)



(i) Number of reaction times = 24

(ii) Athlete with fastest race time is circled (left). Race time: 10.055 s, reaction time: 0.148 s

(iii) There are 16 athletes within one standard deviation of the mean, as shown by the two broken lines.

$\mu = 0.156$ s, $\sigma = 0.0222$ s

$\mu + \sigma = 0.156 + 0.0222 = 0.1782$ s

$\mu - \sigma = 0.156 - 0.0222 = 0.1338$ s

% of athletes within 1 SD

$= \frac{16}{24} \times 100\% = 67\%$

A normal distribution predicts that **68%** of the data lies within one standard deviation of the mean.

(iv) The athlete circled (to the right) had a quick starting reaction time but a slow race.
Race time: 10.26 s,
Reaction time: 0.116 s

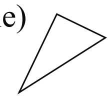
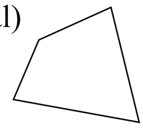
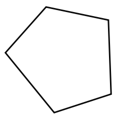
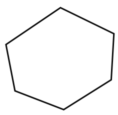
QUESTION 8 (75 MARKS)

Question 8 (a)

(i) A triangle is a three-sided polygon. The sum of the three interior angles in a triangle in degrees is **180°**.

(ii) A triangle with three equal sides and angles is called an **equilateral** triangle.
Each angle in degrees in this triangle is equal to **60°**.

Question 8 (b)

POLYGON	SUM OF THE INTERIOR ANGLES
Three-sided polygon (Triangle) 	180°
Four-sided polygon (Quadrilateral) 	360°
Five-sided polygon 	540°
Six-sided polygon 	720°

A four-sided polygon can be divided into two triangles, giving a sum of $2 \times 180^{\circ} = 360^{\circ}$ for its interior angles.
A five-sided polygon can be divided into three triangles, giving a sum of $3 \times 180^{\circ} = 540^{\circ}$ for its interior angles.

Question 8 (c)

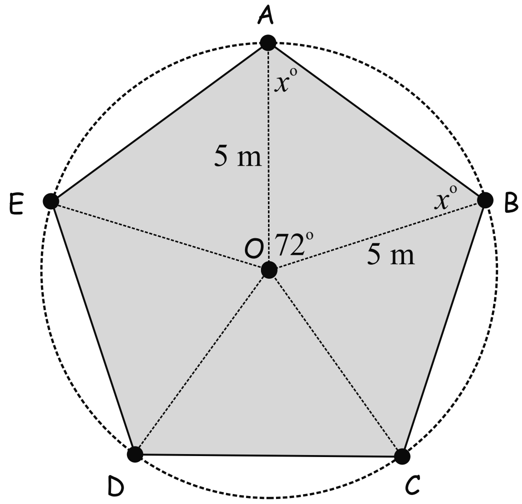
	NAME	SIZE OF EACH INTERIOR ANGLE
Three-sided regular polygon	equilateral triangle	60°
Four-sided regular polygon	square	90°
Five-sided regular polygon	pentagon	108°
Six-sided regular polygon	hexagon	120°

Question 8 (d)

Sum of the interior angles $= (n-2) \times 180^{\circ}$

Measure of each interior angle $= \dfrac{(n-2) \times 180^{\circ}}{n}$

Question 8 (e)



(i) $|\angle AOB| = \dfrac{360^\circ}{5} = 72^\circ$

(ii) Triangle AOB is an isosceles triangle as two sides (the radii) have the same length.
Therefore, the base angles are equal in measure.

$72^\circ + x^\circ + x^\circ = 180^\circ$

$2x^\circ = 180^\circ - 72^\circ = 108^\circ$

$\therefore x = 54^\circ$

$|\angle OAB| = |\angle OBA| = 54^\circ$

(iii) $\dfrac{|AB|}{\sin 72^\circ} = \dfrac{5}{\sin 54^\circ}$ $\boxed{\dfrac{a}{\sin A} = \dfrac{b}{\sin B}}$

