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MATHEMATICS SOLUTIONS
Leaving Certificate
Higher Level

CONTENTS

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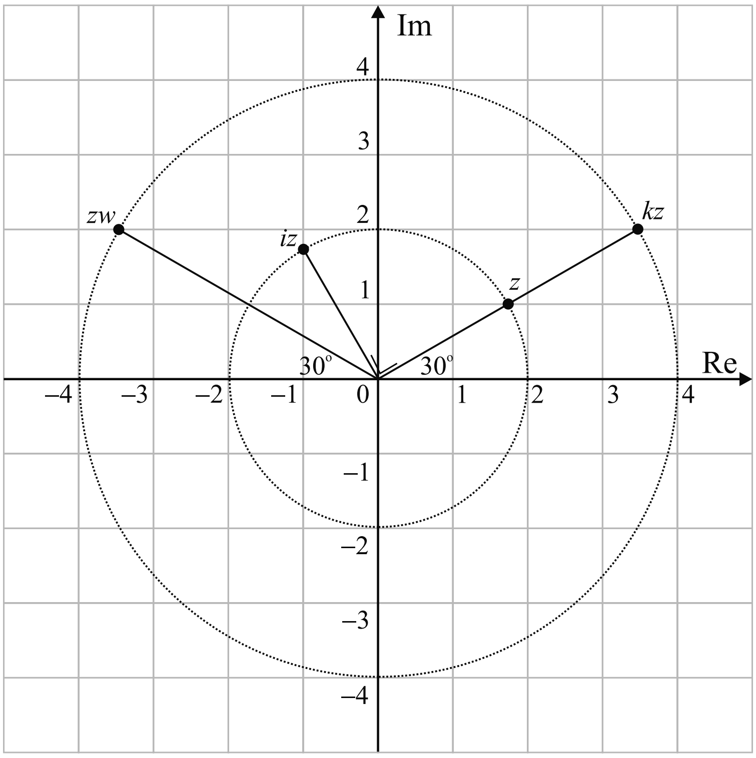
Sample Paper 1: Paper 1

Question 1 (25 marks)

Question 1 (a)

$z=2\left\{\cos\frac{\pi}{6}+i\sin\frac{\pi}{6}\right\}$ $\quad\boxed{z=r(\cos\theta+i\sin\theta)}$

$|z|=2,\ \arg z=\frac{\pi}{6}=30^{\circ}$ $\quad\boxed{|z|=r,\ \arg z=\theta}$



Question 1 (b)

$kz=4\left\{\cos\frac{\pi}{6}+i\sin\frac{\pi}{6}\right\}\Rightarrow|kz|=4,\ \arg kz=\frac{\pi}{6}=30^{\circ}$

Question 1 (c)

Write i as a complex number in polar form: $i=0+1i=1\left(\cos\frac{\pi}{2}+i\sin\frac{\pi}{2}\right)$

$w=iz=1\left\{\cos\frac{\pi}{2}+\sin\frac{\pi}{2}\right\}2\left\{\cos\frac{\pi}{6}+\sin\frac{\pi}{6}\right\}$ [When you multiply complex numbers in polar form you add their arguments.]

$$=2\left\{\cos\left(\frac{\pi}{2}+\frac{\pi}{6}\right)+i\sin\left(\frac{\pi}{2}+\frac{\pi}{6}\right)\right\}$$

$$=2\left\{\cos\frac{2\pi}{3}+i\sin\frac{2\pi}{3}\right\}$$

$|w|=2,\ \arg w=\frac{2\pi}{3}=120^{\circ}$

$\boxed{\text{Multiplying by } i \text{ rotates a complex number by } 90^{\circ} \text{ anticlockwise.}}$

Question 1 (d)

When you multiply complex numbers in polar form you add their arguments.

$$zw = 2\left\{\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right\} 2\left\{\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right\} = 4\left\{\cos\frac{5\pi}{6} + i\sin\frac{5\pi}{6}\right\}$$

$$|zw| = 4, \text{ arg } zw = \frac{5\pi}{6} = 150^{\circ}$$

Question 1 (e)

$$z^{12} = 2^{12}\left\{\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right\}^{12} \quad \boxed{[r(\cos\theta + i\sin\theta)]^n = r^n(\cos n\theta + i\sin n\theta)} \text{ (De Moivre's Theorem)}$$

$$= 2^{12}\left\{\cos\frac{12\pi}{6} + i\sin\frac{12\pi}{6}\right\}$$

$$= 2^{12}\{\cos 2\pi + i\sin 2\pi\}$$

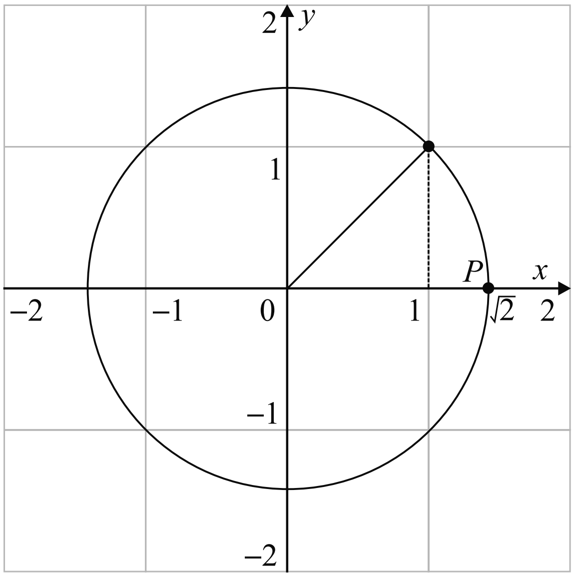
$$= 2^{12}\{1 + i(0)\} = 2^{12}$$

QUESTION 2 (25 MARKS) [THEME: All about irrationals, root 2 and root 3]

Question 2 (a)

Draw a circle with centre (0, 0) through the point (1, 1).
The radius of this circle is
$r = \sqrt{(1-0)^2 + (1-0)^2} = \sqrt{2}.$

$\sqrt{2}$ is the distance of (0, 0) to P on this scale.



Question 2 (b) (i)

$$(\sqrt{3} + \sqrt{2})^2$$
$$= (\sqrt{3})^2 + 2\sqrt{3}\sqrt{2} + (\sqrt{2})^2$$
$$= 3 + 2\sqrt{6} + 2$$
$$= 5 + 2\sqrt{6}$$

Question 2 (b) (ii)

$$(\sqrt{3} - \sqrt{2})(\sqrt{3} + \sqrt{2})$$
$$= 3 + \sqrt{6} - \sqrt{6} - 2 = 1$$

Or (use the difference of 2 squares)

$$(\sqrt{3} - \sqrt{2})(\sqrt{3} + \sqrt{2})$$
$$= (\sqrt{3})^2 - (\sqrt{2})^2$$
$$= 3 - 2 = 1$$

Question 2 (b) (iii)

$$\frac{\sqrt{3} + \sqrt{2}}{\sqrt{3} - \sqrt{2}} = \frac{(\sqrt{3} + \sqrt{2})}{(\sqrt{3} - \sqrt{2})} \times \frac{(\sqrt{3} + \sqrt{2})}{(\sqrt{3} + \sqrt{2})}$$
$$= \frac{5 + 2\sqrt{6}}{1} = 5 + 2\sqrt{6}$$

Question 2 (c) (i)

$$\log_{\sqrt{2}} x + \log_{\sqrt{2}}(x+4) = 4$$

$$\log_{\sqrt{2}} x(x+4) = 4$$

$$x^2 + 4x = (\sqrt{2})^4 = (2^{\frac{1}{2}})^4 = 2^2 = 4$$

$$x^2 + 4x - 4 = 0$$

$$x = \frac{-4 \pm \sqrt{(4)^2 - 4(1)(-4)}}{2(1)} = \frac{-4 \pm \sqrt{16+16}}{2}$$

$$\boxed{x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}}$$

$$= \frac{-4 \pm \sqrt{32}}{2} = \frac{-4 \pm 4\sqrt{2}}{2} = -2 \pm 2\sqrt{2}$$

$$x > 0 \Rightarrow x = 2\sqrt{2} - 2$$

Question 2 (c) (ii)

$$x^2 - 2x > 1$$

$$x^2 - 2x - 1 > 0$$

$x^2 - 2x - 1 = 0$ [Solve the equality using the quadratic formula.]

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-1)}}{2(1)} = \frac{2 \pm \sqrt{4+4}}{2}$$

$$= \frac{2 \pm \sqrt{8}}{2} = \frac{2 \pm 2\sqrt{2}}{2} = 1 \pm \sqrt{2}$$

Carry out the region test, or whatever test you use, to locate the regions that satisfy the inequality:

-1		0		3
$\longleftarrow$	$1-\sqrt{2}$	$\longleftrightarrow$	$1+\sqrt{2}$	$\longrightarrow$
$x^2 - 2x - 1 > 0$	≈ -0.41	$x^2 - 2x - 1 > 0$	≈ 2.41	$x^2 - 2x - 1 > 0$
$(-1)^2 - 2(-1) - 1$		$(0)^2 - 2(0) - 1$		$(3)^2 - 2(3) - 1$
$= 2 > 0$ (True)		$= -1 > 0$ (False)		$= 2 > 0$ (True)

SOLUTIONS: $x < 1 - \sqrt{2},\ x > 1 + \sqrt{2},\ x \in \mathbb{R}$

QUESTION 3 (25 MARKS)

Question 3 (a) (i)

$$\lim_{n\to\infty}\frac{2n}{3n+4}=\lim_{n\to\infty}\frac{2n}{n(3+\frac{4}{n})}=\lim_{n\to\infty}\frac{2}{(3+\frac{4}{n})}=\frac{2}{3}$$

[You can use the technique shown for finding the limit or you can find the limit by inspection.]

Question 3 (a) (ii)

$$\lim_{n\to\infty}2\left(\frac{1}{3}\right)^n=2\lim_{n\to\infty}\left(\frac{1}{3}\right)^n$$

$$=2\times 0=2$$

$$\boxed{\lim_{n\to\infty}r^n=0,\ |r|<1}$$

Question 3 (a) (iii)

$$\lim_{n\to\infty}\frac{1}{3}(2)^n=\frac{1}{3}\lim_{n\to\infty}2^n=\infty$$

$$\boxed{\lim_{n\to\infty}r^n=\infty,\ |r|>1}$$

Question 3 (b) (i)

STEPS FOR PROOF BY INDUCTION

1. Prove result is true for some starting value of $n\in\mathbb{N}$.

2. Assume result is true for $n = k$.

3. Prove result is true for $n = (k+1)$.

Terms of a geometric sequence: $a,\ ar,\ ar^2,\ ar^3,............,\ ar^{n-1}$

1. Prove true for $n = 1$: $S_1=\dfrac{a(1-r^1)}{1-r}=a$ [Therefore, true for $n = 1$.]

2. Assume true for $n = k$: Assume $S_k=a+ar+............+ar^{k-1}=\dfrac{a(1-r^k)}{1-r}$

3. Prove true for $n = k + 1$: Prove $S_{k+1}=a+ar+............+ar^{k-1}+ar^k=\dfrac{a(1-r^{k+1})}{1-r}$

Proof:

$$(a+ar+............+ar^{k-1})+ar^k=\frac{a(1-r^k)}{1-r}+ar^k$$

$$=\frac{a(1-r^k)+ar^k(1-r)}{1-r}=\frac{a-ar^k+ar^k-ar^{k+1}}{1-r}$$

$$=\frac{a(1-r^{k+1})}{1-r}$$

Therefore, assuming true for $n = k$ means it is true for $n = k + 1$. So true for $n = 1$ and true for $n = k$ means it is true for $n = k + 1$. This implies it is true for all $n\in\mathbb{N}$.

Question 3 (b) (ii)

$$\lim_{n\to\infty}S_n=\lim_{n\to\infty}\frac{a(1-r^n)}{1-r}=\frac{a}{1-r}\lim_{n\to\infty}(1-r^n)=\frac{a}{1-r}(1-\lim_{n\to\infty}r^n)$$

$$=\frac{a}{1-r}(1-0)=\frac{a}{1-r}$$

$$\boxed{\lim_{n\to\infty}r^n=0,\ |r|<1}$$

Question 3 (c)

$$0.5\dot{2}\dot{3} = 0.5232323....... = \frac{1}{2} + \left(\frac{23}{1000} + \frac{23}{100\,000} + ...\right) \quad \left[a = \frac{23}{1000},\ r = \frac{1}{100}\right]$$

$$= \frac{1}{2} + \frac{\frac{23}{1000}}{1 - \frac{1}{100}}$$

$$= \frac{1}{2} + \frac{\frac{23}{1000}}{\frac{99}{100}}$$

$$= \frac{1}{2} + \frac{23}{990}$$

$$= \frac{259}{495}$$

QUESTION 4 (25 MARKS)

Question 4 (a)

$$kx^3 - 6x^2 + bx - 6 = (x+1)(x-3)(kx+2)$$
$$kx^3 - 6x^2 + bx - 6 = (x^2 - 2x - 3)(kx+2)$$
$$kx^3 - 6x^2 + bx - 6 = kx^3 + 2x^2 - 2kx^2 - 4x - 3kx - 6$$
$$kx^3 - 6x^2 + bx - 6 = kx^3 + (2-2k)x^2 + (-4-3k)x - 6$$

[A cubic equation equals a linear factor by a linear factor by a linear factor. The coefficients of *x* in each linear factor must multiply together to give *k* and the constants in each linear factor must multiply together to give −6. Find *k* and *b* by lining up the coefficients.]

$$-6 = 2 - 2k$$
$$2k = 8$$
$$\therefore k = 4$$

$$b = -4 - 3k$$
$$b = -4 - 3(4) = -16$$

$$(x+1)(x-3)(4x+2) = 0$$
$$\therefore x = -1,\ -\tfrac{1}{2},\ 3$$

Question 4 (b)

$$\frac{x}{x+1} > 2$$

[Multiply across by $(x + 1)^2$. This is a positive value and so will not reverse the inequality.]

$$\frac{x}{x+1}(x+1)^2 > 2(x+1)^2$$
$$x(x+1) - 2(x+1)^2 > 0$$
$$(x+1)[x - 2(x+1)] > 0$$
$$(x+1)[x - 2x - 2] > 0$$
$$(x+1)[-x-2] > 0$$

Solve the equality:

$$(x+1)(-x-2) = 0$$
$$\therefore x = -2,\ -1$$

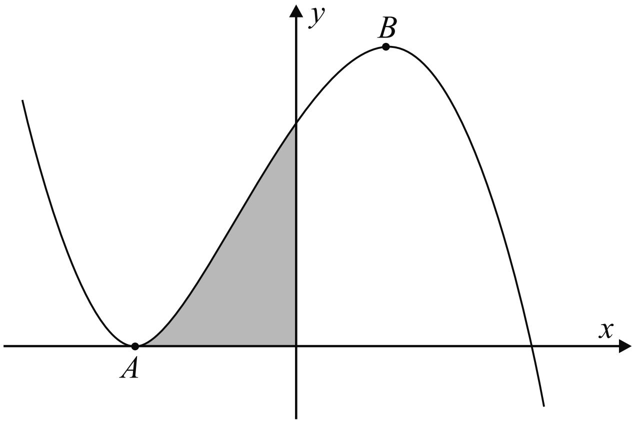
Carry out the region test, or whatever test you use, to locate the regions that satisfy the inequality:

-3		$-\frac{3}{2}$		0
$\longleftarrow$	-2	$\longleftrightarrow$	-1	$\longrightarrow$
$(x+1)(-x-2) > 0$		$(x+1)(-x-2) > 0$		$(x+1)(-x-2) > 0$
$(-2)(1) > 0$ (False)		$(-\frac{1}{2})(-\frac{1}{2}) > 0$ (True)		$(1)(-2) > 0$ (False)

SOLUTIONS: $-2 < x < -1,\ x \in \mathbb{R}$

QUESTION 5 (25 MARKS)

Question 5 (a)



$$f(x) = (x+2)^2(5-2x)$$

$$\begin{aligned} f'(x) &= (x+2)^2(-2) + (5-2x)2(x+2)^1 \\ &= 2(x+2)[-(x+2)+(5-2x)] \\ &= 2(x+2)[-x-2+5-2x] \\ &= 2(x+2)[-3x+3] \\ &= 6(x+2)(1-x) \end{aligned}$$

$$\begin{aligned} f''(x) &= 6\{(x+2)(-1)+(1-x)(1)\} \\ &= 6\{-x-2+1-x\} \\ &= 6\{-2x-1\} \\ &= -6(2x+1) \end{aligned}$$

$f'(x) = 0 \Rightarrow 6(x+2)(1-x) = 0$

$\therefore x = -2, 1$

TO FIND TURNING POINTS (TP):
Put $\dfrac{dy}{dx} = f'(x) = 0$ and solve for x.

$f''(-2) = -6(-3) = 18 > 0 \Rightarrow$ Local Minimum

$f''(1) = -6(3) = -18 > 0 \Rightarrow$ Local Maximum

$\left(\dfrac{d^2y}{dx^2}\right)_{\text{TP}} < 0 \Rightarrow$ Local Maximum

$\left(\dfrac{d^2y}{dx^2}\right)_{\text{TP}} > 0 \Rightarrow$ Local Minimum

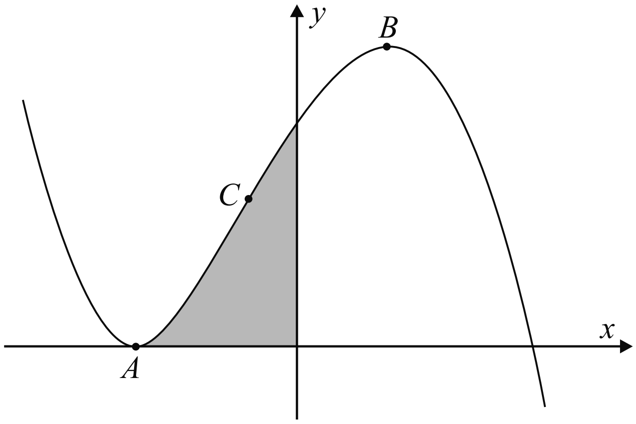
$f(x) = (x+2)^2(5-2x)$

$f(-2) = (-2+2)^2(5-2(-2)) = 0 \Rightarrow A(-2, 0)$ is the local minimum.

$f(1) = (1+2)^2(5-2(1)) = (9)(3) = 27 \Rightarrow B(1, 27)$ is the local maximum.

Question 5 (b)

TO FIND POINT OF INFLECTION:
Put $\dfrac{d^2y}{dx^2} = f''(x) = 0$ and solve for x.



$f''(x) = 0 \Rightarrow -12x - 6 = 0$

$\therefore x = -\tfrac{1}{2}$

$f(x) = (x+2)^2(5-2x)$

$$\begin{aligned} f(-\tfrac{1}{2}) &= (-\tfrac{1}{2}+2)^2(5-2(-\tfrac{1}{2})) \\ &= (\tfrac{3}{2})^2(5+1) \\ &= (\tfrac{9}{4})(6) = \tfrac{27}{2} \end{aligned}$$

$\therefore C(-\tfrac{1}{2}, \tfrac{27}{2})$ is point of inflection.

Question 5 (c)

$m = \frac{27}{2},\ (x_1,\ y_1) = (-\frac{1}{2},\ \frac{27}{2})$

$y - \frac{27}{2} = \frac{27}{2}(x + \frac{1}{2})$

$y - \frac{27}{2} = \frac{27}{2}x + \frac{27}{4}$

$4y - 54 = 54x + 27$

$54x - 4y + 81 = 0$

Question 5 (d)

$$A = \int_{-2}^{0} ((x+2)^2(5-2x))\,dx$$

$$= \int_{-2}^{0} ((x^2+4x+4)(5-2x))\,dx$$

$$= \int_{-2}^{0} (-2x^3 - 3x^2 + 12x + 20)\,dx$$

$$= \left[-\frac{2x^4}{4} - \frac{3x^3}{3} + \frac{12x^2}{2} + 20x\right]_{-2}^{0}$$

$$= \left[-\tfrac{1}{2}x^4 - x^3 + 6x^2 + 20x\right]_{-2}^{0}$$

$$= 0 - \{-\tfrac{1}{2}(-2)^4 - (-2)^3 + 6(-2)^2 + 20(-2)\}$$

$$= +8 - 8 - 24 + 40$$

$$= 16$$

QUESTION 6 (25 MARKS)

Question 6 (a) (i)

$$\int \sin^2 x\ dx$$

$$= \tfrac{1}{2}\int (1 - \cos 2x)\,dx$$

$$= \tfrac{1}{2}(x - \tfrac{1}{2}\sin 2x) + c$$

Question 6 (a) (ii)

$$\int_0^{\ln 2} 3e^{2x}\ dx$$

$$= \tfrac{3}{2}[e^{2x}]_0^{\ln 2}$$

$$= \tfrac{3}{2}(e^{2\ln 2} - e^0)$$

$$= \tfrac{3}{2}(e^{\ln 2^2} - e^0)$$

$$= \tfrac{3}{2}(4-1)$$

$$= \tfrac{9}{2}$$

Question 6 (a) (iii)

$$\boxed{x^3 + 1 = (x+1)(x^2 - x + 1)}$$

$$\int \frac{x^3+1}{x+1}\,dx$$

$$= \int \frac{(x+1)(x^2-x+1)}{(x+1)}\,dx$$

$$= \int (x^2 - x + 1)\,dx$$

$$= \tfrac{1}{3}x^3 - \tfrac{1}{2}x^2 + x + c$$

Question 6 (a) (iv)

$$\int_1^9 \frac{x+1}{\sqrt{x}}\,dx$$

$$= \int_1^9 \left(\sqrt{x} + \frac{1}{\sqrt{x}}\right)dx$$

$$= \int_1^9 (x^{\frac{1}{2}} + x^{-\frac{1}{2}})\,dx$$

$$= [\tfrac{2}{3}x^{\frac{3}{2}} + 2x^{\frac{1}{2}}]_1^9$$

$$= \{\tfrac{2}{3}(9)^{\frac{3}{2}} + 2(9)^{\frac{1}{2}}\} - \{\tfrac{2}{3}(1)^{\frac{3}{2}} + 2(1)^{\frac{1}{2}}\}$$

$$= \tfrac{2}{3}(27) + 2(3) - \tfrac{2}{3}(1) - 2(1)$$

$$= 18 + 6 - \tfrac{2}{3} - 2$$

$$= \tfrac{64}{3}$$

Question 6 (a) (v)

$$\int_0^1 \sqrt{2^x}\ dx$$

$$= \int_0^1 2^{\frac{1}{2}x}\ dx$$

$$= \left[\frac{2 \times 2^{\frac{1}{2}x}}{\ln 2}\right]_0^1$$

$$= \frac{2}{\ln 2}\{2^{\frac{1}{2}(1)} - 2^{\frac{1}{2}(0)}\}$$

$$= \frac{2}{\ln 2}(\sqrt{2} - 1)$$

Question 6 (b)

$f'(x) = 9x^2 - 4x + 5$

$f(x) = \int f'(x)\,dx = \int (9x^2 - 4x + 5)\,dx$

$= 3x^3 - 2x^2 + 5x + c$

$f(1) = 1$

$f(1) = 3(1)^3 - 2(1)^2 + 5(1) + c = 1$

$3 - 2 + 5 + c = 1$

$\therefore c = -5$

$\therefore f(x) = 3x^3 - 2x^2 + 5x - 5$

QUESTION 7 (50 MARKS)

Question 7 (a)

a is the initial amount of Carbon-14

$Q = ae^{bt}$

$t = 0 : Q = ae^0 = a$

$t = 5730 \text{ years}: Q = \frac{1}{2}a$

[At t = 0 years the amount of Carbon-14 present is a. After a half-life of 5730 years, the amount of Carbon-14 present will fall by a half of a.]

$\therefore \frac{1}{2}a = ae^{b(5730)}$

$\frac{1}{2} = e^{b(5730)}$

$\ln(\frac{1}{2}) = b(5730)$

$-\ln 2 = 5730b$

$\therefore b = -\left(\frac{\ln 2}{5730}\right) \text{ yr}^{-1}$

Question 7 (b)

$Q = ae^{bt} \Rightarrow \frac{dQ}{dt} = abe^{bt}$

$$\frac{\left(\frac{dQ}{dt}\right)_{t=4000}}{\left(\frac{dQ}{dt}\right)_{t=1000}} = \frac{abe^{-\left(\frac{\ln 2}{5730}\right)4000}}{abe^{-\left(\frac{\ln 2}{5730}\right)1000}} = \frac{e^{-\left(\frac{\ln 2}{5730}\right)4000}}{e^{-\left(\frac{\ln 2}{5730}\right)1000}} = e^{-\left(\frac{\ln 2}{5730}\right)3000} = 0.696$$

Question 7 (c) (i)

$\frac{Q}{a} = e^{-\left(\frac{\ln 2}{5730}\right)2000} = 0.785 = 78.5\%$

[Q divided by a (the initial amount) will give you the fraction of Carbon-14 present at any time.]

Question 7 (c) (ii)

$$\frac{Q}{a} = 0.93 = e^{-\left(\frac{\ln 2}{5730}\right)t}$$

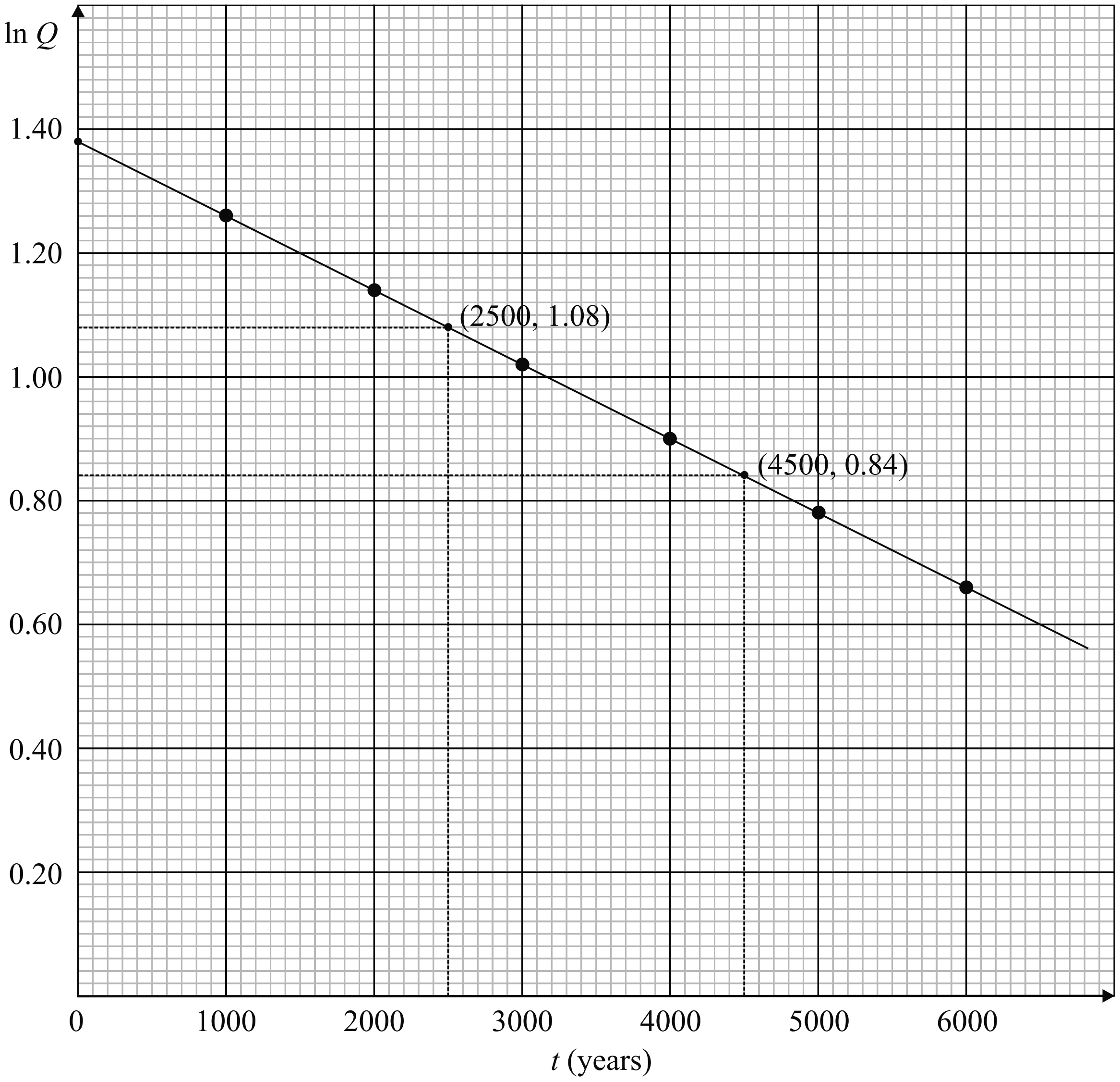
$$\ln(0.93) = -\left(\frac{\ln 2}{5730}\right)t$$

$$\therefore t = -\frac{5730\ln(0.93)}{\ln 2} = 600 \text{ years}$$

Question 7 (d) (i)

t (years)	1000	2000	3000	4000	5000	6000
Q (mg)	3.54	3.14	2.78	2.47	2.18	1.94
ln Q	1.26	1.14	1.02	0.90	0.78	0.66

Question 7 (d) (ii)



Question 7 (e)

$Q = ae^{bt}$

$\ln Q = \ln(ae^{bt})$

$\ln Q = \ln a + \ln(e^{bt})$

$\ln Q = bt + \ln a \; (\rightarrow y = mx + c)$

b is the slope of the graph and $\ln a$ is the y-intercept.

$$b = \frac{1.08 - 0.84}{2500 - 4500} = -1.2 \times 10^{-4} \text{ year}^{-1}$$

$$b = -\frac{\ln 2}{5730} = -1.21 \times 10^{-4} \text{ year}^{-1}$$

$\ln a = 1.38 \Rightarrow a = e^{1.38} = 3.975 \text{ mg}$

Question 7 (f)

$Q = ae^{bt}$

$Q - 0.05Q = ae^{bt_1}$

$Q + 0.05Q = ae^{bt_2}$

$0.95Q = ae^{bt_1}$

$1.05Q = ae^{bt_2}$

$$\frac{0.95Q}{1.05Q} = \frac{e^{bt_1}}{e^{bt_2}} = e^{b(t_1 - t_2)}$$

$$\therefore \ln\left(\frac{0.95}{1.05}\right) = b(t_1 - t_2)$$

$$(t_1 - t_2) = \frac{\ln\left(\frac{0.95}{1.05}\right)}{b} = \frac{\ln\left(\frac{0.95}{1.05}\right)}{-\left(\frac{\ln 2}{5730}\right)} = 827 \text{ years}$$

QUESTION 8 (50 MARKS)

Question 8 (a)

$x = 20\ 000 - 100p$

$100p = 20\ 000 - x$

$p = 200 - 0.01x \geq 0$

$\therefore 20\ 000 \geq x$

$x \leq 20\ 000$

Question 8 (b) (i)

$C = 120\ 000 + 40x$

$$\frac{dC}{dx} = 40 = \text{constant}$$

Question 8 (b) (ii)

It costs €40 to produce one more phone at any level of production.

Question 8 (c) (i)

$R = x \times p$

$p = 200 - 0.01x$

$\therefore R = x(200 - 0.01x) = 200x - 0.01x^2$

Question 8 (c) (ii)

$R = 200x - 0.01x^2 \geq 0$

$20\ 000x - x^2 \geq 0$

$x(20\ 000 - x) \geq 0$

Solve the equality:

$x(20\ 000 - x) = 0$

$\therefore x = 0,\ 20\ 000$

SOLUTION: $0 \leq x \leq 20\ 000$

Question 8 (d)

$R = 200x - 0.01x^2$

$$\frac{dR}{dx} = 200 - 0.02x$$

$$\frac{dR}{dx} = 0 \Rightarrow 200 - 0.02x = 0$$

$200 = 0.02x$

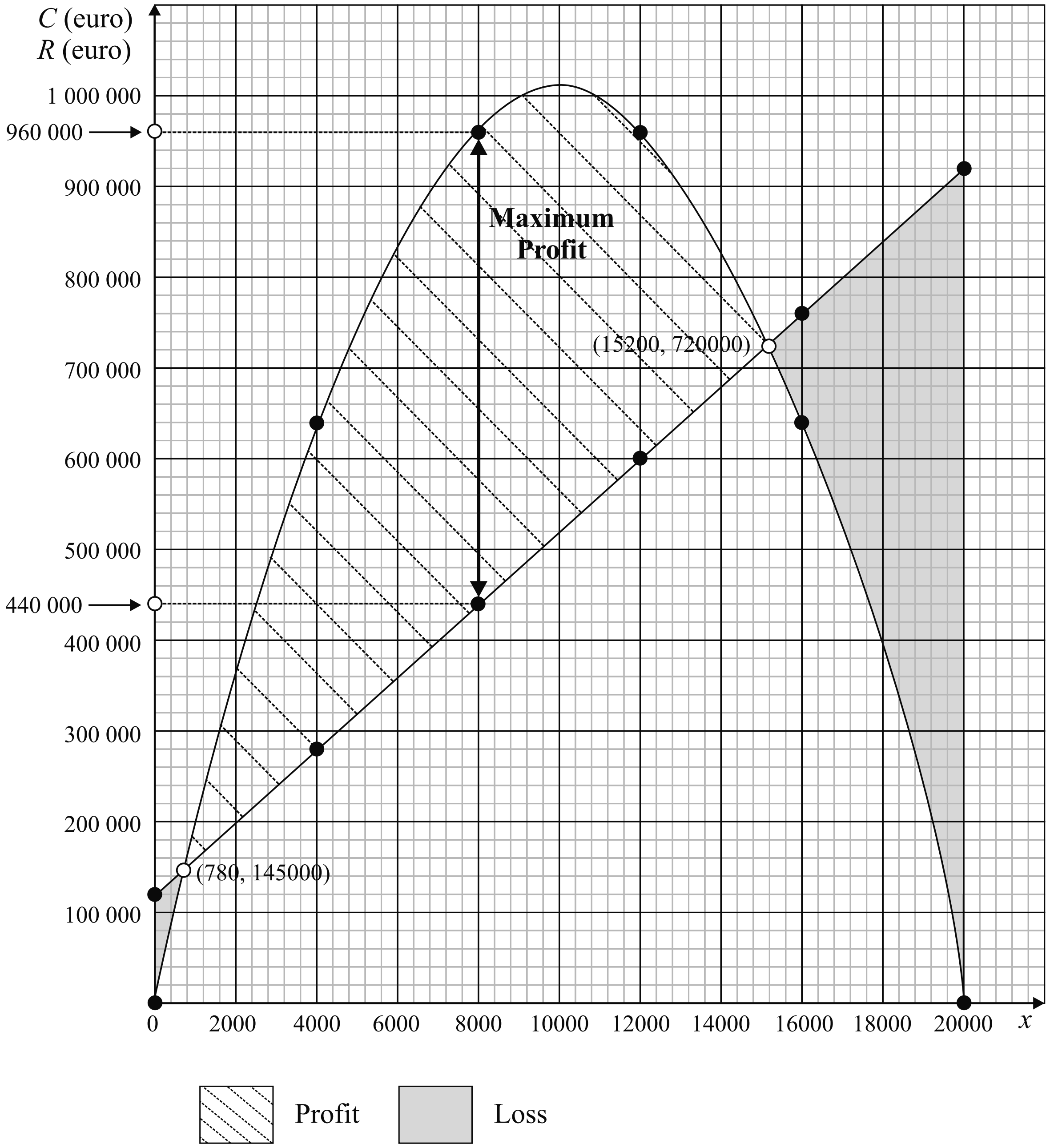
$\therefore x = 10\ 000$

$R_{\text{Max.}} = 200(10\ 000) - 0.01(10\ 000)^2 = €1\ 000\ 000$

Question 8 (e) (i)

x	R (€)	C (€)
0	0	120 000
4000	640 000	280 000
8000	960 000	440 000
12 000	960 000	600 000
16 000	640 000	760 000
20 000	0	920 000

Question 8 (e) (ii)



Question 8 (f) (See graph above)

Question 8 (g)

$R = C$

$200x - 0.01x^2 = 120\ 000 + 40x$

$x^2 - 16\ 000x + 12\ 000\ 000 = 0$

$$x = \frac{-(-16\ 000) \pm \sqrt{(-16\ 000)^2 - 4(12\ 000\ 000)}}{2}$$

$$\boxed{x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}}$$

$\therefore x = 789,\ 15\ 211$

Breakeven points from graph: (780, 145 000), (15 200, 720 000)

Question 8 (h) (i)

Calculus: $P = R - C = 200x - 0.01x^2 - 120\ 000 - 40x$

$$= -0.01x^2 + 160x - 120\ 000$$

$$\frac{dP}{dx} = -0.02x + 160$$

$$\frac{dP}{dx} = 0 \Rightarrow -0.02x + 160 = 0$$

$$160 = 0.02x$$

$$\therefore x = 8000$$

$$P_{\text{Max.}} = -0.01(8000)^2 + 160(8000) - 120\ 000 = €520\ 000$$

Graph: The maximum profit is the greatest distance between the *R* and *C* graphs.
$P_{\text{Max.}}$ = €960 000 – €440 000 = €520 000

Question 8 (h) (ii)

$x = 8{,}000$

$p = 200 - 0.01x$

$\therefore p = 200 - 0.01(8000) = €120$

QUESTION 9 (50 MARKS)

Question 9 (a)

$$\frac{dp}{dt} = kp$$

$$\int \frac{dp}{p} = k\int dt \text{ [Separate the variables.]}$$

$$\ln p = kt + c$$

$$\text{At } t = 0,\ p = N\text{: } \ln N = 0 + c \Rightarrow c = \ln N$$

$$\ln p = kt + \ln N$$

$$\ln p - \ln N = kt$$

$$\ln\left(\frac{p}{N}\right) = kt$$

$$\frac{p}{N} = e^{kt}$$

$$\therefore p = Ne^{kt}$$

Question 9 (b)

$$p = Ne^{3t}$$

$$p = 2N \Rightarrow 2N = Ne^{3t}$$

$$2 = e^{3t}$$

$$\ln 2 = 3t$$

$$t = \frac{\ln 2}{3} = 0.231 \text{ days} = 333 \text{ minutes}$$

Question 9 (c)

$$\int \cos mx\,dx = \tfrac{1}{m}\sin mx + c$$

Question 9 (d)

$$\frac{dp}{dt} = mp\cos mt$$

$$\int \frac{dp}{p} = \int m\cos mt\,dt$$

$$\ln p = \frac{m\sin mt}{m} + c$$

$$\ln p = \sin mt + c$$

$$t = 0,\ p = N : \ln N = \sin 0 + c \Rightarrow c = \ln N$$

$$\ln p = \sin mt + \ln N$$

$$\ln\left(\frac{p}{N}\right) = \sin mt$$

$$p = Ne^{\sin mt}$$

Question 9 (e)

$$p = Ne^{\sin 3t}$$

$$p = 2N : 2N = Ne^{\sin 3t}$$

$$\sin 3t = \ln 2$$

$$t = \tfrac{1}{3}\sin^{-1}(\ln 2) = 0.255 \text{ days} = 368 \text{ minutes}$$

Question 9 (f)

$$p = 1000e^{\sin 3t}$$

$$t = 0 : p = 1000e^{\sin 3(0)} = 1000e^{0} = 1000$$

$$t = \frac{\pi}{6} : p = 1000e^{\sin 3(\frac{1}{6}\pi)} = 1000e \approx 2700$$

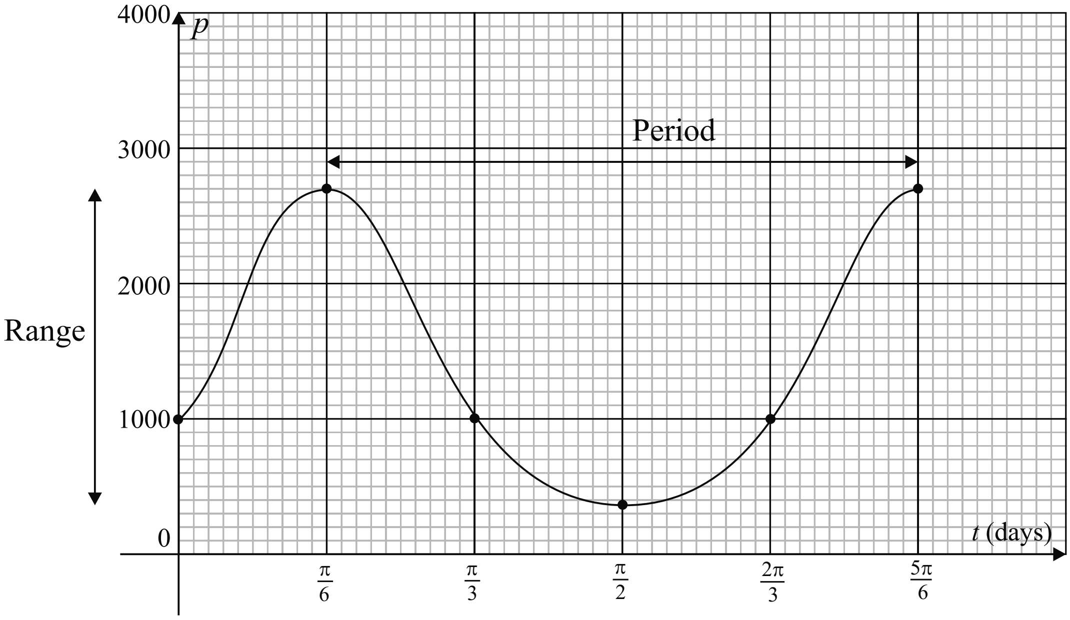
$$t = \frac{\pi}{3} : p = 1000e^{\sin 3(\frac{1}{3}\pi)} = 1000e^{0} = 1000$$

$$t = \frac{\pi}{2} : p = 1000e^{\sin 3(\frac{1}{2}\pi)} = 1000e^{-1} = \frac{1000}{e} \approx 370$$

$$t = \frac{2\pi}{3} : p = 1000e^{\sin 3(\frac{2}{3}\pi)} = 1000e^{0} = 1000$$

$$t = \frac{5\pi}{6} : p = 1000e^{\sin 3(\frac{5}{6}\pi)} = 1000e \approx 2700$$

t (days)	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{5\pi}{6}$
p	1000	$1000e$	1000	$\frac{1000}{e}$	1000	$1000e$
p	1000	2700	1000	370	1000	2700



Question 9 (f) (i)

$\text{Period} = \frac{2\pi}{3} = 2.1$ days

This is the time for the population to return to its original value at any point in the cycle.

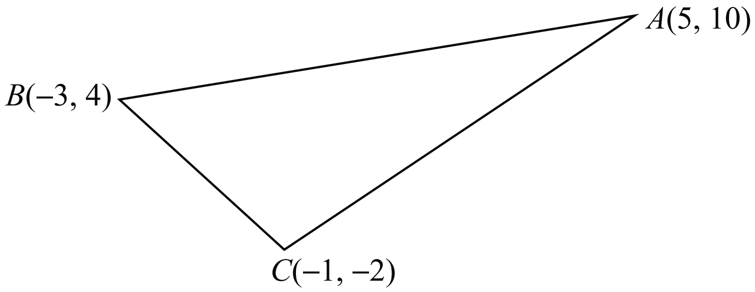
Question 9 (f) (ii)

$\text{Range} = \left[\frac{1000}{e}, 1000e\right]$

This represents the population in a cycle from its maximum to minimum values.

SAMPLE PAPER 1: PAPER 2

QUESTION 1 (25 MARKS)



Question 1 (a)

$A(5, 10)$, $B(-3, 4)$

$$\text{Mid-point} = \left(\frac{5-3}{2}, \frac{10+4}{2}\right) = (1,\ 7)$$

$$m = \frac{4-10}{-3-5} = \frac{-6}{-8} = \frac{3}{4} \Rightarrow m_{\perp} = -\frac{4}{3}$$

$$k: (y-7) = -\tfrac{4}{3}(x-1)$$

$$3(y-7) = -4(x-1)$$

$$3y-21 = -4x+4$$

$$4x+3y-25 = 0$$

$A(5, 10)$, $C(-1, -2)$

$$\text{Mid-point} = \left(\frac{5-1}{2}, \frac{10-2}{2}\right) = (2,\ 4)$$

$$m = \frac{-2-10}{-1-5} = \frac{-12}{-6} = 2 \Rightarrow m_{\perp} = -\frac{1}{2}$$

$$l: (y-4) = -\tfrac{1}{2}(x-2)$$

$$2(y-4) = -1(x-2)$$

$$2y-8 = -x+2$$

$$x+2y-10 = 0$$

Question 1 (b)

$$4x+3y-25=0$$

$$x+2y-10=0(\times -4)$$

[Solve the equations simultaneously for *k* and *l* to find their point of intersection *D*.]

$$\begin{array}{r} 4x+3y-25=0 \\ -4x-8y+40=0 \\ \hline -5y+15=0 \Rightarrow y=3 \end{array}$$

$$x+2(3)-10=0$$

$$x+6-10=0$$

$$\therefore x=4$$

Circumcentre $D(4, 3)$

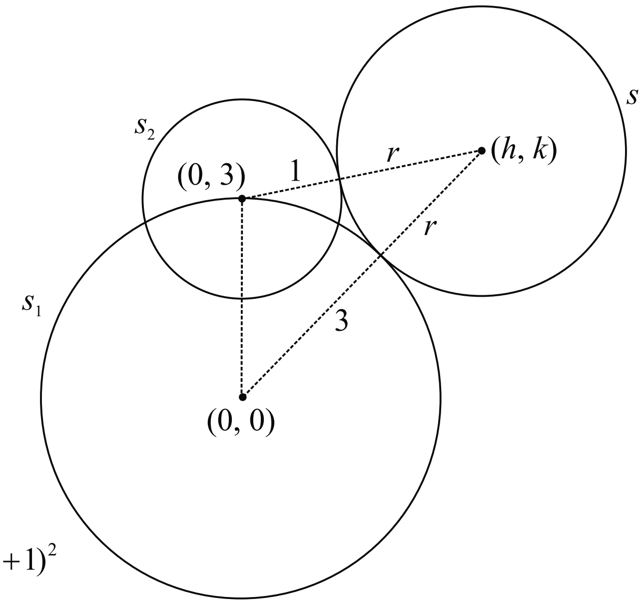
$$r = |AD| = \sqrt{(5-4)^2+(10-3)^2} = \sqrt{1+49} = \sqrt{50} = 5\sqrt{2}$$

Equation of circumcircle: $(x-4)^2+(y-3)^2 = (\sqrt{50})^2$

$$(x-4)^2+(y-3)^2 = 50$$

QUESTION 2 (25 MARKS)

Question 2 (a)



$s_1 : x^2 + y^2 - 9 = 0$

$x^2 + y^2 = 9$

Centre $(0, 0)$, $r = 3$

$s_2 : x^2 + y^2 - 6y + 8 = 0$

Centre$(-g, -f) = (0, 3)$

$r = \sqrt{g^2 + f^2 - c} = \sqrt{0+9-8} = \sqrt{1} = 1$

$$\sqrt{(h-0)^2 + (k-3)^2} = (r+1) \Rightarrow h^2 + (k-3)^2 = (r+1)^2$$

$$\sqrt{(h-0)^2 + (k-0)^2} = (r+3) \Rightarrow \underline{h^2 + k^2 = (r+3)^2}$$

$$(k-3)^2 - k^2 = (r+1)^2 - (r+3)^2$$

$(k-3+k)(k-3-k) = (r+1+r+3)(r+1-r-3)$ $\boxed{a^2 - b^2 = (a-b)(a+b)}$

$(2k-3)(-3) = (2r+4)(-2)$

$-6k + 9 = -4r - 8$

$4r + 17 = 6k$

Question 2 (b)

$h^2 + k^2 = (r+3)^2$

$4r + 17 = 6k \Rightarrow r = \dfrac{6k-17}{4}$

$h^2 + k^2 = \left(\dfrac{6k-17}{4} + 3\right)^2$

$h^2 + k^2 = \left(\dfrac{6k-17+12}{4}\right)^2 = \left(\dfrac{6k-5}{4}\right)^2$

$h^2 = \dfrac{(6k-5)^2}{16} - k^2$

$16h^2 = (6k-5)^2 - 16k^2$ $\boxed{a^2 - b^2 = (a-b)(a+b)}$

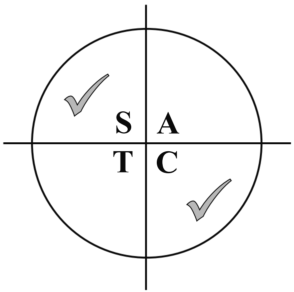
$16h^2 = (6k-5+4k)(6k-5-4k)$

$16h^2 = (10k-5)(2k-5)$

$\therefore 16h^2 = 5(2k-1)(2k-5)$

QUESTION 3 (25 MARKS)

Question 3 (a) (i)



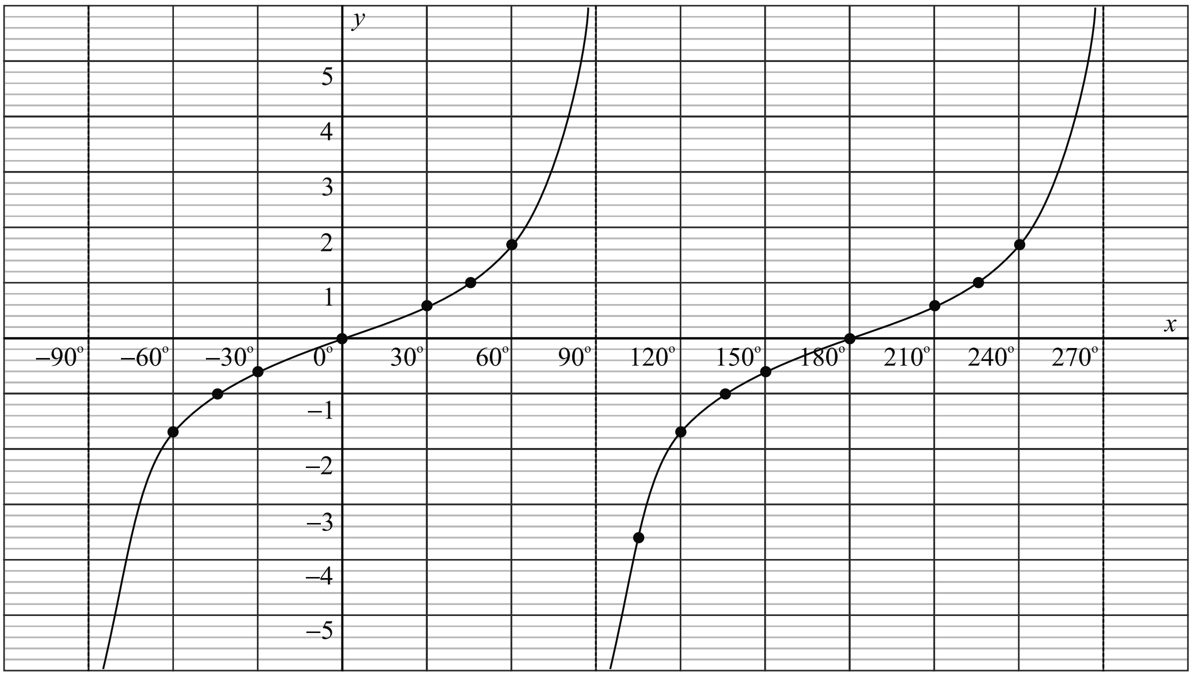
Question 3 (a) (ii)

$$\tan 330^\circ$$
$$= \tan(360^\circ - 30^\circ)$$
$$= -\tan 30^\circ$$
$$= -\frac{1}{\sqrt{3}}$$

Question 3 (b) (i)

x°	–90	–60	–45	–30	0	30	45	60	90
y	$-\infty$	–1.7	–1	–0.6	0	0.6	1	1.7	∞

x°	105	120	135	150	180	210	225	240	270
y	–3.7	–1.7	–1	–0.6	0	0.6	1	1.7	$-\infty$



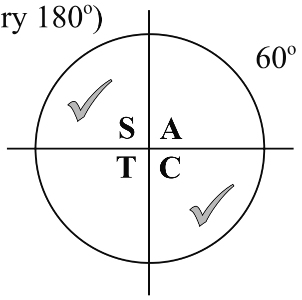
Question 3 (b) (ii)

Yes, because each value of x only gives one value of y.