$\therefore |AB| = \dfrac{5 \sin 72^\circ}{\sin 54^\circ} = 5.9 \text{ m}$

Perimeter $= 5 \times 5.9 = 29.4$ m

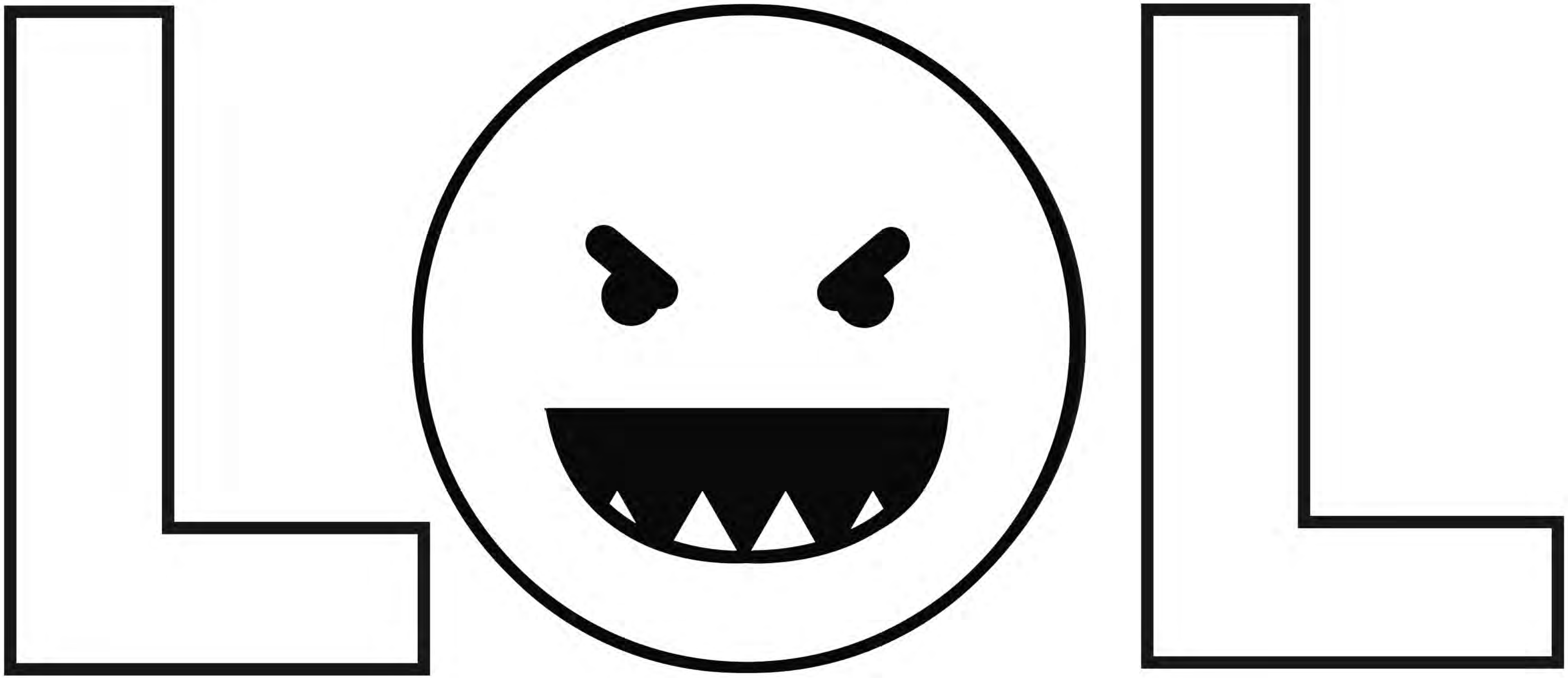
(iv) Area of $\Delta AOB = \frac{1}{2}(5)(5)\sin 72^\circ = 11.88 \text{ m}^2$ $\boxed{\text{Area} = \frac{1}{2} ab \sin C}$

Area of flower bed $= 5 \times 11.88 = 59.4 \text{ m}^2$

(v) Number of kilograms of fertiliser $= 59.4 \times 0.75 = 44.55$ kg

Number of bags $= \dfrac{44.55}{1.5} = 29.7 \approx 30$

Cost of fertiliser $= 30 \times €3.45 = €103.50$



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