Question 3 (b) (iii)

Period = 180° (function repeats itself every 180°)
Range = $[-\infty, \infty]$

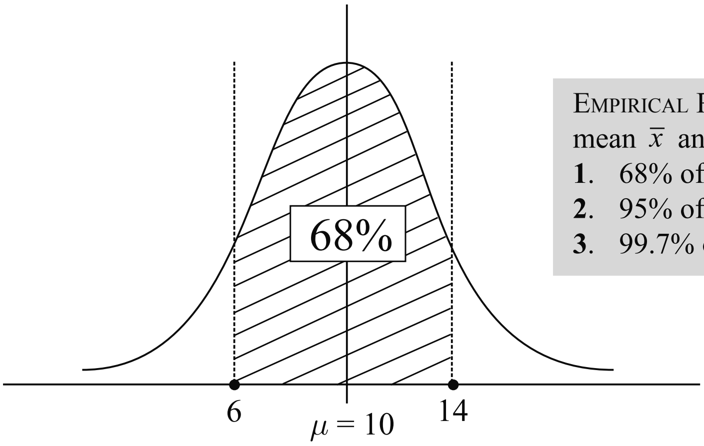
Question 3 (b) (iv)



$\tan x = \sqrt{3} \Rightarrow x = \tan^{-1}(\sqrt{3}) = 60^\circ$

Second quadrant: $x = 180^\circ - 60^\circ = 120^\circ$
Fourth quadrant: $x = 360^\circ - 60^\circ = 300^\circ$

QUESTION 4 (25 MARKS)



EMPIRICAL RULE: In any normal distribution with mean $\bar{x}$ and standard deviation σ.
1. 68% of the data falls within 1σ of the mean $\bar{x}$.
2. 95% of the data falls within 2σ of the mean $\bar{x}$.
3. 99.7% of the data falls within 3σ of the mean $\bar{x}$.

Question 4 (a) (i)

Standard deviation = 4
Median = 10

Question 4 (a) (ii)

$$x = 10 : z = \frac{10-10}{4} = 0$$

$$x = 14 : z = \frac{14-10}{4} = 1$$

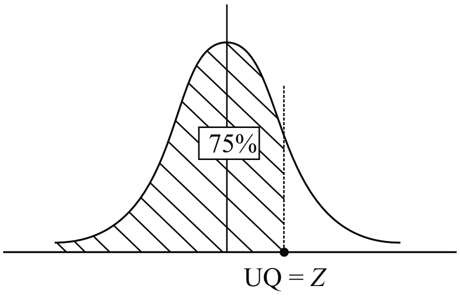
$$x = 6 : z = \frac{6-10}{4} = -1$$

$$\boxed{z = \frac{x-\mu}{\sigma}}$$

Question 4 (b)

The standard normal distribution has a mean of **0** and standard deviation of **1** and has the same shape and area distribution as the original normal distribution.

Question 4 (c) (i)



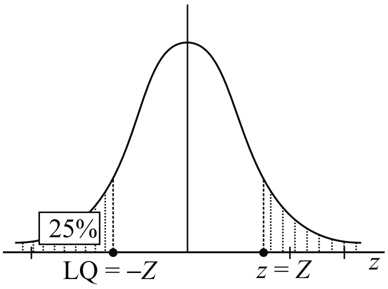
$P(z < Z) = 0.75$

$\therefore z = 0.68 = \dfrac{x-10}{4}$

$2.72 = x - 10$

$\therefore x = 12.72$ (Upper Quartile)

Question 4 (c) (ii)



$z = -0.68 = \dfrac{x-10}{4}$

$-2.72 = x - 10$

$\therefore x = 7.28$ (Lower Quartile)

Question 4 (c) (iii)

Interquartile range = UQ – LQ = 12.72 – 7.28 = 5.44

QUESTION 5 (25 MARKS)

$$\boxed{9}\ \boxed{9}\ \boxed{9}\ \boxed{9}\ \boxed{x}\ \boxed{y}$$

Question 5 (a)

(i) Median $= \frac{9+9}{2} = 9$

(ii) Mode = 9

(iii) Range cannot be worked out without knowing the values of x and y.

Possibilities for arranging the numbers in ascending order:
$9, 9, 9, 9, x, y$
$x, y, 9, 9, 9, 9$
$x, 9, 9, 9, 9, y$
$y, 9, 9, 9, 9, x$
$9, 9, 9, 9, 9, 9$

Question 5 (b)

Mean = 9
Therefore sum = $9 \times 6 = 54$
$x + y = 54 - 36 = 18$

Possibilities for x and y:

x	1	2	3	4	5	6	7	8	9
y	17	16	15	14	13	12	11	10	9

OR

x	17	16	15	14	13	12	11	10	9
y	1	2	3	4	5	6	7	8	9

Greatest range = [1, 17]
$R = 17 - 1 = 16$

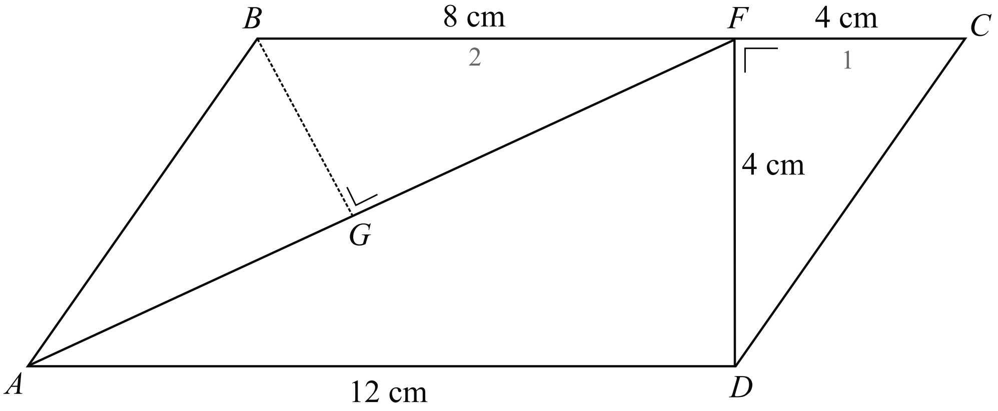
Question 5 (c)

$R = x - y = 6$

$\therefore 2x = 24 \Rightarrow x = 12$

$\therefore y = 6$

QUESTION 6 (25 MARKS)



Question 6 (a)

$|AD| = |BC| = 12$ cm

$|BF| = 2|FC|$

$\therefore |BF| = 8$ cm, $|FC| = 4$ cm

Question 6 (b)

Triangle FCD: $A = \frac{1}{2}(4)(4) = 8 \text{ cm}^2$

Triangle AFD: $A = \frac{1}{2}(12)(4) = 24 \text{ cm}^2$

Parallelogram $ABCD$: $A = (12)(4) = 48 \text{ cm}^2$

Triangle BFA: $A = 48 - 8 - 24 = 16 \text{ cm}^2$

Question 6 (c)

$|AF|^2 = 4^2 + 12^2 = 16 + 144 = 160 \Rightarrow |AF| = \sqrt{160} = 4\sqrt{10}$

$\frac{1}{2}(4\sqrt{10})|BG| = 16$

$\therefore |BG| = \dfrac{4\sqrt{10}}{5}$ cm

QUESTION 7 (75 MARKS)

Question 7 (a) (i)

$\mu = 120$

$\sigma = 15$

$135 = 120 + 15 = \mu + 1\sigma$

$105 = 120 - 15 = \mu - 1\sigma$

EMPIRICAL RULE: 68% of young adults have a blood pressure reading of between 105 mm of Hg and 135 mm of Hg.

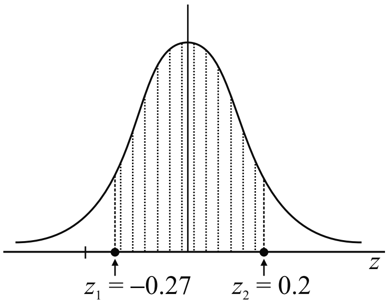
Question 7 (a) (ii)

$\mu = 120$

$\sigma = 15$

$x_1 = 116 : z_1 = \dfrac{116 - 120}{15} = -0.27$

$x_2 = 123 : z_1 = \dfrac{123 - 120}{15} = 0.2$



$$\begin{aligned}P &= P(-\infty \text{ to } 0.2) - P(-\infty \text{ to } -0.27)\\ &= P(-\infty \text{ to } 0.2) - P(z > 0.27)\\ &= P(-\infty \text{ to } 0.2) - \{1 - P(z < 0.27)\}\\ &= 0.5793 - 1 + 0.6064\\ &= 0.1857\end{aligned}$$

From a sample of 100 people the number will be $100 \times 0.1857 = 18$ or 19 people

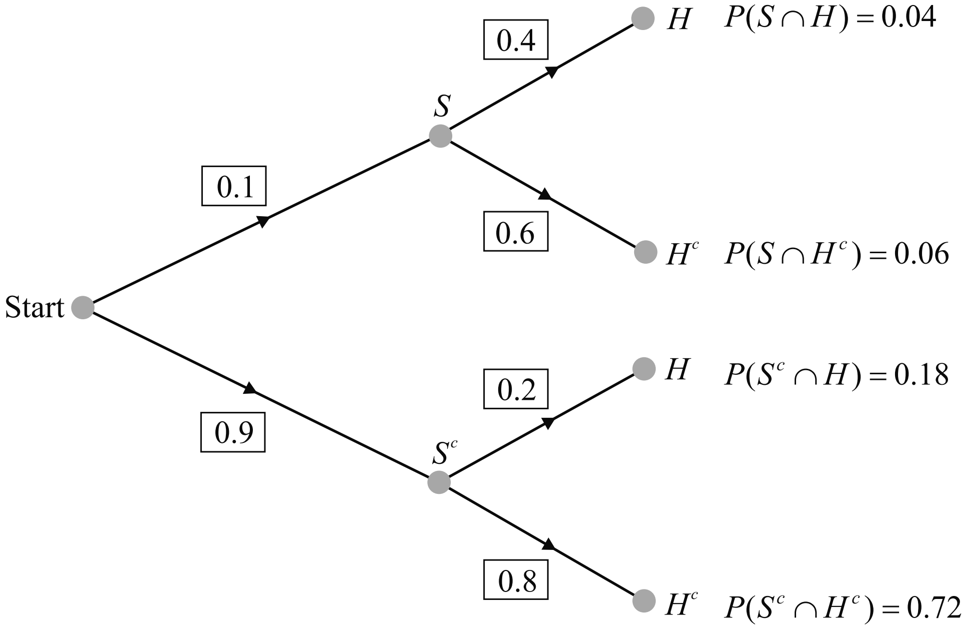
Question 7 (b)

(i) $P(S) = 0.1$
(ii) $P(H|S) = 0.4$
(iii) $P(H|S^c) = 0.2$

Question 7 (c)

$$P(S\,|\,H) = \frac{P(H \cap S)}{P(H)}$$

Question 7 (d)



Question 7 (e)

$$\begin{aligned}P(H) &= P(S \cap H) + P(S^c \cap H)\\ &= 0.04 + 0.18\\ &= 0.22\end{aligned}$$

Question 7 (f)

$$P(S\,|\,H) = \frac{P(S \cap H)}{P(H)} = \frac{0.04}{0.22} = 0.182$$

Question 7 (g)

$P(\text{Having a stroke}) = 0.182$

$P(\text{Not having a stroke}) = 0.818$

$P(\text{At least 2 out of 10 people will suffer a stroke})$

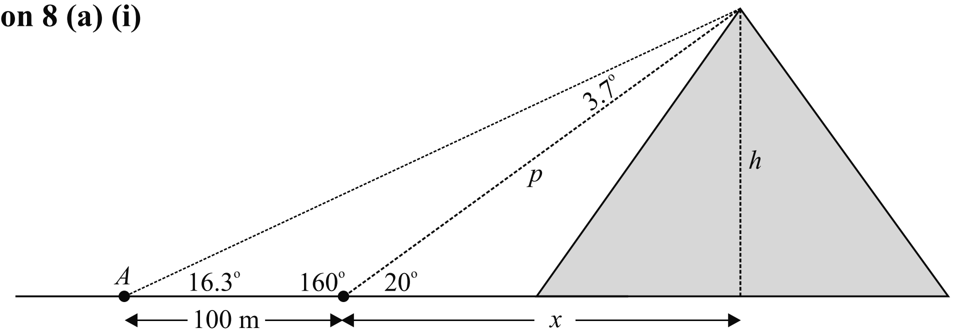
$= 1 - P(\text{2 out of 10 people will suffer a stroke})$

$= 1 - \{{}^{10}C_0(0.182)^0(0.818)^{10} + {}^{10}C_1(0.182)^1(0.818)^9\}$

$= 0.57$

QUESTION 8 (75 MARKS)

Question 8 (a) (i)



Question 8 (a) (ii)

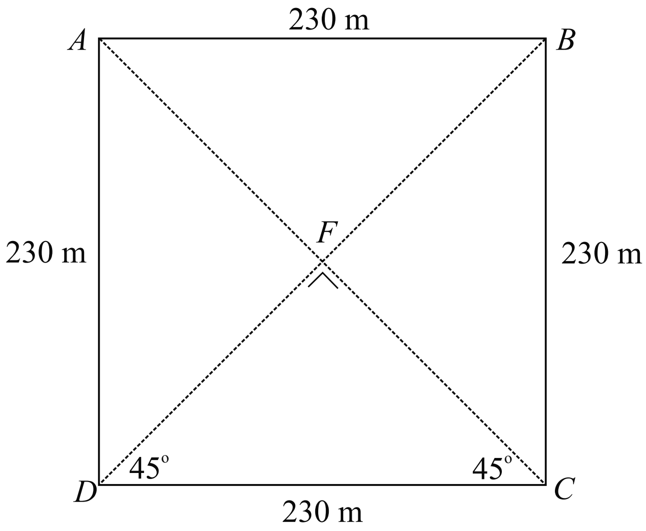
$$\frac{100}{\sin 3.7^\circ}=\frac{p}{\sin 16.3^\circ} \qquad \boxed{\frac{a}{\sin A}=\frac{b}{\sin B}}$$

$$\therefore p=\frac{100\sin 16.3^\circ}{\sin 3.7^\circ}=434.92 \text{ m}$$

$$\sin 20^\circ=\frac{h}{434.92}$$

$$\therefore h=434.92\sin 20^\circ=148.75\approx 149 \text{ m}$$

Question 8 (b)

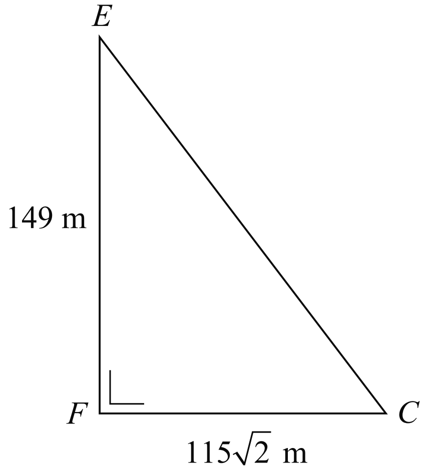


$$\cos 45^\circ=\frac{|FC|}{230}$$

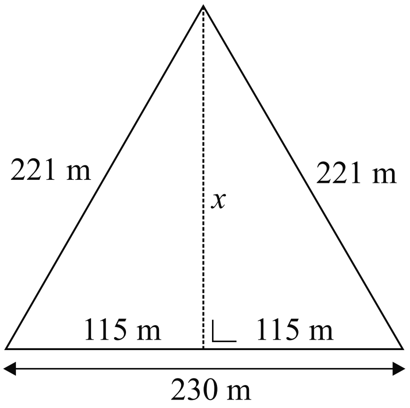
$$|FC|=230\cos 45^\circ=115\sqrt{2} \text{ m}$$

Question 8 (c)

$$|EC|=\sqrt{149^2+(115\sqrt{2})^2}=221 \text{ m}$$

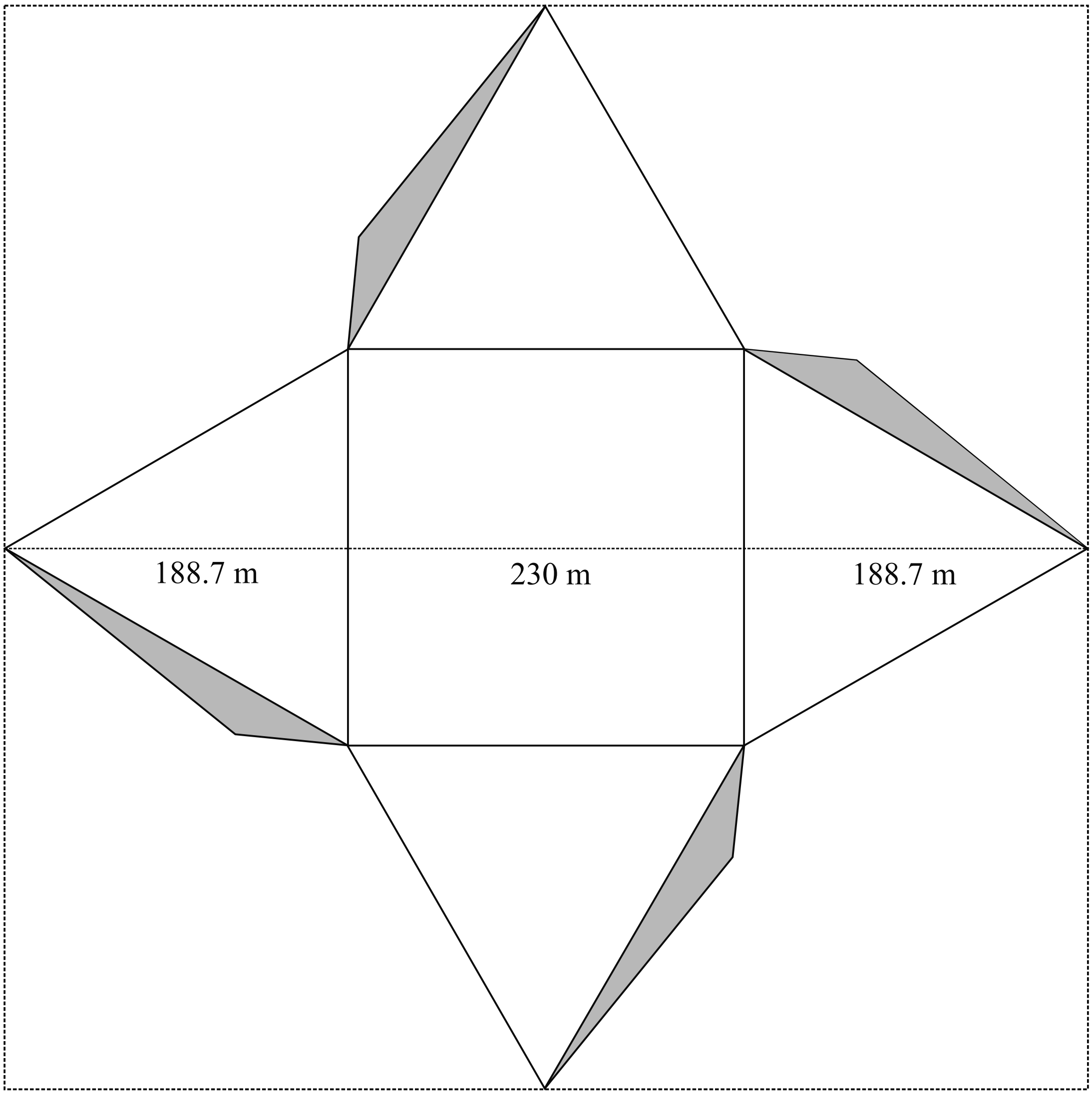


Question 8 (d)



$$115^2 + x^2 = 221^2$$

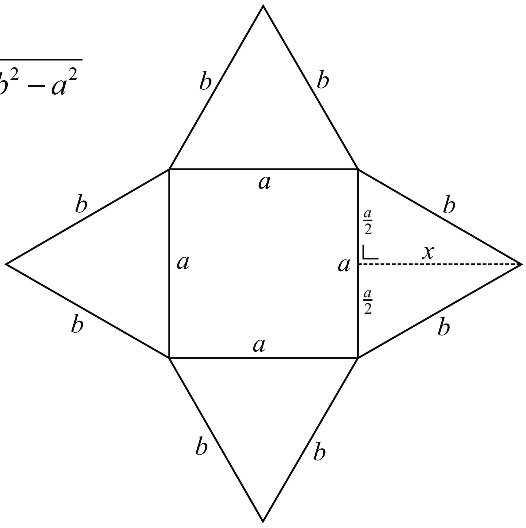
$$x = \sqrt{221^2 - 115^2} = 188.7 \text{ m}$$



Length of square $= \dfrac{2(188.7)+230}{50} = 12.15 \text{ m}$

Question 8 (e)

$$\left(\frac{a}{2}\right)^2 + x^2 = b^2 \Rightarrow x = \sqrt{b^2 - \frac{a^2}{4}} = \sqrt{\frac{4b^2 - a^2}{4}} = \tfrac{1}{2}\sqrt{4b^2 - a^2}$$



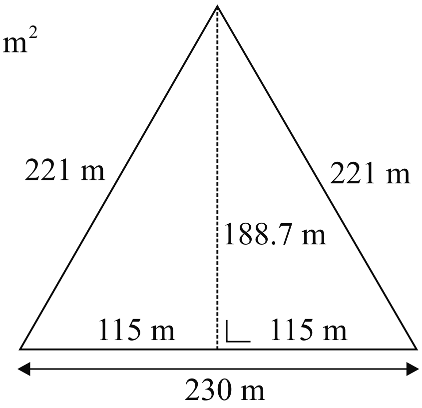
Area of 4 triangles:

$$A = 4(\tfrac{1}{2} \times \text{base} \times \text{height}) = 2 \times \text{base} \times \text{height}$$

$$= 2a \times \tfrac{1}{2}\sqrt{4b^2 - a^2} = a\sqrt{4b^2 - a^2}$$

Question 8 (f) (i)

Surface area: $A = a\sqrt{4b^2 - a^2} = 230\sqrt{4(221)^2 - 230^2} = 86\ 812\ \text{m}^2$



Question 8 (f) (ii)

Area of a casing stone = (0.86 m)(0.86 m) = 0.7396 m^2

Number of stones $= \dfrac{86\ 812}{0.7396} \approx 117\ 000$

SAMPLE PAPER 2: PAPER 1

QUESTION 1 (25 MARKS)

Question 1 (a)

$$\log_9(x+2)-\log_3 x=0 \qquad \boxed{\log_b x=\frac{\log_a x}{\log_a b}}$$

$$\log_9(x+2)-\frac{\log_9 x}{\log_9 3}=0$$

$$\log_9(x+2)-\frac{\log_9 x}{\frac{1}{2}}=0$$

$$\log_9(x+2)-2\log_9 x=0 \qquad \boxed{\log_a\left(\frac{x}{y}\right)=\log_a x-\log_a y}$$

$$\log_9\frac{x+2}{x^2}=0$$

$$\frac{x+2}{x^2}=9^0=1 \qquad \boxed{a^x=y \Leftrightarrow \log_a y=x}$$

$$x+2=x^2$$

$$x^2-x-2=0$$

$$(x-2)(x+1)=0$$

$x=2$ [$x=-1$ is not allowed as $x>0$.]

Question 1 (b) (i)

$$\binom{5}{3}=\frac{5\times4\times3}{3\times2\times1}=10$$

$$\binom{5}{2}=\frac{5\times4}{2\times1}=10$$

Question 1 (b) (ii)

$$(p+q)^5=\binom{5}{0}p^5q^0+\binom{5}{1}p^4q^1+\binom{5}{2}p^3q^2+\binom{5}{3}p^2q^3+\binom{5}{4}p^1q^4+\binom{5}{5}p^0q^5$$

$$=p^5+5p^4q^1+10p^3q^2+10p^2q^3+5p^1q^4+q^5$$

Question 1 (b) (iii)

$$T_3=10p^3q^2=10(0.6)^3(0.4)^2=0.3456$$

QUESTION 2 (25 MARKS)

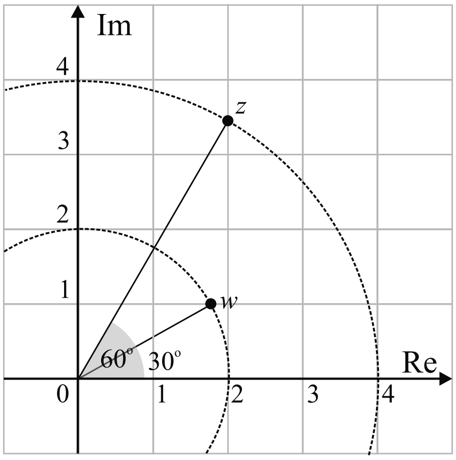
Question 2 (a)

$$z = 2 + 2\sqrt{3}i$$

$$|z| = \sqrt{2^2 + (2\sqrt{3})^2} = \sqrt{4+12} = \sqrt{16} = 4 \quad \boxed{z = r(\cos\theta + i\sin\theta)}$$

$$\tan\theta = \frac{2\sqrt{3}}{2} = \sqrt{3} \Rightarrow \theta = \tan^{-1}(\sqrt{3}) = 60^\circ = \frac{\pi}{3} \quad \boxed{|z| = r,\ \arg z = \theta}$$

$$\therefore z = 4\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$$



$$w = \sqrt{3} + i$$

$$|w| = \sqrt{(\sqrt{3})^2 + (1)^2} = \sqrt{3+1} = \sqrt{4} = 2$$

$$\tan\theta = \frac{1}{\sqrt{3}} \Rightarrow \theta = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = 30^\circ = \frac{\pi}{6}$$

$$\therefore w = 2\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)$$

Question 2 (b) (i)

$$\begin{aligned} zw &= 4\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)2\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right) \\ &= 8\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right) \\ &= 8(0+i) \\ &= 8i \end{aligned}$$

Question 2 (b) (ii)

'According to De Moivre's theorem, when two complex numbers are multiplied together, the arguments are <u>added</u> and the moduli are <u>multiplied</u> to form the new complex number.'

Question 2 (c) (i)

$$\begin{aligned} \frac{z}{w} &= \frac{4\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)}{2\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)} = 2\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right) \\ &= 2\left(\frac{\sqrt{3}}{2} + \frac{1}{2}i\right) = \sqrt{3} + i \end{aligned}$$

Question 2 (c) (ii)

'According to De Moivre's theorem, when two complex numbers are divided, the arguments are <u>subtracted</u> and the moduli are <u>divided</u> to form the new complex number.'

Question 2 (c) (iii)

$$\left(\frac{z}{w}\right)^{14} = 2^{14}\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)^{14} \quad \boxed{[r(\cos\theta + i\sin\theta)]^n = r^n(\cos n\theta + i\sin n\theta)} \text{ (De Moivre's Theorem)}$$

$$= 2^{14}\left(\cos\frac{14\pi}{6} + i\sin\frac{14\pi}{6}\right)$$

$$= 2^{14}\left(\cos\frac{7\pi}{3} + i\sin\frac{7\pi}{3}\right) = 2^{14}\left(\frac{1}{2} + \frac{\sqrt{3}}{2}i\right) = 2^{13}(1 + \sqrt{3}i)$$

Question 3 (25 marks)

Question 3 (a)

100, 90, 81,......
$a = 100$, $r = 0.9$

Which term is equal to 50, half of its original mass?

$$50 = 100(0.9)^{n-1}$$

$$0.5 = 0.9^{n-1}$$

$$\log_{10}(0.5) = (n-1)\log_{10}(0.9)$$

$$\therefore n = \frac{\log_{10}(0.5)}{\log_{10}(0.9)} + 1 = 7.58 \text{ days}$$

Seven days must elapse before it is less than half its original mass.

Question 3 (b)

$$S_n = \frac{108(1-(\frac{1}{3})^n)}{\frac{2}{3}} = 162(1-(\tfrac{1}{3})^n)$$

$$S_\infty = \frac{108}{\frac{2}{3}} = 162$$

$$S_n - S_\infty = 0.05$$

$$162(1-(\tfrac{1}{3})^n) - 162 = 0.05$$

$$162(\tfrac{1}{3})^n = 0.05$$

$$3^n = 3240$$

$$n\log_{10} 3 = \log_{10} 3240$$

$$\therefore n = \frac{\log_{10} 3240}{\log_{10} 3} = 7.36$$

Eight terms must be added together so that the sum differs from the sum to infinity by less than 0.05.

Question 4 (25 marks)

Question 4 (a)

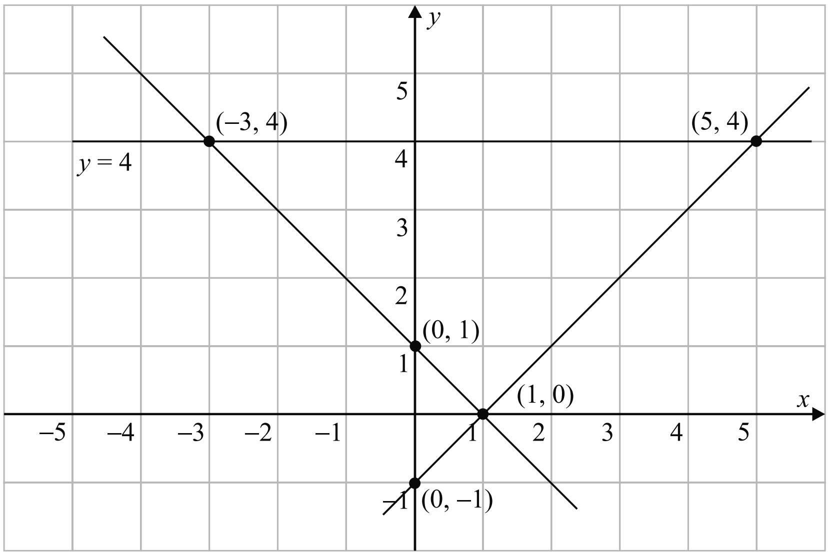
Draw each of the straight line graphs by plotting their intercepts.

$y = |x-1| \Rightarrow y = \pm(x-1)$

$y = x-1 \rightarrow$ Intercepts: (0, −1), (1, 0)

$y = -x+1 \rightarrow$ Intercepts: (0, 1), (1, 0)

Point of intersection: (1, 0)



Question 4 (b)

Points of intersection: (–3, 4), (5, 4)

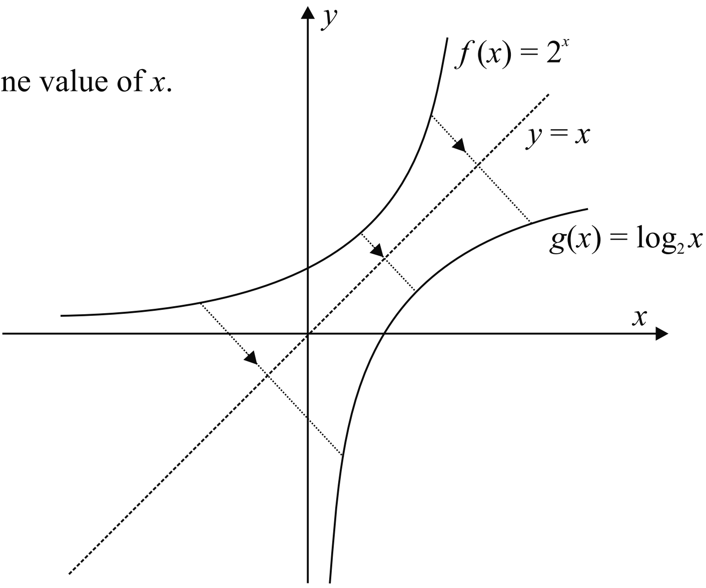
Question 4 (c)

$x < -3$, $x > 5$

QUESTION 5 (25 MARKS)

Question 5 (a)

Yes. Each value of y corresponds to only one value of x.



Question 5 (b)

$f(x) = y = 2^x$

$\log_2 y = \log_2 2^x$

$\log_2 y = x \log_2 2$

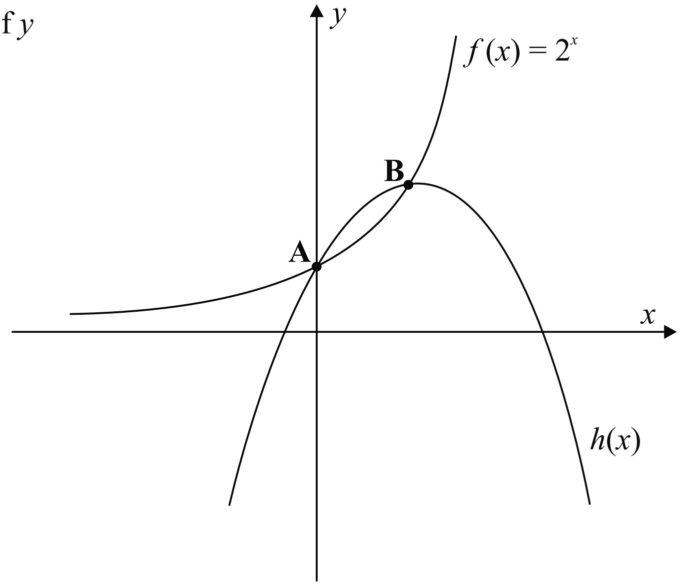
$\log_2 y = x$

$\therefore f^{-1}(x) = \log_2 y = g(x)$

$g(x)$ is the inverse function of $f(x)$.
Reflect the curve for $f(x)$ by an axial symmetry through the line $y = x$ to get its inverse function.

Question 5 (c)

It is not injective because certain values of y correspond to two values of x.



Question 5 (d)

$x = 0 \Rightarrow y = 2^0 = 1$

$\therefore A(0, 1)$

$(0, 1) \in h(x) \Rightarrow 1 = -(-1)^2 + a$

$\therefore a = 2$

$h(x) = 2 - (x-1)^2$

Question 5 (e)

Local maximum is at $y = 2$.

$h(x) = 2 - (x-1)^2$

$\therefore 2 = 2 - (x-1)^2$

$(x-1)^2 = 0$

$x - 1 = 0$

$\therefore x = 1$

Local Maximum $B(1, 2)$

$B(1, 2) \in f(x)$?

$f(x) = 2^x$

$2 = 2^1$ (True)

QUESTION 6 (25 MARKS)

Question 6 (a)

$f(x)=g(x)$

$5x-x^2=x+3$

$0=x^2-4x+3$

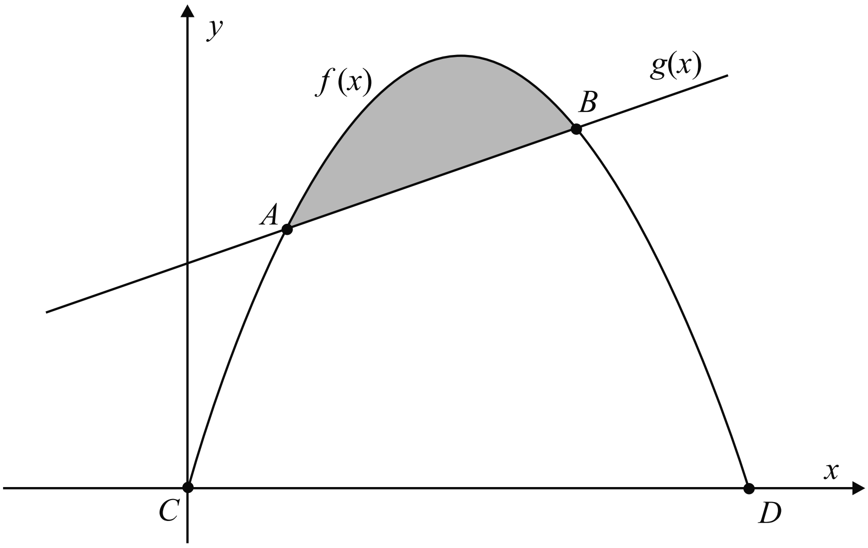
$0=(x-1)(x-3)$

$\therefore x=1,\ 3$

$g(1)=1+3=4$

$g(3)=3+3=6$

$A(1,\ 4),\ B(3,\ 6)$



Question 6 (b)

$$|A|=\int_1^3 (f(x)-g(x))\,dx$$

$$=\int_1^3 (-x^2+4x-3)\,dx$$

$$=[-\tfrac{1}{3}x^3+2x^2-3x]_1^3$$

$$=\{-\tfrac{1}{3}(3)^3+2(3)^2-3(3)\}-\{-\tfrac{1}{3}(1)^3+2(1)^2-3(1)\}$$

$$=-9+18-9+\tfrac{1}{3}-2+3$$

$$=\tfrac{4}{3}$$

Question 6 (c)

$C(0,\ 0),\ D(5,\ 0)$

$$\text{Average value}=\frac{1}{5-0}\int_0^5 f(x)\,dx$$

$$=\tfrac{1}{5}\int_0^5 (5x-x^2)\,dx$$

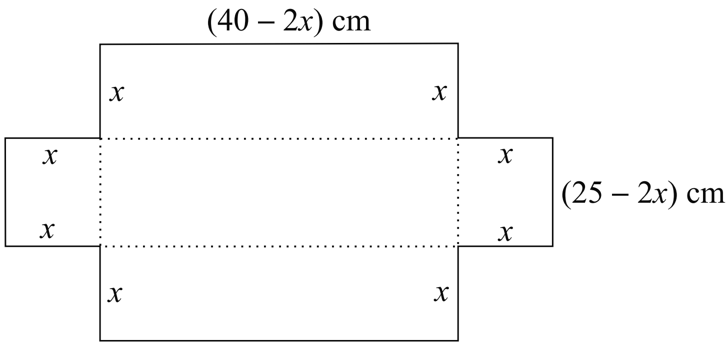
$$=\tfrac{1}{5}[\tfrac{5}{2}x^2-\tfrac{1}{3}x^3]_0^5$$

$$=\tfrac{1}{5}\{\tfrac{5}{2}(5)^2-\tfrac{1}{3}(5)^3\}-\{0\}$$

$$=\tfrac{1}{5}(\tfrac{125}{2}-\tfrac{125}{3})=\tfrac{25}{6}$$

QUESTION 7 (50 MARKS)

Question 7 (a) (i)



$l = 40 - 2x,\ b = 25 - 2x,\ h = x$

Question 7 (a) (ii)

$$V=(40-2x)(25-2x)x$$

$$=(1000-80x-50x+4x^2)x$$

$$=(1000-130x+4x^2)x$$

$$=1000x-130x^2+4x^3$$

Question 7 (b) (i)

$$S=2(40-2x)x+2(25-2x)x+(40-2x)(25-2x)$$

$$=80x-4x^2+50x-4x^2+1000-80x-50x+4x^2$$

$$=1000-4x^2$$

Question 7 (b) (ii)

$600=1000-4x^2$

$4x^2=400$

$x^2=100$

$\therefore x=10$ cm

Question 7 (c)

$V = 4x^3 - 130x^2 + 1000x$

$4x^3 - 130x^2 + 1000x = 1512$

$2x^3 - 65x + 500x - 756 = 0$

$x = 1 : 2(1)^3 - 65(1) + 500(1) - 756 = -319 \neq 0$

$x = 2 : 2(2)^3 - 65(2) + 500(2) - 756 = 0$

Question 7 (d)

$V = 4x^3 - 130x^2 + 1000x$

$\frac{dV}{dx} = 12x^2 - 260x + 1000$

Maximum Volume: $\frac{dV}{dx} = 0$

$\frac{dV}{dx} = 0 \Rightarrow 12x^2 - 260x + 1000 = 0$

$3x^2 - 65x + 250 = 0$

$(3x - 50)(x - 5) = 0$

$x = 5, \frac{50}{3}$ cm [Ignore the second solution as it will cause one of the sides to be negative.]

$V_{Max.} = 4(5)^3 - 130(5)^2 + 1000(5) = 500 - 3250 + 5000 = 2250 \text{ cm}^3$

$l = 40 - 2(5) = 30$ cm, $b = 25 - 2(5) = 15$ cm, $h = 5$ cm

Question 7 (e)

$\frac{dV}{dt} = -(2t + 5)$

$V = -\int (2t + 5)\, dt = -t^2 - 5t + c$

$V = 2250 \text{ when } t = 0 : 2250 = c$

$\therefore V = -t^2 - 5t + 2250$

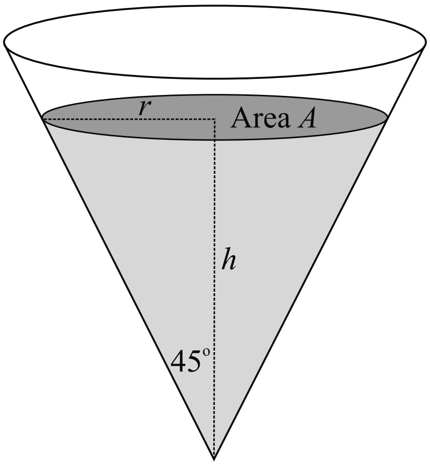
$-t^2 - 5t + 2250 = 0$

$t^2 + 5t - 2250 = 0$

$(t - 45)(t + 50) = 0$

$\therefore t = 45 \text{ s}$

Question 7 (f) (i)



$\tan 45° = \frac{r}{h} \Rightarrow 1 = \frac{r}{h}$

$\therefore r = h$

$V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi r^3$

Question 7 (f) (ii)

$V = \frac{1}{3}\pi r^3$

$\frac{dV}{dt} = \pi r^2 \times \frac{dr}{dt} = 3 \Rightarrow \frac{dr}{dt} = \frac{3}{\pi r^2}$

$A = \pi r^2$

$\frac{dA}{dt} = 2\pi r \times \frac{dr}{dt} = 2\pi r \times \frac{3}{\pi r^2} = \frac{6}{r}$

$\left(\frac{dA}{dt}\right)_{r=2} = \frac{6}{2} = 3 \text{ cm}^2\text{/s}$

QUESTION 8 (50 MARKS)

Question 8 (a)

$$M=\log_{10}\left(\frac{s}{s}\right)=\log_{10}1=0$$

Question 8 (b)

$$M_1=\log_{10}\left(\frac{A_1}{s}\right)=8.3$$

$$M_2=\log_{10}\left(\frac{4A_1}{s}\right)$$

$$M_2-M_1=\log_{10}\left(\frac{4A_1}{s}\right)-\log_{10}\left(\frac{A_1}{s}\right)$$

$$M_2-8.3=\log_{10}\left(\frac{4A_1}{s}\times\frac{s}{A_1}\right)$$

$$M_2-8.3=\log_{10}4$$

$$\therefore M_2=\log_{10}4+8.3=8.9$$

Question 8 (c)

$$M_1=\log_{10}\left(\frac{A_1}{s}\right)=8.3$$

$$M_2=\log_{10}\left(\frac{A_2}{s}\right)=4$$

$$M_1-M_2=\log_{10}\left(\frac{A_1}{s}\right)-\log_{10}\left(\frac{A_2}{s}\right)$$

$$4.3=\log_{10}\left(\frac{A_1}{s}\times\frac{s}{A_2}\right)=\log_{10}\left(\frac{A_1}{A_2}\right)$$

$$\therefore\frac{A_1}{A_2}=10^{4.3}=19953$$

Question 8 (d)

$$M_1=5.9, A=A_1$$

$$M_1=\log_{10}\left(\frac{A_1}{s}\right)=5.9\Rightarrow\frac{A_1}{s}=10^{5.9}$$

$$M_2=6.5, A=A_2$$

$$M_2=\log_{10}\left(\frac{A_2}{s}\right)=6.5\Rightarrow\frac{A_2}{s}=10^{6.5}$$

$$\text{Fractional change in amplitude}=\frac{A_2-A_1}{A_1}=\frac{A_2}{A_1}-1$$

$$=\frac{10^{6.5}\times s}{10^{5.9}\times s}-1=10^{0.6}-1=2.98$$

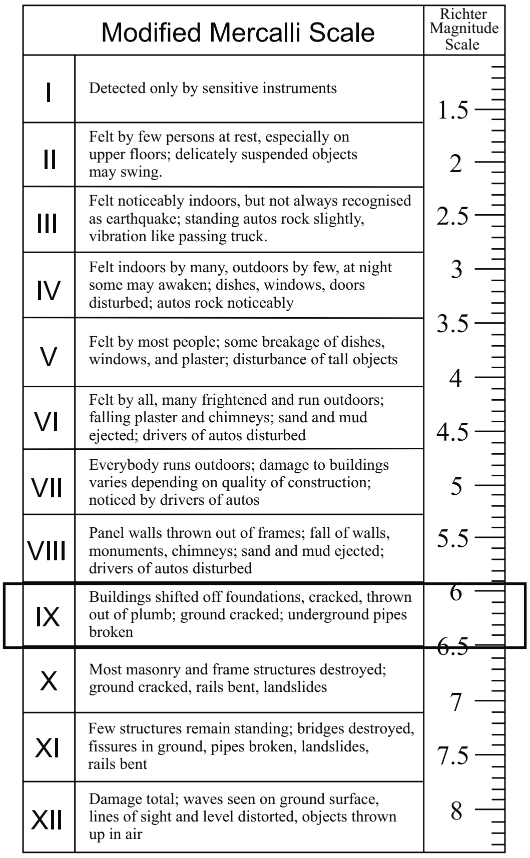
$$\%\text{ change}=298\%$$

Question 8 (e)

$$6.5=\log_{10}\left(\frac{A}{10^{-4}}\right)$$

$$10^{6.5}=\frac{A}{10^{-4}}$$

$$\therefore A=10^{6.5}\times10^{-4}=10^{2.5}=316\text{ cm}$$

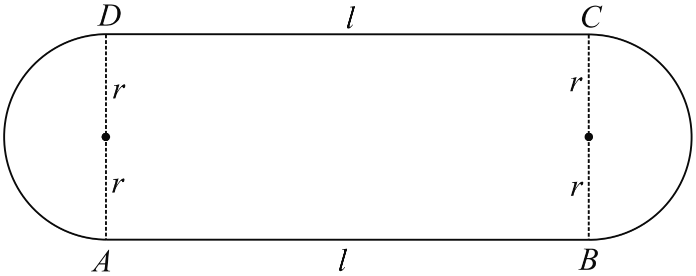


Modified Mercalli Scale	
I	Detected only by sensitive instruments
II	Felt by few persons at rest, especially on upper floors; delicately suspended objects may swing.
III	Felt noticeably indoors, but not always recognised as earthquake; standing autos rock slightly, vibration like passing truck.
IV	Felt indoors by many, outdoors by few, at night some may awaken; dishes, windows, doors disturbed; autos rock noticeably
V	Felt by most people; some breakage of dishes, windows, and plaster; disturbance of tall objects
VI	Felt by all, many frightened and run outdoors; falling plaster and chimneys; sand and mud ejected; drivers of autos disturbed
VII	Everybody runs outdoors; damage to buildings varies depending on quality of construction; noticed by drivers of autos
VIII	Panel walls thrown out of frames; fall of walls, monuments, chimneys; sand and mud ejected; drivers of autos disturbed
IX	Buildings shifted off foundations, cracked, thrown out of plumb; ground cracked; underground pipes broken
X	Most masonry and frame structures destroyed; ground cracked, rails bent, landslides
XI	Few structures remain standing; bridges destroyed, fissures in ground, pipes broken, landslides, rails bent
XII	Damage total; waves seen on ground surface, lines of sight and level distorted, objects thrown up in air

Question 8 (f)

$$E=1.74\times10^{(5+1.44\times4.3)}=2.7\times10^{11}\text{ J}$$

QUESTION 9 (50 MARKS)



Question 9 (a)

$P = 2l + 2\pi r$

Question 9 (b)

$400 = 2l + 2\pi r$

$200 = l + \pi r$

$r = \dfrac{200 - l}{\pi}$

Question 9 (c)

$A = 2rl = \dfrac{2l(200 - l)}{\pi} = \dfrac{400}{\pi}l - \dfrac{2}{\pi}l^2$

Question 9 (d)

$\dfrac{dA}{dl} = \dfrac{400}{\pi} - \dfrac{4}{\pi}l$

$\dfrac{dA}{dl} = 0 \Rightarrow \dfrac{400}{\pi} - \dfrac{4}{\pi}l = 0$

$\dfrac{400}{\pi} = \dfrac{4}{\pi}l$

$\therefore l = 100 \text{ m}$

Question 9 (e) (i)

A: $s = ut \Rightarrow 100 = 9.1t$

$\therefore t = 10.99 \text{ s}$

B: $s = 5t + 0.56t^2$

$100 = 5t + 0.56t^2$

$\therefore 0.56t^2 + 5t - 100 = 0$

$t = \dfrac{-5 \pm \sqrt{25 + 224}}{2 \times 0.56} = 9.62 \text{ s}$

Question 9 (e) (ii)

B wins the race.

Question 9 (e) (iii)

$9.1t = 5t + 0.56t^2$

$0.56t^2 - 4.1t = 0$

$t(0.56t - 4.1) = 0$

$t = \dfrac{4.1}{0.56} = 7.32 \text{ s}$

$s = 9.1t = 9.1(7.32) = 66.6 \text{ m}$

SAMPLE PAPER 2: PAPER 2

QUESTION 1 (25 MARKS)

Question 1 (a)

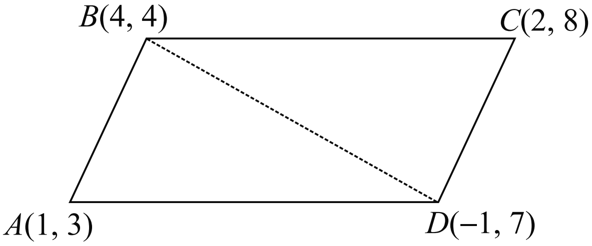
The diagonal bisects the area of a parallelogram. Find the area of triangle *ABD* and multiply the answer by 2.

$A(1, 3) \to (0, 0)$

$B(4, 4) \to (3, 1)$

$D(-1, 7) \to (-2, 4)$

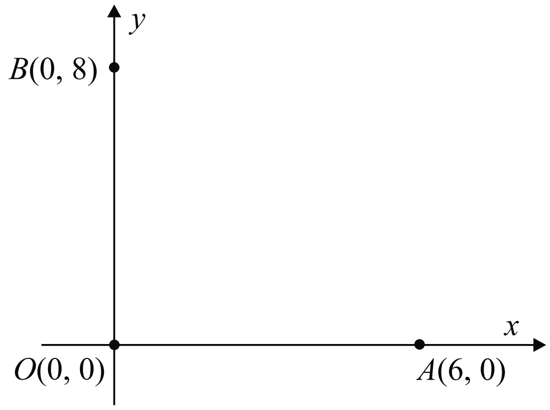
$$A = \tfrac{1}{2}|x_1y_2 - x_2y_1|$$
$$= \tfrac{1}{2}|3(4) - (-2)(1)|$$
$$= \tfrac{1}{2}|12 + 2|$$
$$= 7$$



Area of parallelogram *ABCD* = 14

NOTE: The diagram in the question is a rough sketch showing the relative positions of the points. A grid is drawn whenever we wish to display the absolute positions of the points.

Question 1 (b)



Intercepts of $2x + 3y = c$: $D(\tfrac{1}{2}c, 0)$, $E(0, \tfrac{1}{3}c)$

$O(0, 0)$, $D(\tfrac{1}{2}c, 0)$, $E(0, \tfrac{1}{3}c)$

Area $|\Delta ODE| = \tfrac{1}{2}|(0)(0) - (\tfrac{1}{2}c)(\tfrac{1}{3}c)| = \tfrac{1}{2}|\tfrac{1}{6}c^2|$

$O(0, 0)$, $A(6, 0)$, $B(0, 8)$

Area $|\Delta OAB| = \tfrac{1}{2}|(0)(0) - (6)(8)| = \tfrac{1}{2}|48|$

Area $|\Delta ODE| = \tfrac{1}{2}$ Area $|\Delta OAB|$

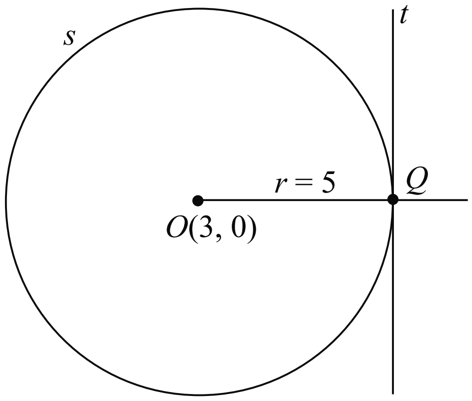
$\therefore \tfrac{1}{2}|\tfrac{1}{6}c^2| = \tfrac{1}{2} \times \tfrac{1}{2}|48|$

$\tfrac{1}{6}c^2 = 24$

$c^2 = 144$

$\therefore c = \pm\sqrt{144} = \pm 12$

Question 2 (25 marks)



Slope of t: $m=-\frac{4}{3}$

Circle s: Centre $O(3, 0)$, $r = 5$

Equation of OQ: Point $O(3, 0)$, $m=\frac{3}{4}$

$m=\frac{3}{4}$

$y-0=\frac{3}{4}(x-3)$

$4y=3x-9$

$3x-4y-9=0$

Find the point of intersection of lines t and OQ.

$4x+3y+13=0(\times 3)$

$3x-4y-9=0(\times -4)$

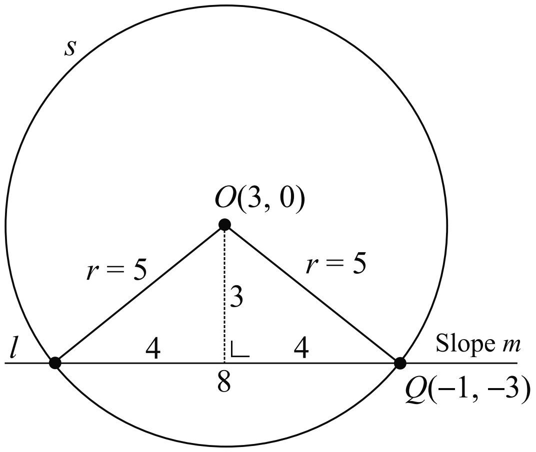
$$\begin{array}{r} 12x+9y+39=0 \\ -12x+16y+36=0 \\ \hline 25y+75=0 \Rightarrow y=-3 \end{array}$$

$y=-3: 4x+3(-3)+13=0 \Rightarrow 4x=-4$

$\therefore x=-1$

Point of contact $Q(-1, -3)$

The perpendicular line from O bisects the chord.
The distance from the centre to the chord is 3.



The chord passing through Q is called l.

$l: mx-y+k=0$

$Q(-1,-3)\in l: -m+3+k=0 \Rightarrow k=m-3$

$l: mx-y+m-3=0$

$d=\frac{|ax_1+by_1+c|}{\sqrt{a^2+b^2}}$

$d=3, (x_1, y_1)=(3, 0)$

$l: mx-y+m-3=0$

$3=\frac{|m(3)-(0)+m-3|}{\sqrt{m^2+1}}$ [The perpendicular distance of the chord l from the centre O is 3.]

$3\sqrt{m^2+1}=|3m+m-3|$

$3\sqrt{m^2+1}=|4m-3|$

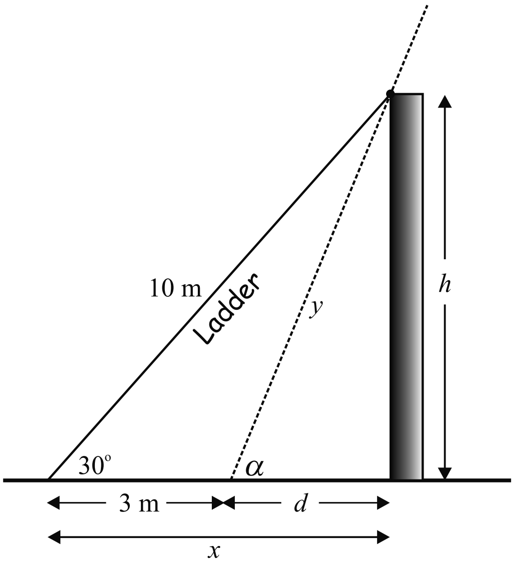
$9m^2+9=16m^2-24m+9$

$7m^2-24m=0$

$m(7m-24)=0$

$m=0, \frac{24}{7}$

QUESTION 3 (25 MARKS)



Question 3 (a)

$$\sin 30^{\circ} = \frac{h}{10} \Rightarrow h = 10\sin 30^{\circ} = 5 \text{ m}$$

Question 3 (b)

$$\tan 30^{\circ} = \frac{5}{x} \Rightarrow x = \frac{5}{\tan 30^{\circ}} = 5\sqrt{3} \text{ m}$$

$$d = 5\sqrt{3} - 3 = 5.66 \text{ m}$$

Question 3 (c)

$$\tan\alpha = \frac{5}{5.66} \Rightarrow \alpha = \tan^{-1}\left(\frac{5}{5.66}\right) = 41.46^{\circ}$$

Question 3 (d)

$$\sin\alpha = \frac{5}{y}$$

$$\sin 41.46^{\circ} = \frac{5}{y}$$

$$y = \frac{5}{\sin 41.46^{\circ}} = 7.55 \text{ m}$$

Length of ladder protruding beyond the top of the wall = 10 − 7.55 = 2.45 m

QUESTION 4 (25 MARKS)

Question 4 (a)

Condition: There are only two possible outcomes (success or failure) in each trial.
Condition: There is a fixed number of trials *n*.
Condition: The probability of success *p* is fixed from trial to trial.
Condition: The trials are independent.
Condition: The binomial random variable is the number of successes in *n* trials.

Question 4 (b) (i)

$$P(\text{Income} < €32\,000) = \tfrac{1}{2},\ P(\text{Income} > €32\,000) = \tfrac{1}{2}$$

$$P(x \geq 8) = {}^{10}C_8(\tfrac{1}{2})^2(\tfrac{1}{2})^8 + {}^{10}C_9(\tfrac{1}{2})^1(\tfrac{1}{2})^9 + {}^{10}C_{10}(\tfrac{1}{2})^0(\tfrac{1}{2})^{10} = 0.055$$

Question 4 (b) (ii)

$$P(x \geq 2) = 1 - P(\text{None or } 1) = 1 - {}^{10}C_0(\tfrac{1}{2})^{10}(\tfrac{1}{2})^0 - {}^{10}C_1(\tfrac{1}{2})^9(\tfrac{1}{2})^1 = 0.99$$

Question 5 (25 marks)

Question 5 (a)

Life expectancy (years)	Number of countries	Life expectancy (years)	Cumulative number of countries
30–35	2	< 35	2
35–40	3	< 40	5
40–45	9	< 45	14
45–50	12	< 50	26
50–55	22	< 55	48
55–60 (LQ)	46	< 60	94
60–65 (M)	58	< 65	152
65–70	50	< 70	202
70–75 (UQ)	40	< 75	242
75–80	32	< 80	274
80–85	16	< 85	290
	290		

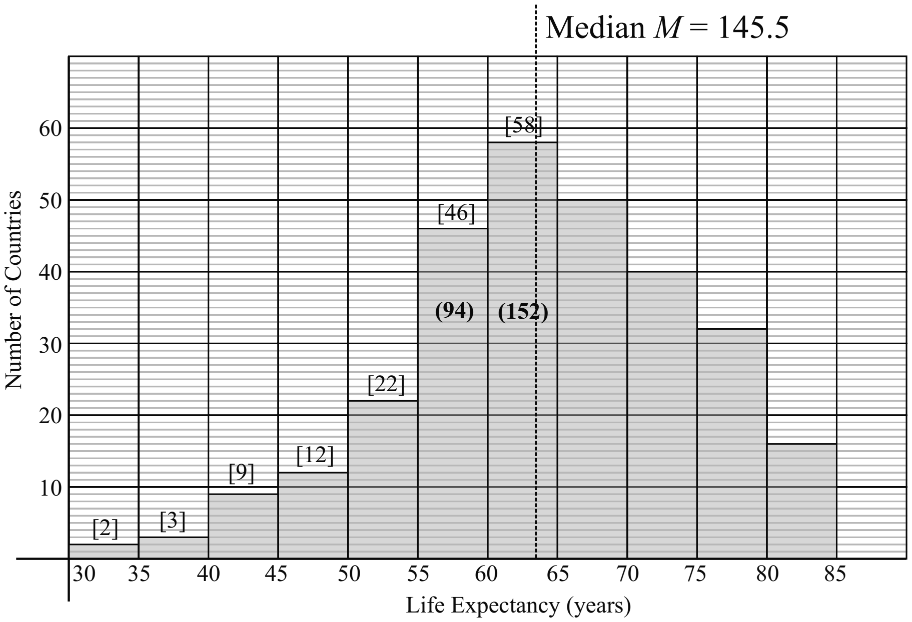
Question 5 (b)

Number of countries = 290

Median class: 290(0.5) + 0.5 = 145.5 (Class 60–65)

Upper quartile class: 290(0.75) + 0.75 = 218.25 (Class 70–75)

Lower quartile class: 290(0.25) + 0.25 = 72.75 (Class 55–60)



Question 5 (c)

$$M = 60 + \left(\frac{145.5 - 94}{58}\right)5 = 64.4$$

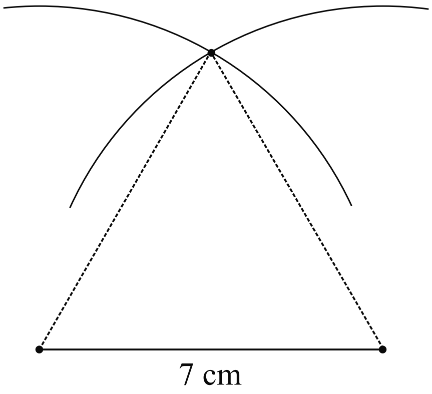
Question 5 (d)

Modal class: 60–65

Shape: Skewed left

QUESTION 6 (25 MARKS)

Question 6 (a)



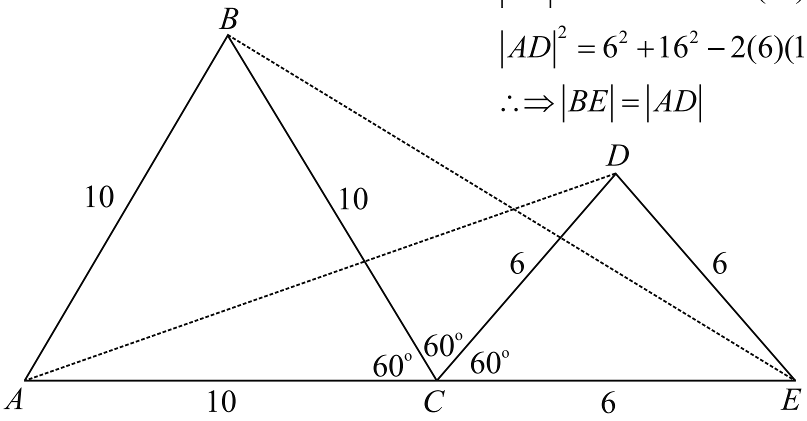
Draw a line of length 7 cm with your ruler.
Open your compass to a radius of 7 cm using this line.
Placing the point of the compass on each end point of the line draw arcs of radius 7 cm.
Draw lines from the end points of the line to the point of intersection of the two arcs.
An equilateral triangle of side 7 cm has been constructed.

Question 6 (b)

$$|BE|^2 = 10^2 + 16^2 - 2(10)(16)\cos 60^\circ = 196 \Rightarrow |BE| = 14$$

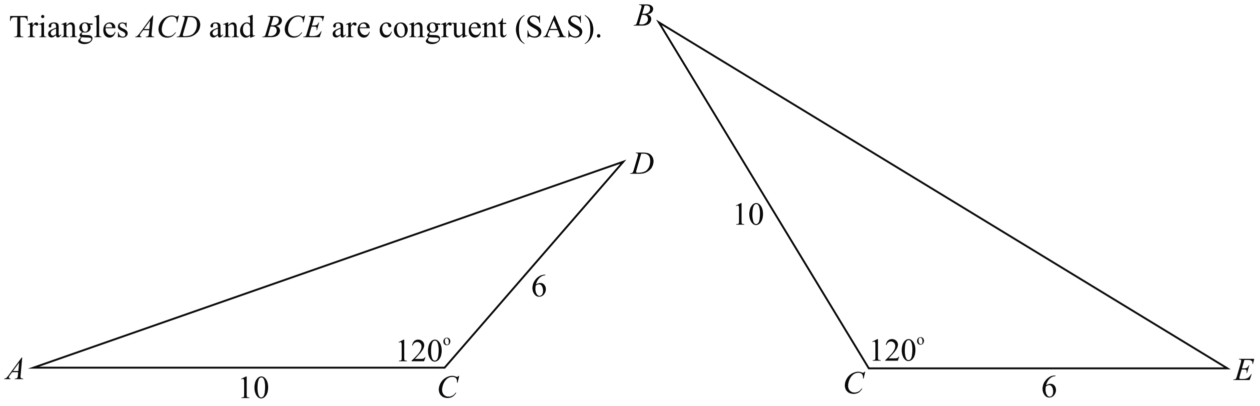
$$|AD|^2 = 6^2 + 16^2 - 2(6)(16)\cos 60^\circ = 196 \Rightarrow |AD| = 14$$

$$\therefore \Rightarrow |BE| = |AD|$$



Question 6 (c)

Triangles *ACD* and *BCE* are congruent (SAS).



Question 7 (75 marks)

Question 7 (a) (i)

$$R = \frac{100I}{\text{GDP}}$$

$R = 10.1\%$, GDP = €159 billion, $I = ?$

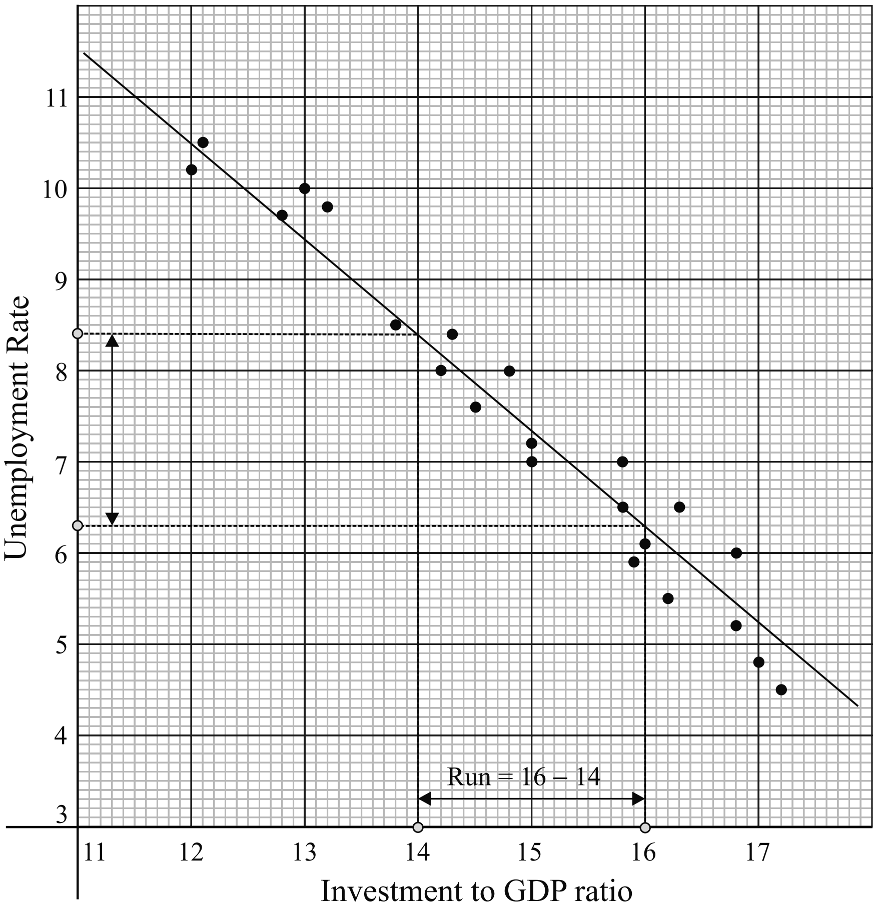
$$10.1 = \frac{100I}{159}$$

$$\therefore I = \frac{159 \times 10.1}{100} = €16.1 \text{ billion}$$

Question 7 (a) (ii)

$$\text{GDP per capita } = \frac{€159\,000\,000\,000}{4\,588\,252} = €34\,654$$

Question 7 (b)



Question 7 (c) (i)

The data shows that the most effective way to reduce *the unemployment rate* is to *increase investment to GDP ratio*.

Question 7 (c) (ii)

A. Linear,
B. Negative shape,
C. Very strong correlation

Question 7 (d)

CASIO CALCULATOR (*fx-85GT PLUS*)
Steps to find r:
Press Mode.
Press 2: Stat
Press 2: $A + Bx$
Input your x and y values
Press AC Button
Press Shift followed by the Number 1
Press 5: Reg
Press 3: r
Press =

$r = –0.9767$

Question 7 (e) (i)

(14, 8.4), (16, 6.3)

$$m = \frac{8.4 - 6.3}{14 - 16} = -1.1$$

Question 7 (e) (ii)

$m = -1.1,\ (x_1,\ y_2) = (14,\ 8.4)$

$(y - 8.4) = -1.1(x - 14)$

$y - 8.4 = -1.1x + 15.4$

$1.1x + y - 23.8 = 0$

Question 7 (e) (iii)

$y = 3 : 1.1x + (3) - 23.8 = 0$

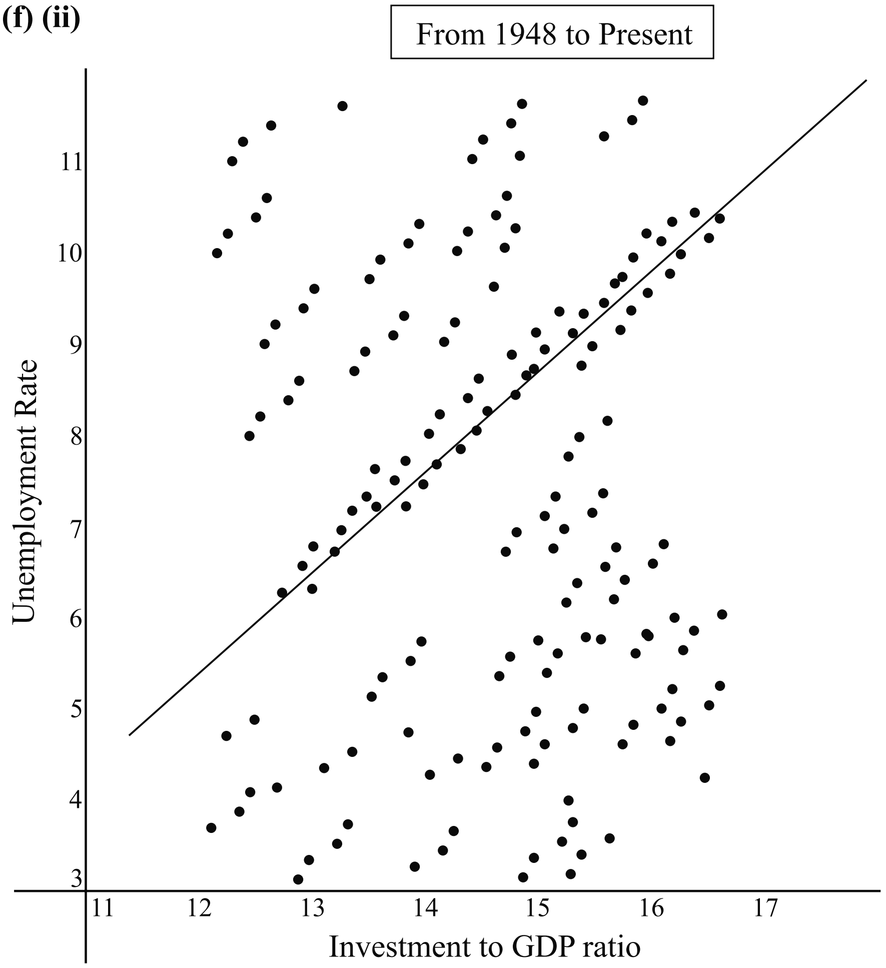
$\therefore x = 18.9$

Question 7 (f) (i)

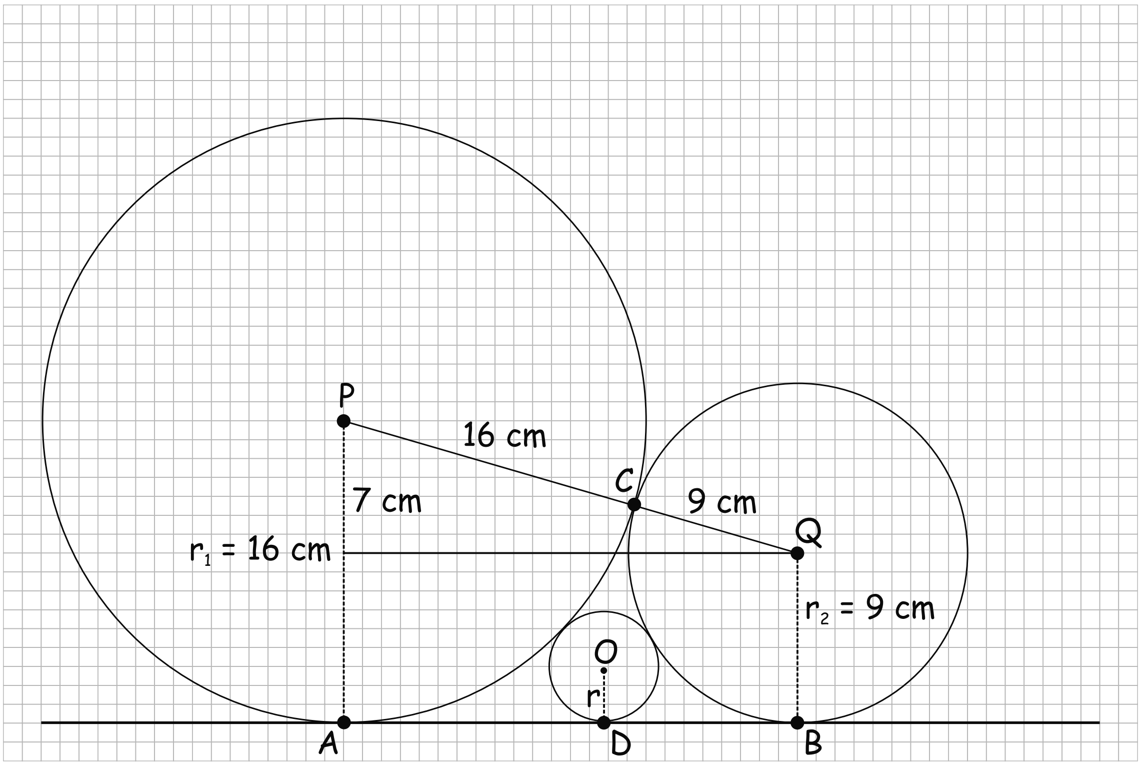
The 11 year plot is misleading as the 64 year plot shows little correlation between the unemployment rate and investment to GDP ratio. It is a prime example of someone using statistics to support their own position.

Question 7 (f) (ii)

Yes



QUESTION 8 (75 MARKS)



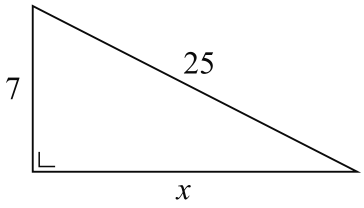
Question 8 (a) (i)

$r_1 = 16$ cm
$r_2 = 9$ cm

Question 8 (a) (ii)

$|PQ| = |PC| + |CQ| = r_1 + r_2$
$= 16$ cm $+ 9$ cm $= 25$ cm

Question 8 (b) (i)



$$x^2 + 7^2 = 25^2$$

$$x = \sqrt{25^2 - 7^2} = 24 \text{ cm}$$

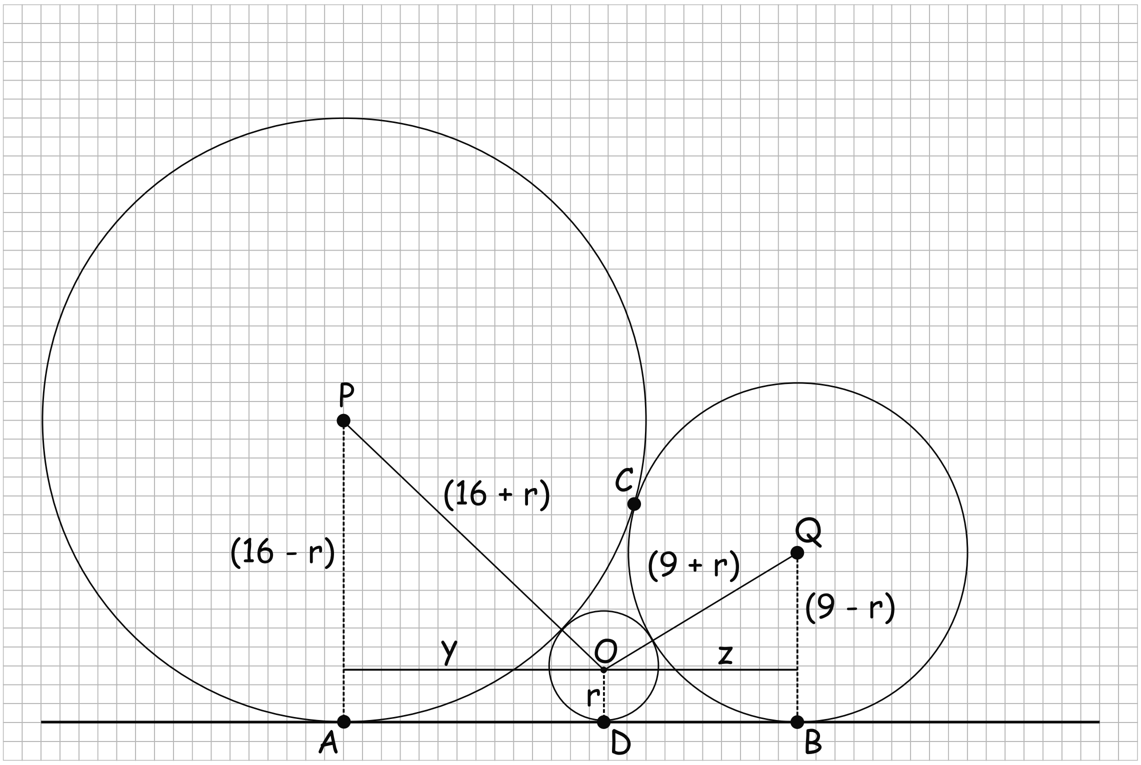
Question 8 (b) (ii)

$$\tan(|\angle APQ|) = \frac{24}{7}$$

$$\therefore |\angle APQ| = \tan^{-1}\left(\frac{24}{7}\right)$$

$$= 73.74^\circ = 1.287 \text{ rads}$$

$$A = \tfrac{1}{2} r^2\theta = \tfrac{1}{2}(16)^2(1.287) = 164.7 \text{ cm}^2$$



Question 8 (c) (i)

$y^2 + (16 - r)^2 = (16 + r)^2$

$y^2 = (16 + r)^2 - (16 - r)^2$

$y^2 = (16 + r + 16 - r)(16 + r - 16 + r)$

$y^2 = (32)(2r) = 64r$

$y = 8\sqrt{r}$

Question 8 (c) (ii)

$z^2 + (9 - r)^2 = (9 + r)^2$

$z^2 = (9 + r)^2 - (9 - r)^2$

$z^2 = (9 + r + 9 - r)(9 + r - 9 + r)$

$z^2 = (18)(2r) = 36r$

$z = 6\sqrt{r}$

Question 8 (d)

$y + z = 24$

$8\sqrt{r} + 6\sqrt{r} = 24$

$14\sqrt{r} = 24$

$\sqrt{r} = \dfrac{24}{14} = \dfrac{12}{7}$

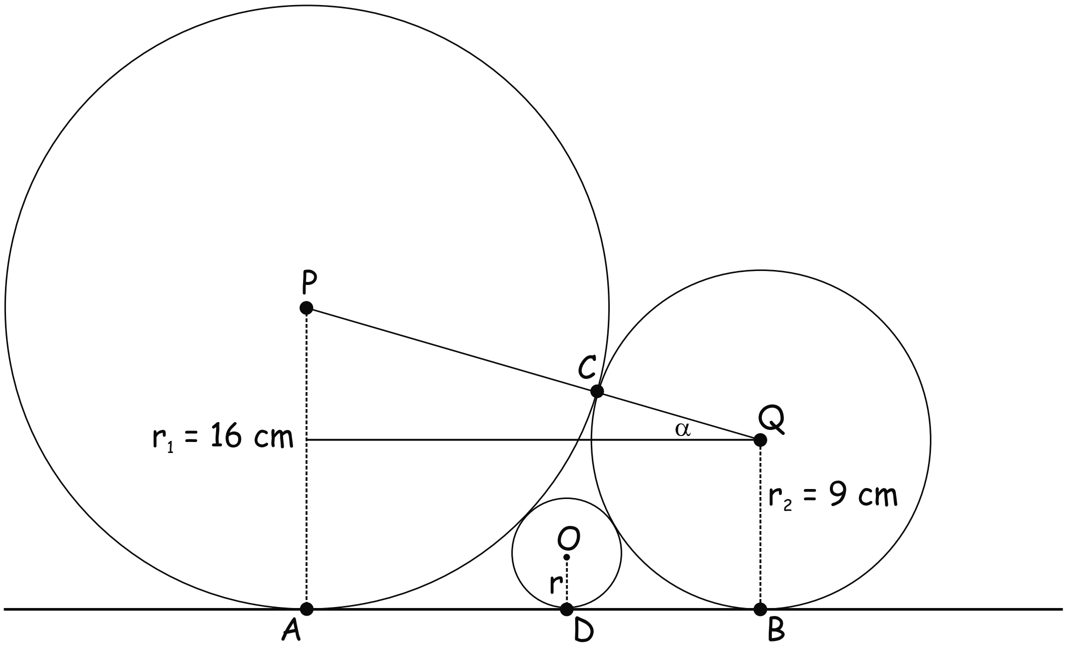
$\therefore r = \dfrac{144}{49}$

Question 8 (e)

Area of trapezium *ABQP*: $A = \frac{1}{2}(16 + 9)(24) = 25(12) = 300 \text{ cm}^2$

Question 8 (f)

Area of space between wheels
= Area of trapezium – Area of sector *APC* – Area of sector *CQB* – Area of smallest wheel



Area of sector *CQB*:

$\alpha = 90^{\circ} - 73.74^{\circ} = 16.26^{\circ}$

$|\angle CQB| = 16.26^{\circ} + 90^{\circ} = 106.26^{\circ} = 1.855$ rads

$A = \frac{1}{2}r^2\theta = \frac{1}{2}(9)^2(1.855) = 75.13$ cm^2

Area of space between wheels $= 300 - 164.7 - 75.13 - \pi\left(\frac{144}{49}\right)^2 \approx 33$ cm^2

% of air space to area of three wheels $= \dfrac{33}{\pi\left[16^2 + 9^2 + \left(\frac{144}{49}\right)^2\right]} \times 100\% \approx 3\%$

Sample Paper 3: Paper 1

Question 1 (25 marks)

Question 1 (a)

$$\left(1-\frac{1}{x}\right)^2=2$$

$$1-\frac{1}{x}=\pm\sqrt{2}$$

$$1\pm\sqrt{2}=\frac{1}{x}$$

$$x=\frac{1}{1+\sqrt{2}},\ \frac{1}{1-\sqrt{2}}$$

Rationalise the denominators by multiplying above and below by the conjugate of the denominator.

$$\frac{1}{(1+\sqrt{2})}\times\frac{(1-\sqrt{2})}{(1-\sqrt{2})}=\frac{1-\sqrt{2}}{1-2}=\sqrt{2}-1$$

$$\frac{1}{(1-\sqrt{2})}\times\frac{(1+\sqrt{2})}{(1+\sqrt{2})}=\frac{1+\sqrt{2}}{1-2}=-\sqrt{2}-1$$

Question 1 (b)

$x^3+3x^2+x-2=0$

$x=-2:(-2)^3+3(-2)^2-2-2=0$

$\therefore (x+2)$ is a factor

$\therefore x^3+3x^2+x-2=(x+2)(x^2+kx-1)$

$\qquad\qquad\qquad\quad =x^3+(k+2)x^2+(2k-1)x-2$ [Line up the coefficients.]

$\therefore 3=k+2\Rightarrow k=1$

$\therefore x^3+3x^2+x-2=(x+2)(x^2+x-1)=0$

$x^2+x-1=0$ [Solve the quadratic equation.]

$$x=\frac{-1\pm\sqrt{1^2-4(1)(-1)}}{2(1)}$$

$$=\frac{-1\pm\sqrt{1+4}}{2}=\frac{-1\pm\sqrt{5}}{2}$$

$$\therefore x=-2,\ \frac{-1\pm\sqrt{5}}{2}$$

QUESTION 2 (25 MARKS)

Question 2 (a)

$$\log_4(6x+1)-2=2\log_4 x$$

$$\log_4(6x+1)-2\log_4 x=2$$

$$\boxed{\log_a\left(\frac{x}{y}\right)=\log_a x-\log_a y}$$

$$\log_4\left(\frac{6x+1}{x^2}\right)=2$$

$$\boxed{a^x=y \Leftrightarrow \log_a y=x}$$

$$\frac{6x+1}{x^2}=4^2$$

$$6x+1=16x^2$$

$$16x^2-6x-1=0$$

$(8x+1)(2x-1)=0$ [$x=-\frac{1}{8}$ is not allowed as $\log_4(-\frac{1}{8})$ is not defined.]

$$x=\tfrac{1}{2}$$

Question 2 (b) (i)

$$F\left(\frac{a}{b}\right)=F(a)-F(b)$$

$$F(1)\Rightarrow a=b$$

$$F(1)=F(a)-F(a)=0$$

Question 2 (b) (ii)

$$F\left(\frac{1}{x}\right)=F(1)-F(x)$$

$$=0-F(x)$$

$$=-F(x)$$

Question 2 (b) (iii)

$$F(y^2)=F\left(\frac{y}{\frac{1}{y}}\right)=F(y)-F\left(\frac{1}{y}\right)$$

$$=F(y)-[F(1)-F(y)]$$

$$=F(y)-0+F(y)$$

$$=2F(y)$$

QUESTION 3 (25 MARKS)

Question 3 (a)

Three consecutive numbers: $n - 1, n, n + 1$

$S = (n-1)^2 + n^2 + (n+1)^2$

$= n^2 - 2n + 1 + n^2 + n^2 + 2n + 1$

$= 3n^2 + 2$

$$\frac{3n^2 + 2}{3} = n^2 + \tfrac{2}{3}$$

Question 3 (b)

STEPS FOR PROOF BY INDUCTION

1. Prove result is true for some starting value of $n \in \mathbb{N}$.
2. Assume result is true for $n = k$.
3. Prove result is true for $n = (k + 1)$.

1. Prove true for $n = 1$: $n = 1 : 7^1 + 3(1) + 8 = 18$ [Therefore, true for $n = 1$.]

2. Assume true for $n = k$: Assume $n = k : 7^k + 3k + 8 = 9a,\ a \in \mathbb{N}$

3. Prove true for $n = k + 1$: Prove $7^{k+1} + 3(k+1) + 8 = 9b,\ b \in \mathbb{N}$

Proof:

$7^{k+1} + 3(k+1) + 8$

$= 7(7^k) + 3k + 3 + 8$

$= 7(9a - 3k - 8) + 3k + 11$

$= 63a - 21k - 56 + 3k + 11$

$= 63a - 18k - 45$

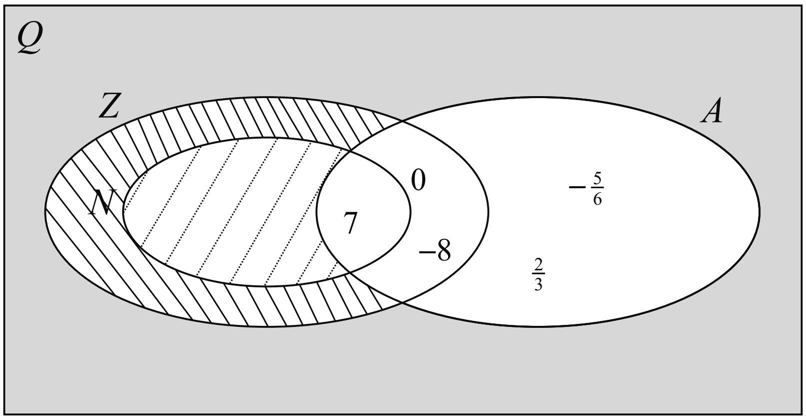
$= 9[7a - 2k - 5]$

$= 9b,\ b \in \mathbb{N}$

Therefore, assuming true for $n = k$ means it is true for $n = k + 1$. So true for $n = 1$ and true for $n = k$ means it is true for $n = k + 1$. This implies it is true for all $n \in \mathbb{N}$.

QUESTION 4 (25 MARKS)

Question 4 (a) (i) & (ii)



Question 4 (b) (i)

$S = \{6, 12, 18, 24,.....\}$
$S_1 = \{3, 6, 9, 12,...\}$
No. 9 is an element of S_1 but not an element of S.

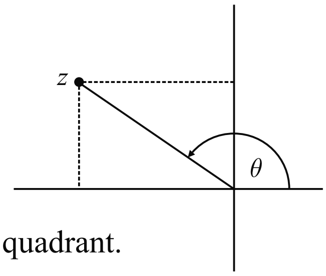
Question 4 (b) (iii)

$S = \{6, 12, 18, 24,.....\}$
$S_3 = \{12, 24, 36, 48,...\}$
Yes, all elements of S_3 are elements of S.

Question 4 (b) (ii)

$S = \{6, 12, 18, 24,.....\}$
$S_2 = \{9, 18, 27, 36,...\}$
No. 9 is an element of S_2 but not an element of S.

Question 4 (c)



$z = -4\sqrt{3} + 4i$

$$|z| = \sqrt{(-4\sqrt{3})^2 + 4^2} = \sqrt{48+16} = \sqrt{64} = 8 \quad \boxed{|z| = r,\ \arg z = \theta}$$

$$|\tan\theta| = \left|\frac{4}{-4\sqrt{3}}\right| = \frac{1}{\sqrt{3}} = \tan\alpha,$$ where α is the related angle in the first quadrant.

$$\therefore \alpha = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = 30^\circ = \frac{\pi}{6}$$

Second quadrant: $\theta = \pi - \frac{\pi}{6} = \frac{5\pi}{6}$ $\quad \boxed{z = r(\cos\theta + i\sin\theta)}$

$$\therefore z = 8\left\{\cos\left(\frac{5\pi}{6} + 2n\pi\right) + i\sin\left(\frac{5\pi}{6} + 2n\pi\right)\right\} = 8\left\{\cos\left(\frac{5\pi + 12n\pi}{6}\right) + i\sin\left(\frac{5\pi + 12n\pi}{6}\right)\right\}$$

$$z^{\frac{1}{3}} = 8^{\frac{1}{3}}\left\{\cos\left(\frac{5\pi + 12n\pi}{6}\right) + i\sin\left(\frac{5\pi + 12n\pi}{6}\right)\right\}^{\frac{1}{3}} = 2\left\{\cos\left(\frac{5\pi + 12n\pi}{18}\right) + i\sin\left(\frac{5\pi + 12n\pi}{18}\right)\right\}$$

$$n = 0 : z_1 = 2\left\{\cos\left(\frac{5\pi}{18}\right) + i\sin\left(\frac{5\pi}{18}\right)\right\}$$

$$n = 1 : z_2 = 2\left\{\cos\left(\frac{17\pi}{18}\right) + i\sin\left(\frac{17\pi}{18}\right)\right\}$$

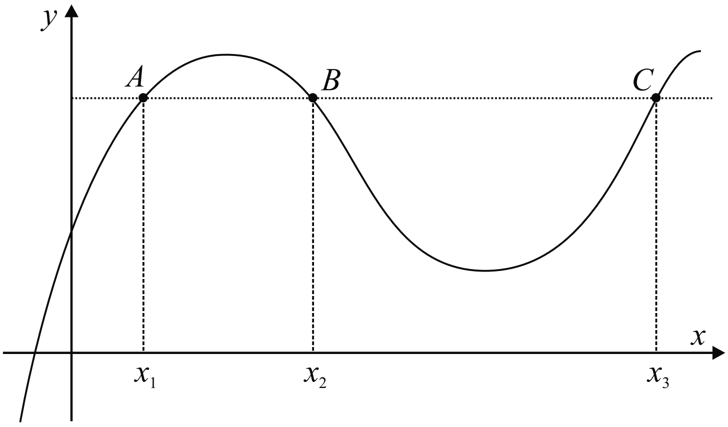
$$n = 2 : z_3 = 2\left\{\cos\left(\frac{29\pi}{18}\right) + i\sin\left(\frac{29\pi}{18}\right)\right\}$$

QUESTION 5 (25 MARKS)

Question 5 (a)

It is not injective because there is more than one x value mapping on to the same y value.

It is surjective because every y value has a corresonding x value.



Question 5 (b)

$$f(x) = y = \frac{\ln x}{x}$$

$$\frac{dy}{dx} = \frac{x(\frac{1}{x}) - (\ln x)(1)}{x^2} = \frac{1 - \ln x}{x^2}$$

$$\frac{dy}{dx} = 0 \Rightarrow \frac{1 - \ln x}{x^2} = 0$$

$$1 - \ln x = 0$$

$$1 = \ln x \Rightarrow x = e$$

$$f(e) = \frac{\ln e}{e} = \frac{1}{e}$$

$$\text{TP} : \left(e, \frac{1}{e} \right)$$

Question 5 (c)

$$\frac{dy}{dx} = \frac{1 - \ln x}{x^2}$$

$$\frac{d^2y}{dx^2} = \frac{x^2(-\frac{1}{x}) - (1 - \ln x)(2x)}{(x^2)^2}$$

$$= \frac{-x - 2x + 2x \ln x}{x^4}$$

$$= \frac{-3x + 2x \ln x}{x^4}$$

$$\left(\frac{d^2y}{dx^2} \right)_{x=e} = \frac{-3e + 2e \ln e}{e^4} = \frac{-3e + 2e}{e^4} = \frac{-e}{e^4} = -\frac{1}{e^3} < 0$$

$$\therefore \left(e, \frac{1}{e} \right) \text{ is a local maximum}$$

Question 5 (d)

$$a^b = b^a$$

$$\ln a^b = \ln b^a$$

$$b \ln a = a \ln b$$

$$\therefore \frac{\ln a}{a} = \frac{\ln b}{b}$$

Question 5 (e) (i)

$$2^4 = 4^2 \Rightarrow \frac{\ln 2}{2} = \frac{\ln 4}{4}$$

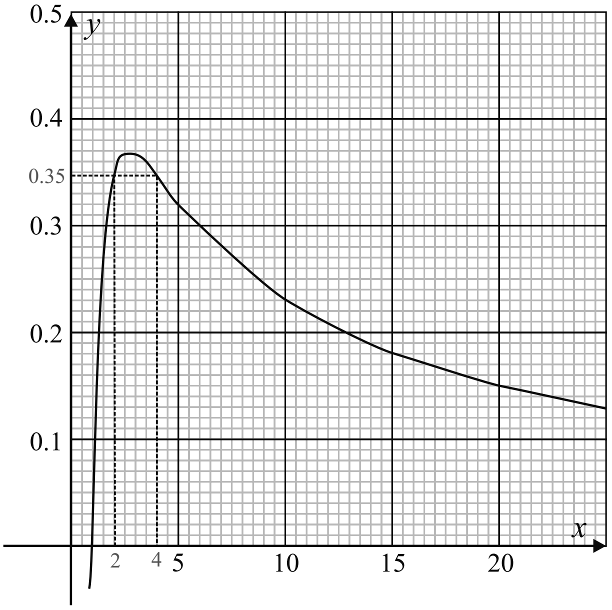
$$f(x) = \frac{\ln x}{x}$$

$$f(2) = \frac{\ln 2}{2}$$

$$f(4) = \frac{\ln 4}{4}$$

$$\therefore f(2) = f(4) = 0.35$$

Calculator: $\frac{\ln 2}{2} = \frac{\ln 4}{4} = 0.3466$



Question 5 (e) (ii)

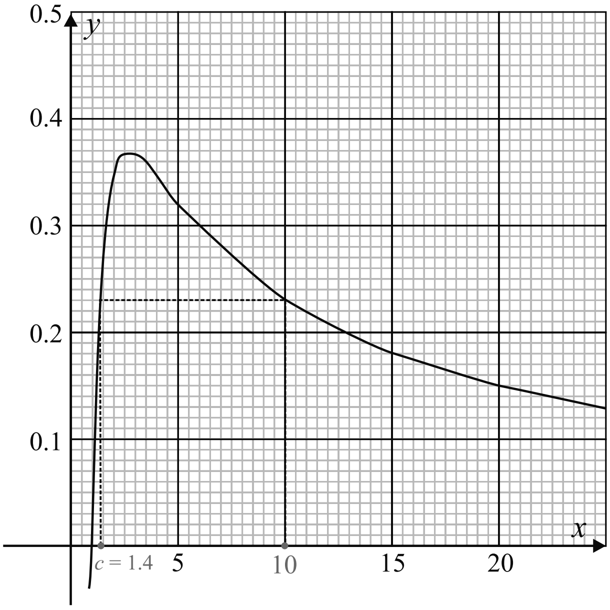
$$10^c = c^{10} \Rightarrow \frac{\ln 10}{10} = \frac{\ln c}{c}$$

$$f(x) = \frac{\ln x}{x}$$

$$f(10) = \frac{\ln 10}{10}$$

$$f(c) = \frac{\ln c}{c}$$

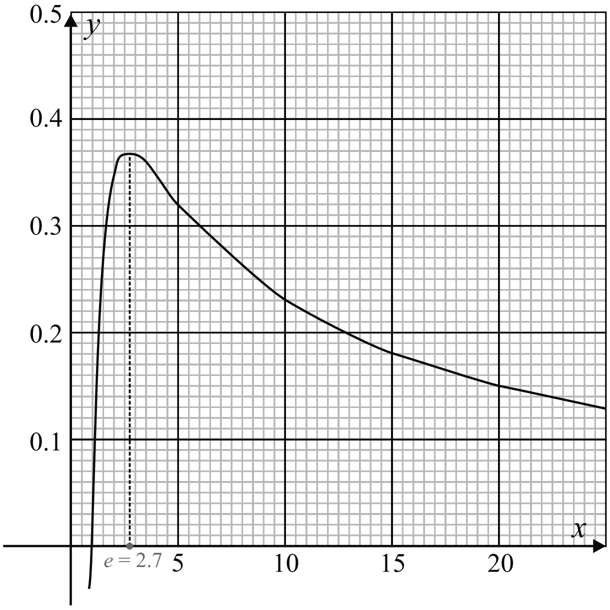
$$\therefore f(10) = f(c) \Rightarrow c \approx 1.4$$



Question 5 (e) (iii)

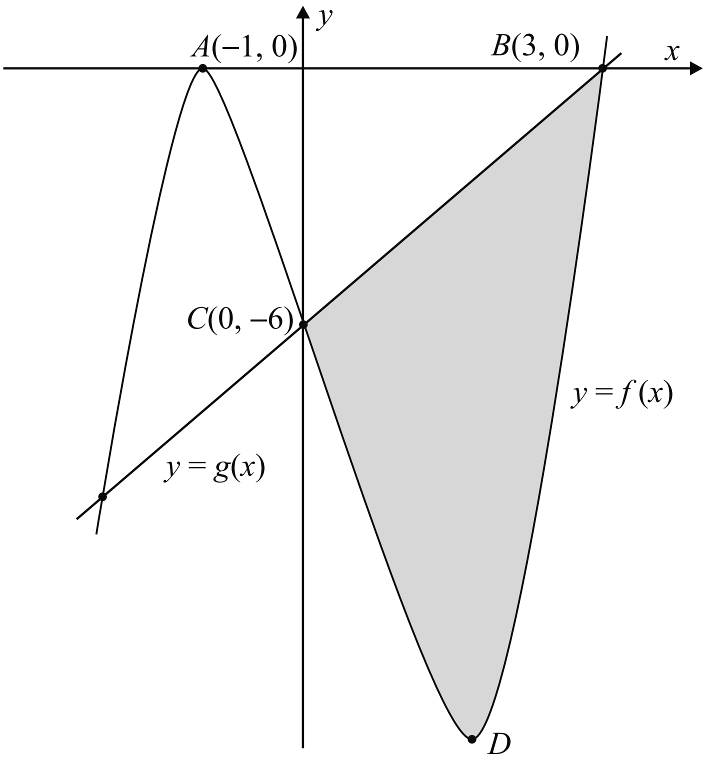
The maximum value of $f(x)$ as calculated in **part (b)** is *e*.

$\therefore e \approx 2.7$



QUESTION 6 (25 MARKS)

Question 6 (a)



$f(x)=ax^3+bx^2+cx+d$

$A(-1,\ 0)\in f(x): f(-1)=a(-1)^3+b(-1)^2+c(-1)+d=0$

$\therefore -a+b-c+d=0...(\mathbf{1})$

$B(3,\ 0)\in f(x): f(3)=a(3)^3+b(3)^2+c(3)+d=0$

$\therefore 27a+9b+3c+d=0...(\mathbf{2})$

$C(0,\ -6)\in f(x): f(0)=a(0)^3+b(0)^2+c(0)+d=-6$

$\therefore d=-6$

$\therefore -a+b-c=6...(\mathbf{1})$

$\therefore 27a+9b+3c=6\Rightarrow 9a+3b+c=2...(\mathbf{2})$

$f(x)=ax^3+bx^2+cx+d$

$f'(x)=3ax^2+2bx+c$

$A(-1,\ 0)$ is a turning point $\Rightarrow f'(-1)=3a(-1)^2+2b(-1)+c=0$

$\therefore 3a-2b+c=0...(\mathbf{3})$

Solve equations (**1**), (**2**) and (**3**) simultaneously.

$-a+b-c=6.....(\mathbf{1})$

$9a+3b+c=2...(\mathbf{2})$

$3a-2b+c=0...(\mathbf{3})$

$(\mathbf{1})+(\mathbf{2}): 8a+4b=8 \Rightarrow 2a+b=2...(\mathbf{4})$

$(\mathbf{1})+(\mathbf{3}): 2a-b=6.....(\mathbf{5})$

$\therefore 4a=8...(\mathbf{4})+(\mathbf{5})$

$\therefore a=2$

Substitute into equation $(\mathbf{4}): 2(2)+b=2 \Rightarrow b=-2$

Substitute into equation $(\mathbf{1}): -2-2-c=6 \Rightarrow c=-10$

$\therefore f(x)=2x^3-2x^2-10x-6$

Question 6 (b)

$f'(x)=6x^2-4x-10$

D is a turning point $\Rightarrow f'(x)=0 \Rightarrow 6x^2-4x-10=0$

$3x^2-2x-5=0$

$(3x-5)(x+1)=0$

$\therefore x=-1, \frac{5}{3}$

x-coordinate of $D=\frac{5}{3}$

Question 6 (c)

$B(3,\ 0)$ and $C(0,\ -6) \in g(x)$

Slope $m=\dfrac{0+6}{3-0}=2$

Equation of $g(x)$: $2x-y+k=0$

$B(3,\ 0) \in g(x): 6-0+k=0 \Rightarrow k=-6$

$2x-y-6=0$

Equation of $g(x)$: $g(x)=y=2x-6$

Question 6 (d)

$$A=\int_0^3 (g(x)-f(x))\,dx$$

$$=\int_0^3 (2x-6-2x^3+2x^2+10x+6)\,dx$$

$$=\int_0^3 (-2x^3+2x^2+12x)\,dx$$

$$=\left[-\frac{2x^4}{4}+\frac{2x^3}{3}+\frac{12x^2}{2}\right]_0^3$$

$$=\left[-\tfrac{1}{2}x^4+\tfrac{2}{3}x^3+6x^2\right]_0^3$$

$$=\{-\tfrac{1}{2}(3)^4+\tfrac{2}{3}(3)^3+6(3)^2\}-0$$

$$=\tfrac{63}{2}$$

QUESTION 7 (50 MARKS)

Question 7 (a)

$$P = \frac{F}{\left(1+\frac{r}{100}\right)^n}$$

Question 7 (c) (i)

$$P = \frac{200}{(1.06)^7} = €133 \text{ million}$$

The exploration company **should sell in 2020** because you would need €133 million now to make this money. However, the multinational company is only offering you €120 million now.

Question 7 (c) (iii)

$$120 = \frac{200}{(1+i)^7}$$

$$(1+i)^7 = \frac{200}{120} = \frac{5}{3}$$

$$1+i = \left(\frac{5}{3}\right)^{\frac{1}{7}}$$

$$\therefore i = \left(\frac{5}{3}\right)^{\frac{1}{7}} - 1 = 0.076 = 7.6\%$$

Question 7 (b)

$$P = \frac{50\,000}{(1.03)^6} = €41874$$

Question 7 (c) (ii)

$$P = \frac{200}{(1.08)^7} = €116.7 \text{ million}$$

The exploration company should **take the offer now** because you would need €116.7 million now to make this money. However, the multinational company is offering you €120 million now.

Question 7 (d) (i)

$$P = \frac{3}{1.06} = €2.83 \text{billion}$$

Question 7 (d) (ii)

Year	Reduction in Billions
1	3
2	3
3	3
4	3
5	3
6	2
7	2
8	2
9	2
10	2

$$P = \frac{3}{1.06} + \frac{3}{1.06^2} + \frac{3}{1.06^3} + \frac{3}{1.06^4} + \frac{3}{1.06^5} + \frac{2}{1.06^6} + \frac{2}{1.06^7} + \frac{2}{1.06^8} + \frac{2}{1.06^9} + \frac{2}{1.06^{10}}$$

$$= \frac{3}{1.06}\left[1 + \frac{1}{1.06} + \frac{1}{1.06^2} + \frac{1}{1.06^3} + \frac{1}{1.06^4}\right] + \frac{2}{1.06^6}\left[1 + \frac{1}{1.06} + \frac{1}{1.06^2} + \frac{1}{1.06^3} + \frac{1}{1.06^4}\right]$$

$$= \left(\frac{3}{1.06} + \frac{2}{1.06^6}\right)\left[1 + \frac{1}{1.06} + \frac{1}{1.06^2} + \frac{1}{1.06^3} + \frac{1}{1.06^4}\right]$$

$$= \left(\frac{3}{1.06} + \frac{2}{1.06^6}\right)\left[\frac{1\left(1-\left(\frac{1}{1.06}\right)^5\right)}{1-\left(\frac{1}{1.06}\right)}\right]$$

$$= €18.9 \text{ billion}$$

QUESTION 8 (50 MARKS)

Pure silver is, of course, 100% pure silver.
Britannia silver contains 95.8% pure silver.
Sterling silver contains 92.5% pure silver.

Question 8 (a)

Pure silver: 14 g
Mass of coin = 15 g

$$\text{Purity } = \frac{14 \text{ g}}{15 \text{ g}} = 93.3\%$$

Question 8 (b)

Let x = Mass of Sterling silver
Let y = Mass of Britannia silver
Mass of ring = 28 g

$$x + y = 28 \ (\times 0.925)$$
$$0.925x + 0.958y = 26.3224$$

$$0.925x + 0.925y = 25.9$$
$$0.925x + 0.958y = 26.3224$$
$$0.033y = 0.4224 \Rightarrow y = 12.8 \text{ g}$$

$$\therefore x = 28 - 12.8 = 15.2 \text{ g}$$

Question 8 (c)

$$1 \text{ T} = 31.1034768 \text{ g}$$
$$\frac{1}{31.1034768} \text{ T} = 1 \text{ g}$$
$$\frac{40}{31.1034768} \text{ T} = 40 \text{ g}$$

$$\therefore 40 \text{ g} = 1.286 \text{ T}$$

Question 8 (d)

$$S = C \times T \times p$$
$$C = 0.94$$
$$T = \frac{28}{31.1034768} = 0.9$$
$$p = \$26$$
$$S = 0.94 \times 0.9 \times 26 = \$22$$

Question 8 (e)

Question 8 (e) (i)

$S = C \times T \times p$

$C = 0.96$

$T = \dfrac{60}{31.1034768} = 1.93$

$p = \$12.50$

$S = 0.96 \times 1.93 \times 12.5 = \$23.16$

Question 8 (e) (ii)

$S = C \times T \times p$

$C = 0.96$

$T = 1.93$

$p = \$27.50$

$S = 0.96 \times 1.93 \times 27.5 = \$50.95$

Question 8 (f)

Minimum value = \$8.92

Maximum value = \$48.48

% change $= \dfrac{(48.48 - 8.92)}{8.92} \times 100\% = 443.5\%$

Question 8 (g)

$S = C \times T \times p$

$C = ?$

$T = 1$

$p = \$30$

$S = \$25$

$25 = C \times 1 \times 30$

$\therefore C = \dfrac{25}{30} = 83.33\%$

$\dfrac{90}{25} = 3.6$ $\text{T} = 3.6 \times 31.1034768 = 111.97$ g

$\dfrac{93.2}{111.97} = 0.83 = 83.3\%$

You thought you were not getting the full market value for pure silver. The scrap dealer was right.

QUESTION 9 (50 MARKS)

Question 9 (a) (i)

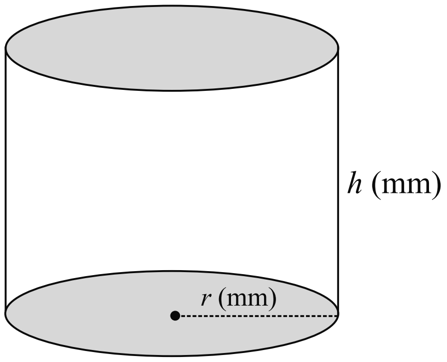
$$V = 16\pi = \pi r^2 h \Rightarrow h = \frac{16}{r^2}$$

Question 9 (a) (ii)

$$A = 2\pi r^2 + 2\pi rh$$
$$= 2\pi r^2 + 2\pi r\left(\frac{16}{r^2}\right)$$
$$= 2\pi r^2 + \frac{32\pi}{r}$$

Question 9 (a) (iii)

$$A = 2\pi r^2 + 32\pi r^{-1}$$
$$\frac{dA}{dr} = 0 \Rightarrow 4\pi r - 32\pi r^{-2} = 0$$
$$4\pi r - \frac{32\pi}{r^2} = 0 \Rightarrow r^3 = 8$$
$$\therefore r = 2 \text{ mm}$$



Question 9 (a) (iv)

$$A = 2\pi r^2 + \frac{32\pi}{r}$$
$$A_{\text{Min.}} = 2\pi(2)^2 + \frac{32\pi}{2} = 8\pi + 16\pi = 24\pi \text{ mm}^2$$

Question 9 (b) (i)

$$\frac{dm}{dt} = -e^{-\frac{1}{10}t}$$
$$\int dm = -\int e^{-\frac{1}{10}t}\, dt$$
$$m = 10e^{-\frac{1}{10}t} + c$$
$$t = 0,\ m = 10 : 10 = 10e^0 + c \Rightarrow c = 0$$
$$\therefore m = 10e^{-\frac{1}{10}t} \text{ mg}$$

Question 9 (b) (iv)

$$2 = 10e^{-\frac{1}{10}T} + 10e^{-\frac{1}{10}(T+5)}$$
$$0.2 = e^{-\frac{1}{10}T}(1 + e^{-\frac{1}{2}})$$
$$e^{-\frac{1}{10}T} = \frac{0.2}{1 + \frac{1}{\sqrt{e}}} = 0.12449$$
$$-\tfrac{1}{10}T = \ln(0.12449)$$
$$\therefore T = -10\ln(0.12449) = 20.835 \text{ hours}$$

Question 9 (b) (ii)

$$t = 5 : m = 10e^{-\frac{1}{10}(5)} = 10e^{-0.5} = 6.065 \text{ mg}$$

Question 9 (b) (iii)

$$m = 10e^{-\frac{1}{10}} + 10e^{-\frac{6}{10}} = 14.536 \text{ mg}$$

SAMPLE PAPER 3: PAPER 2

QUESTION 1 (25 MARKS)

Question 1 (a)

$s:(x-2)^2+(y+3)^2=25$

Centre $O(2,\ -3)$, $r=\sqrt{25}=5$

Centre $(h,\ k)$, Radius r
$(x-h)^2+(y-k)^2=r^2$

Substitute A into s to see if it satisfies the equation of the circle.

$A(6,\ -6)\in s$?

$(6-2)^2+(-6+3)^2$

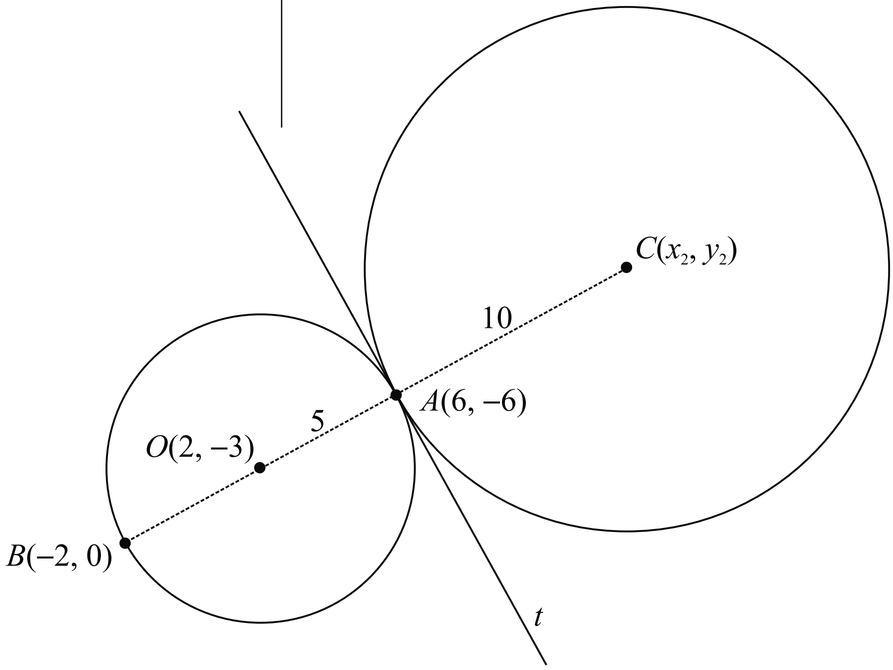
$=16+9$

$=25$

Pass A through O by a central symmetry to find B.

$A(6,\ -6)\rightarrow O(2,\ -3)\rightarrow B(-2,\ 0)$

Question 1 (b)



Slope of OA: $m_1=\dfrac{-6-(-3)}{6-2}=\dfrac{-3}{4}=-\dfrac{3}{4}$

Slope of t: $m_2=\dfrac{4}{3}$ [Perpendicular slope]

Equation of t:

$m=\frac{4}{3}$, $(x_{1,}\ y_1)=A(6,\ -6)$

$y+6=\frac{4}{3}(x-6)$

$3y+18=4x-24$

$t:4x-3y-42=0$

Question 1 (c)

OC is divided in the ratio $a{:}b$ = 1:2.

$O(2,\ -3)=(x_1,\ y_1)$, $C(x_2,\ y_2)$, $A(6,\ -6)$

Ratio: $a:b=1:2$

Point dividing $[PQ]$ in the ratio $a{:}b$ $\left(\dfrac{bx_1+ax_2}{b+a},\ \dfrac{by_1+ay_2}{b+a}\right)$

$\therefore \dfrac{2(2)+1x_2}{2+1}=6\Rightarrow 4+x_2=18\Rightarrow x_2=14$

$\therefore \dfrac{2(-3)+1y_2}{2+1}=-6\Rightarrow -6+y_2=-18\Rightarrow y_2=-12$

$\therefore C(14,\ -12)$

Equation of second circle: Centre $C(14,\ -12)$, $r=10$

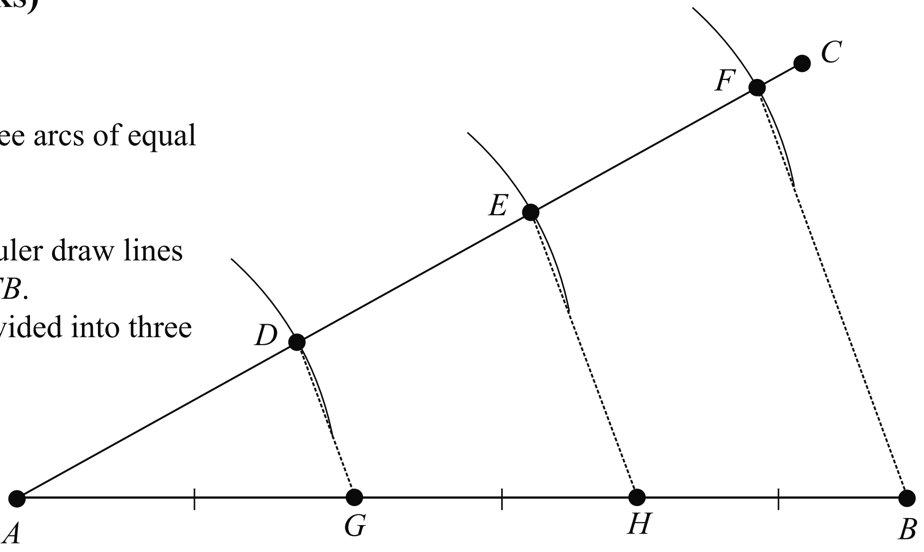
$(x-14)^2+(y+12)^2=100$

Centre $(h,\ k)$, Radius r
$(x-h)^2+(y-k)^2=r^2$

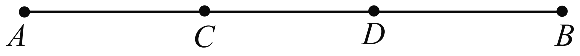
QUESTION 2 (25 MARKS)

Question 2 (a)

Draw a line AC.
On line AC mark out three arcs of equal radius with a compass.
Draw the line FB.
Using a set square and ruler draw lines EH and DG parallel to FB.
The line AB has been divided into three equal parts.



Question 2 (b)



$A(-1, 2) = (x_1, y_1)$, $B(5, -4) = (x_2, y_2)$, $C = ?$

Ratio: $a : b = 1 : 2$

$$\therefore \frac{2(-1)+1(5)}{2+1} = \frac{-2+5}{3} = 1$$

$$\therefore \frac{2(2)+1(-4)}{2+1} = \frac{4-4}{3} = 0$$

$\therefore C(1, 0)$

D is the midpoint of BC:

$B(5, -4)$, $C(1, 0)$

$$D = \left(\frac{5+1}{2}, \frac{-4+0}{2}\right) = (3, -2)$$

Question 2 (c)

$O(2, 0)$

$c : y^2 = 3x$

$P(a, b) \in c \Rightarrow b^2 = 3a$

$|PO| = \sqrt{10}$

$\therefore \sqrt{(a-2)^2 + (b-0)^2} = \sqrt{10}$

$(a-2)^2 + b^2 = 10$

$(a-2)^2 + 3a = 10$

$a^2 - 4a + 4 + 3a - 10 = 0$

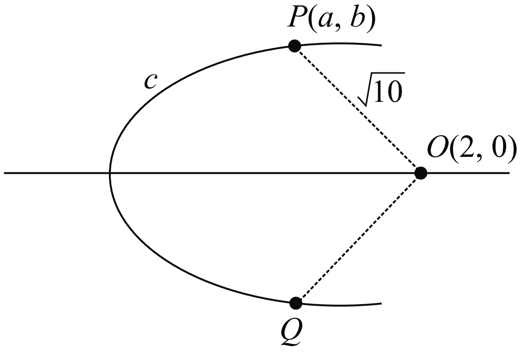
$a^2 - a - 6 = 0$

$(a+2)(a-3) = 0$

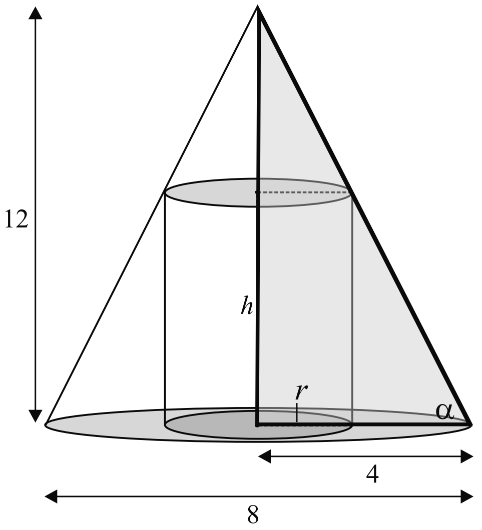
$\therefore a = \cancel{-2}, 3$

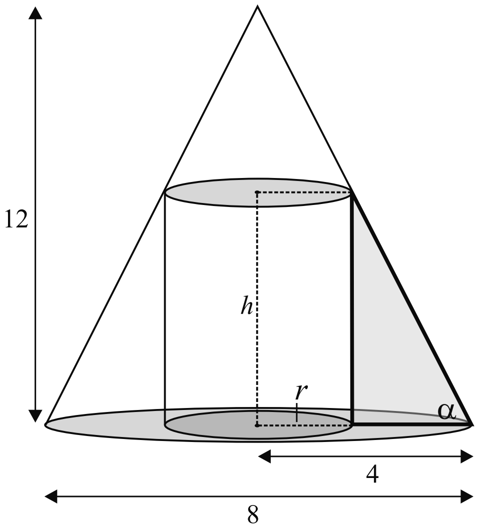
$b^2 = 3(3) = 9 \Rightarrow b = \pm 3$

$P(3, 3)$, $Q(3, -3)$



QUESTION 3 (25 MARKS)





Question 3 (a)

The two highlighted triangles are similar triangles.

$$\frac{12}{4} = \frac{h}{4-r}$$

$$3 = \frac{h}{4-r}$$

$$\therefore 12 - 3r = h$$

Question 3 (b)

$$V = \pi r^2 h = \pi r^2 (12 - 3r) = 24\pi$$

$$12r^2 - 3r^3 = 24$$

$$r^3 - 4r^2 + 8 = 0$$

Question 3 (c)

$$f(r) = r^3 - 4r^2 + 8$$

$$f(1) = 1 - 4 + 8 \neq 0$$

$$f(2) = 8 - 16 + 8 = 0$$

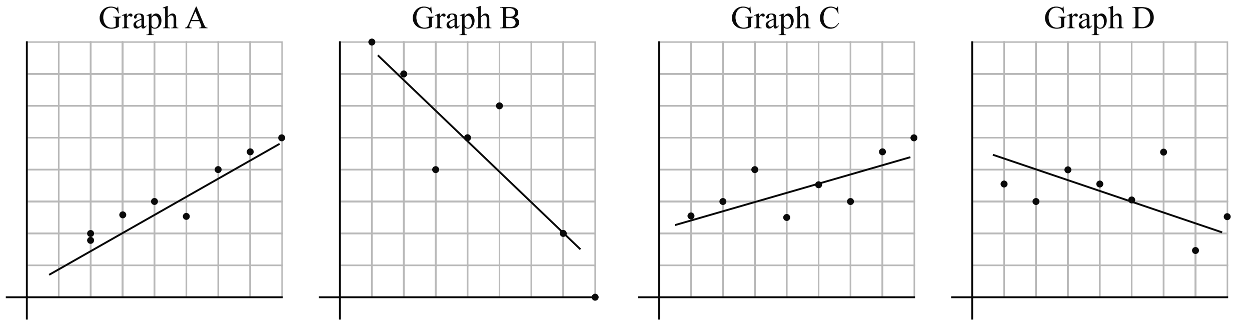
$$\therefore r = 2$$

$$h = 12 - 3r$$

$$\therefore h = 12 - 6 = 6$$

QUESTION 4 (25 MARKS)

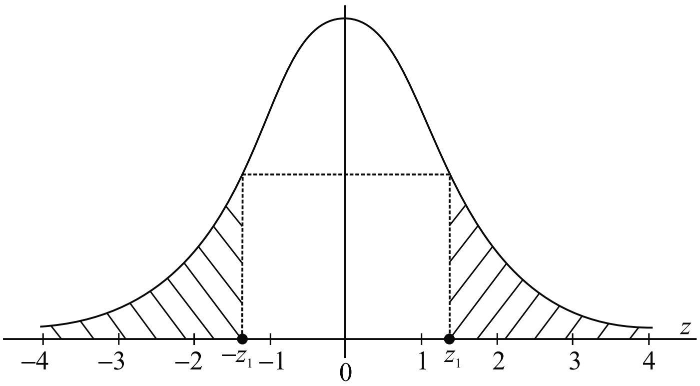
Question 4 (a)



Correlation	0.72	–0.90	0.96	–0.42
Graph	C	B	A	D

GRAPH A: Very tight clustering indicating *r* is close to 1 and a positive slope.
GRAPH B: Mostly tight clustering indicating *r* is close enough to 1 and a negative slope.
GRAPH C: Clustering is moderate indicating *r* is somewhere around 0.75 and a positive slope.
GRAPH D: Weaker clustering indicating *r* is less than 0.5 and a negative slope.

Question 4 (b) (i)



Question 4 (b) (ii)

Is $P(z > z_1) = P(z < -z_1)$? Yes

Question 4 (b) (iii)

$P(0) = 0$

$P(-\infty < z < +\infty) = 1$

Question 4 (b) (iv)

$P(z > z_1) = 1 - P(z \leq z_1)$

Question 4 (b) (v)

$P(z > 0) = 0.5$ $P(z < 0) = 0.5$

$P(z \geq 0) = 0.5$ $P(z \leq 0) = 0.5$

QUESTION 5 (25 MARKS)

G: Germinate, *NG*: Not germinate
$P(G) = 0.6$, $P(NG) = 0.4$

Question 5 (a) (i)

$$P(G, G, G, G) = (0.6)(0.6)(0.6)(0.6) = (0.6)^4 = \frac{81}{625}$$

Question 5 (a) (ii)

$$P(G, NG, NG, NG) = (0.6)(0.4)(0.4)(0.4) \times \frac{4!}{3!} = \frac{96}{625}$$

Question 5 (a) (iii)

$$P(NG, NG, NG, NG) = (0.4)(0.4)(0.4)(0.4) = (0.4)^4 = \frac{16}{625}$$

Question 5 (b)

Independence is necessary to do the calculation in **(a)** as the probability would change depending on whether or not the other seeds germinate. Independence may not be valid as the growing conditions (temperature, moisture and soil type) may vary from seed to seed.

Question 5 (c)

Expected value = $xP(x) = 4(0.6) = 2.4$

Question 5 (d)

Null hypothesis H_o: $P = 0.9$
Alternative hypothesis H_1: $P < 0.9$
Sample proportion = $p = \frac{97}{120}$

True population proportion = Sample proportion ± 1.96(Standard error of the proportion)

$$\text{Standard error of the proportion} = \sqrt{\frac{p(1-p)}{n}}$$

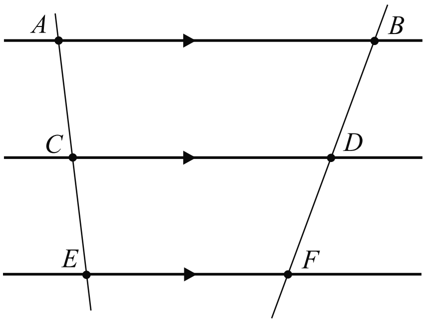
$$\text{True population proportion} = \frac{97}{120} \pm 1.96\sqrt{\frac{\frac{97}{120}(1-\frac{97}{120})}{120}} = 0.738,\ 0.879$$

Confidence interval: 0.738 ↔ 0.879
There is evidence to support the gardener's claim that less than 90% of the seeds germinate because, based on the sample data, any values in the range 73.8% – 87.9% are possible values for the proportion of seeds in the sample that germinate.
$P = 0.9$ is not in the confidence interval. At the 5% significance level, we accept the alternative hypothesis and agree that the gardener's suspicions are well-founded.

Question 6 (25 marks)

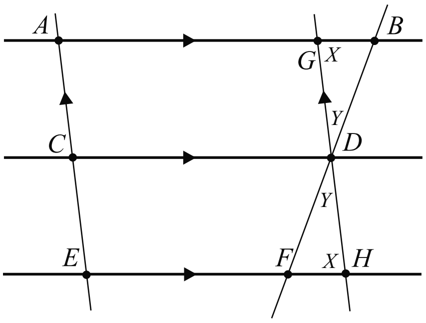
Question 6 (a)

Given: Three parallel lines AB, CD and EF such that C is on AE and D is on BF with $|AC| = |CE|$.



To Prove: $|BD| = |DF|$.

Construction: Draw a line GH through D parallel to AE such that G is on AB and H is on EF.



Proof: $ACDG$ is a parallelogram $\Rightarrow |AC| = |GD| = |CE|$

$CEHD$ is a parallelogram $\Rightarrow |CE| = |DH|$

$\therefore |GD| = |DH|$

Now triangle GDB and triangle FDH are congruent (**ASA**) because:

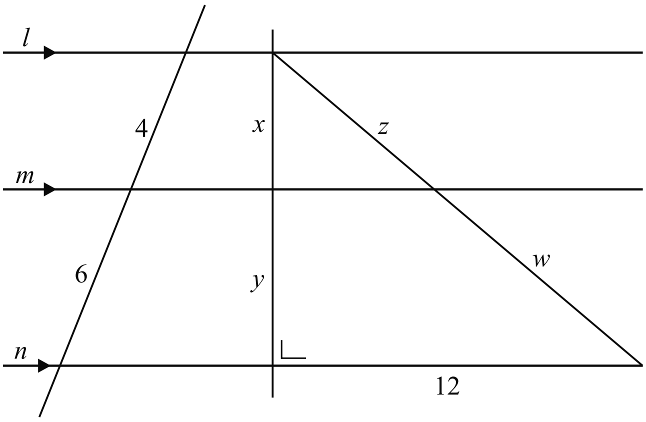
$|\angle BGD| = |\angle FHD| = X$ [Alternate angles]

$|\angle GDB| = |\angle FDH| = Y$ [Vertically opposite angles]

$|GD| = |DH|$ [Already proved]

$\therefore |BD| = |DF|$

Question 6 (b)



$x + y = 5 \Rightarrow y = 5 - x$

$\frac{x}{4} = \frac{y}{6} \Rightarrow x = \frac{4y}{6} = \frac{2y}{3} = \frac{2(5-x)}{3}$

$\therefore 3x = 10 - 2x$

$5x = 10$

$\therefore x = 2,\ y = 3$

$z + w = 13$ (Pythagoras) $\Rightarrow z = 13 - w$

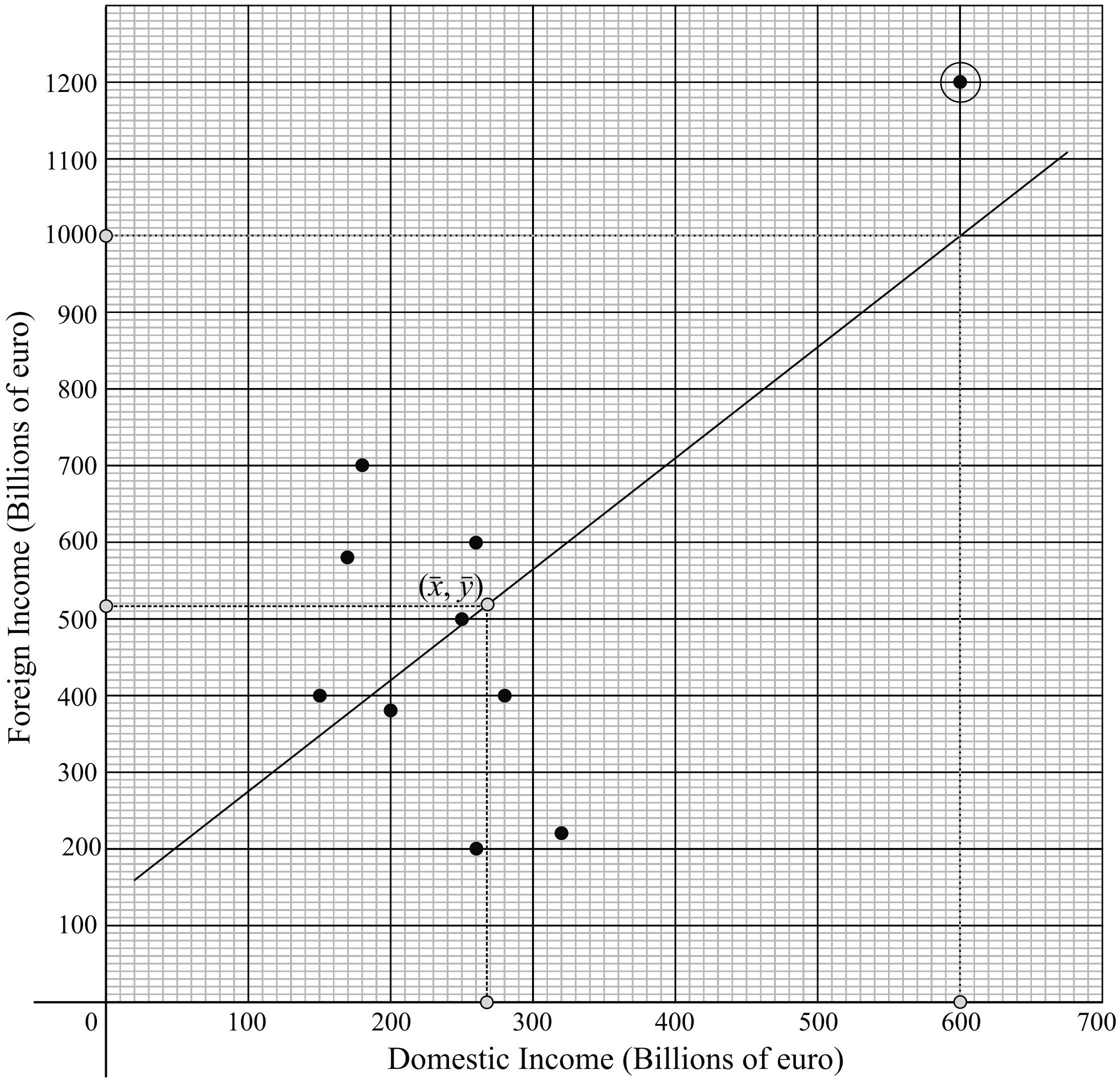
$\frac{x}{y} = \frac{z}{w}$

$\frac{2}{3} = \frac{13-w}{w} \Rightarrow 2w = 39 - 3w \Rightarrow 5w = 39$

$\therefore w = \frac{39}{5},\ z = \frac{26}{5}$

QUESTION 7 (75 MARKS)

Question 7 (a)



Question 7 (b)

CASIO CALCULATOR (*fx-85GT PLUS*)
Steps to find r:
Press Mode.
Press 2: Stat
Press 2: $A + Bx$
Input your x and y values
Press AC Button
Press Shift followed by the Number 1
Press 5: Reg
Press 3: r
Press =

$r = 0.6378$

Question 7 (c)

$$\overline{x}=\frac{150+170+180+200+250+260+280+320+600+260}{10}=267$$

$$\overline{y}=\frac{400+580+700+380+500+200+400+220+1200+600}{10}=518$$

$(\overline{x},\ \overline{y})=(267,\ 518)$

$(\overline{x},\ \overline{y})=(267,\ 518),\ (x_2,\ y_2)=(600,\ 1000)$

$$m=\frac{1000-518}{600-267}=1.4$$

As the yearly domestic income *increases* the foreign yearly income *increases*.

Question 7 (d)

Outlier (600, 1200);
Possible answer: A boom for the economy due to a huge tourist influx and a huge increase in computer/pharmaceutical exports due to the exchange rate of the euro.

Question 7 (e)

Remove (600, 1200) from the table and input the results again into your calculator. After pressing the AC button do the following:

Question 7 (e) (i)

Casio Calculator (*fx-85GT PLUS*)
Press Shift followed by the Number 1
Press 4: Var
Press 2: $\overline{x}$
Press = [*Write down the answer*]
Press Shift followed by the Number 1
Press 4: Var
Press 3: σx
Press = [*Write down the answer*]

$\overline{x}=230$

$\sigma_x=54$

Question 7 (e) (iii)

Casio Calculator (*fx-85GT PLUS*)
Press Shift followed by the Number 1
Press 5: Reg
Press 3: r
Press = [*Write down the answer*]

$r=-0.5085$

Question 7 (e) (ii)

Casio Calculator (*fx-85GT PLUS*)
Press Shift followed by the Number 1
Press 4: Var
Press 3: $\overline{y}$
Press = [*Write down the answer*]
Press Shift followed by the Number 1
Press 4: Var
Press 3: σy
Press = [*Write down the answer*]

$\overline{y}=442$

$\sigma_y=160$

Question 7 (f) (i)

$$y=\overline{y}+\frac{r\sigma_y}{\sigma_x}(x-\overline{x})$$

$$y=442+\left[\frac{(-0.5085)160}{54}\right](x-230)$$

$$y=442-1.51(x-230)$$

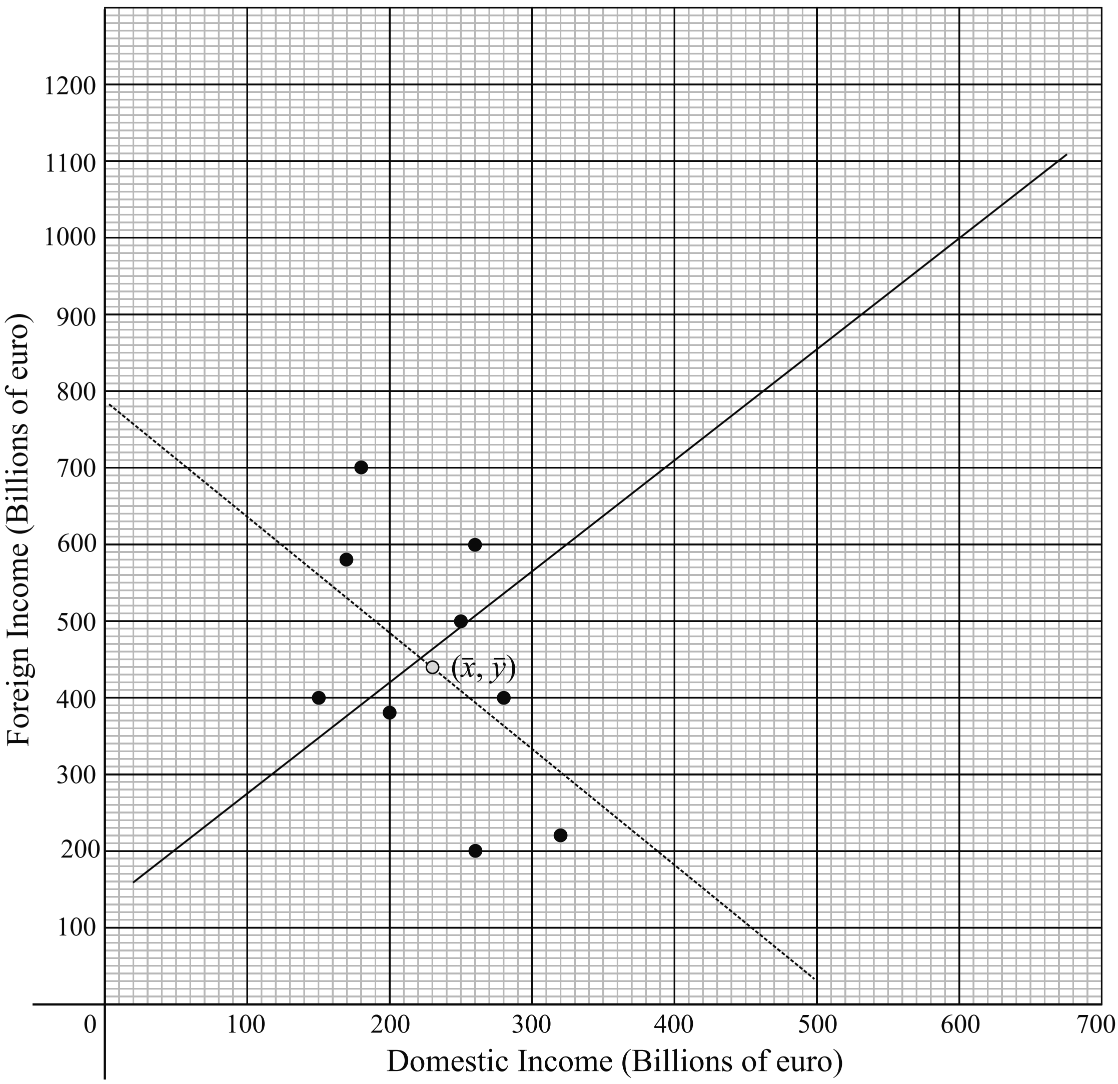
$$y=442-1.51x+347.3$$

$$y=-1.51x+789.3$$

Question 7 (f) (ii)

Slope $m=-1.5$

Question 7 (f) (iii)



Question 7 (f) (iv)

The slope is now negative, there is a negative moderate correlation. The difference is that as domestic yearly income increases the foreign yearly income decreases.

QUESTION 8 (50 MARKS)

Question 8 (a)

$P = 100 + 20\cos 6t$

Maximum value : $\cos 6t = 1$

$\therefore P = 100 + 20(1) = 120$ mm Hg (SP)

$P = 100 + 20\cos 6t$

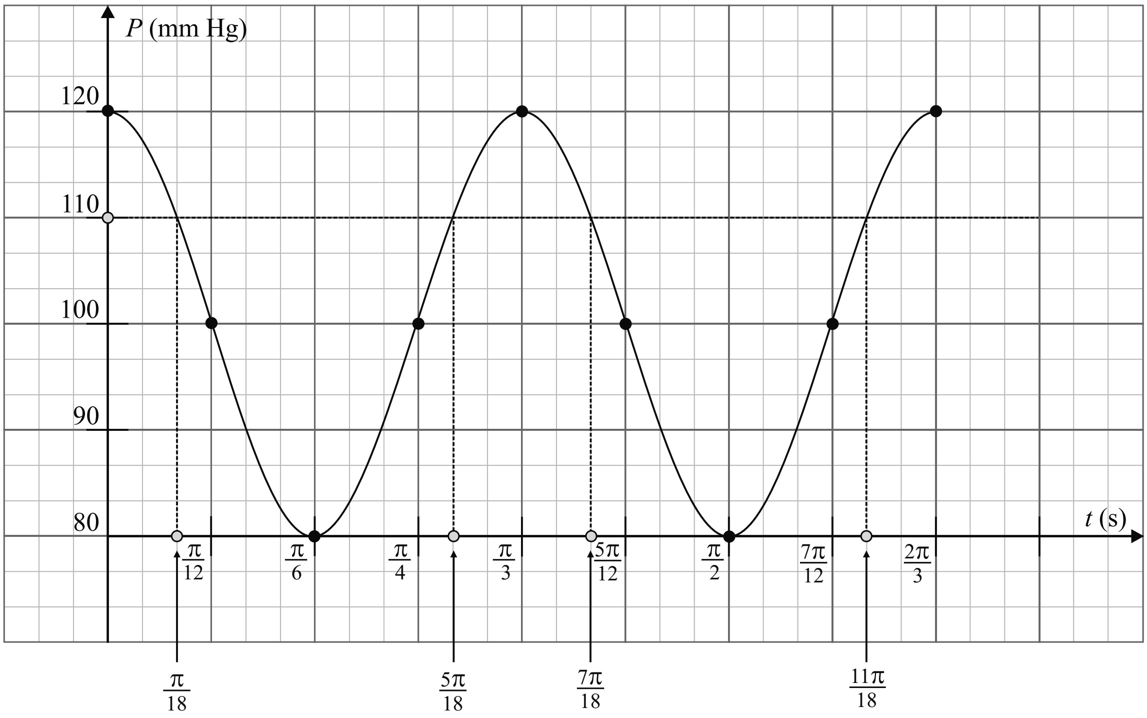
Minimum value : $\cos 6t = -1$

$\therefore P = 100 + 20(-1) = 80$ mm Hg (DP)

Question 8 (b) (i)

t (s)	0	$\frac{\pi}{12}$	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{5\pi}{12}$	$\frac{\pi}{2}$	$\frac{7\pi}{12}$	$\frac{2\pi}{3}$
P (mm Hg)	120	100	80	100	120	100	80	100	120

Question 8 (b) (ii)



Question 8 (b) (iii)

Range = [80, 120]

Period $= \dfrac{\pi}{3}$

Question 8 (b) (iv)

Duration $= \dfrac{\pi}{3}$ s

Number of beats per minute $= \dfrac{60}{\frac{1}{3}\pi} = \dfrac{180}{\pi} = 57$

Question 8 (c) (i)

Draw a line through P = 100 mm Hg and read off the times as shown above.

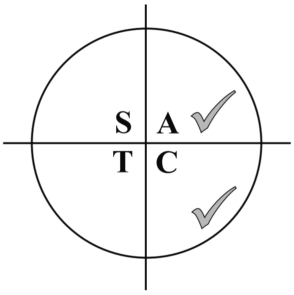
P = 110 mm Hg $\Rightarrow t = \dfrac{\pi}{18}$ s, $\dfrac{5\pi}{18}$ s, $\dfrac{7\pi}{18}$ s, $\dfrac{11\pi}{18}$ s

Question 8 (c) (ii)

$$P = 100 + 20\cos 6t = 110$$

$$20\cos 6t = 10$$

$$\cos 6t = \tfrac{1}{2} \Rightarrow 6t = \cos^{-1}(\tfrac{1}{2}) = 60^{\circ} = \frac{\pi}{3}$$



$6t$	$= \frac{\pi}{3}, \frac{7\pi}{3}$ [First quadrant]	t	$= \frac{\pi}{18}, \frac{7\pi}{18}$
	$= \frac{5\pi}{3}, \frac{11\pi}{3}$ [Fourth quadrant]		$= \frac{5\pi}{18}, \frac{11\pi}{18}$

Question 8 (d)

$$P = 100 + 20\cos 6t$$

$$\frac{dP}{dt} = -20\sin 6t \times 6 = -120\sin 6t$$

$$\left(\frac{dP}{dt}\right)_{t=0} = -120\sin 6(0) = 0$$

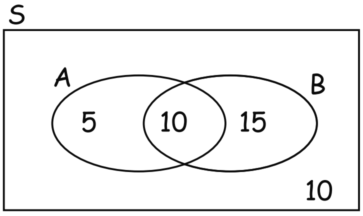
$$\left(\frac{dP}{dt}\right)_{t=\frac{\pi}{15}} = -120\sin 6\left(\frac{\pi}{15}\right) = -120\sin\left(\frac{2\pi}{5}\right) = -114 \text{ mm Hg}$$

$$\left(\frac{dP}{dt}\right)_{t=\frac{\pi}{5}} = -120\sin 6\left(\frac{\pi}{5}\right) = -120\sin\left(\frac{6\pi}{5}\right) = 70.5 \text{ mm Hg}$$

$$\left(\frac{dP}{dt}\right)_{t=\frac{\pi}{3}} = -120\sin 6\left(\frac{\pi}{3}\right) = -120\sin 2\pi = 0 \text{ mm Hg}$$

QUESTION 9 (25 MARKS)

A: People with high blood pressure
B: High level of cholesterol



(a) People with high blood pressure and high level of cholesterol = 10

(b) $P(\text{High BP}) = \frac{15}{40} = \frac{3}{8}$

(c) $P(\text{High level of cholesterol}) = \frac{25}{40} = \frac{5}{8}$

(d) $P(\text{High BP } \textit{and} \text{ high level of cholesterol}) = \frac{10}{40} = \frac{1}{4}$

(e) $P(\text{High BP } \textit{or} \text{ high level of cholesterol}) = \frac{30}{40} = \frac{3}{4}$

(f) $P(A \text{ or } B) = \frac{3}{4}$

$$P(A) + P(B) - P(A \text{ and } B) = \frac{3}{8} + \frac{5}{8} - \frac{1}{4} = \frac{3}{4}$$

Sample Paper 4: Paper 1

Question 1 (25 marks)

Question 1 (a) (i)

$z=-3-i\Rightarrow \overline{z}=-3+i$ $\boxed{z=a+bi\Rightarrow \overline{z}=a-bi}$

Question 1 (a) (ii)

$z=-3-i\Rightarrow (z+3+i)$ is a factor

$\overline{z}=-3+i\Rightarrow (z+3-i)$ is a factor

$\therefore (z+3+i)(z+3-i)=0$

$z^2+3z-iz+3z+9-3i+iz+3i-i^2=0$

$z^2+6z+9+1=0$

$z^2+6z+10=0$

OR

Roots: $-3-i,\ -3+i$

Sum $S=-6$

Product $P=10$

$z^2-Sz+P=0$

$\therefore z^2+6z+10=0$

Question 1 (b)

If z is a root of the cubic equation, its conjugate is also a root. This is because the coefficients in the cubic are all real. Therefore, $z^2+6z+10=0$ is a factor of the cubic equation.

$az^3+22z^2+bz+40=(z^2+6z+10)(az+4)$

$az^3+22z^2+bz+40=az^3+(6a+4)z^2+(10a+24)z+40$

$\therefore 22=6a+4\Rightarrow a=3$

$10a+24=b\Rightarrow 54=b$

$\therefore 3z^3+22z^2+54z+40=(z^2+6z+10)(3z+4)=0$

$z=-3-i,\ -3+i,\ -\tfrac{4}{3}$

QUESTION 2 (25 MARKS)

Question 2 (a)

Let $\dfrac{ax}{b-c}=\dfrac{by}{c-a}=\dfrac{cz}{a-b}=k$

$ax=k(b-c)$

$by=k(c-a)$

$cz=k(a-b)$

$ax+by+cz=kb-kc+kc-ka+ka-kb=0$

Question 2 (b) (i)

$x+\sqrt{5x+19}=-1$

$\sqrt{5x+19}=-(x+1)$

$5x+19=x^2+2x+1$

$x^2-3x-18=0$

$(x+3)(x-6)=0$

$x=-3,\ 6$

Check solutions:

$x=-3:-3+\sqrt{4}=-3+2=-1$

$x=6:6+\sqrt{49}=6+7=13$

ANS: $x=-3$

Question 2 (b) (ii)

$\log_2 x+\dfrac{12}{\log_2 x}=7$

$(\log_2 x)^2-7\log_2 x+12=0$

$(\log_2 x-3)(\log_2 x-4)=0$

$\therefore \log_2 x=3\Rightarrow x=2^3=8$

$\therefore \log_2 x=4\Rightarrow x=2^4=16$

QUESTION 3 (25 MARKS)

Question 3 (a)

$2x-y+3z=20.....(\mathbf{1})$

$7x+y+z=23.......(\mathbf{2})$

$3x+y-z=3.........(\mathbf{3})$

$(\mathbf{1})+(\mathbf{2}):9x+4z=43....(\mathbf{4})$

$(\mathbf{2})-(\mathbf{3}):4x+2z=20....(\mathbf{5})(\times-2)$

$$\begin{array}{rl} 9x+4z & =43 \\ -8x-4z & =-40 \\ \hline x \quad & =3 \end{array}$$

$27+4z=43$

$4z=16\Rightarrow z=4$

$6-y+12=20$

$\therefore y=-2$

ANSWER: $(3,-2,4)$

Question 3 (b)

$(2-3k)x^2+(4-k)x+2=0$

No real roots: $b^2-4ac<0$

$(4-k)^2-4(2-3k)(2)<0$

$16-8k+k^2-16+24k<0$

$k^2+16k<0$

$k(k+16)<0$

$k(k+16)=0\Rightarrow k=-16,\ 0$

ANS: $-16<k<0$

QUESTION 4 (25 MARKS)

Question 4 (a)

$1, \frac{1}{3}, \frac{1}{9}, \frac{1}{27}, \ldots\ldots$

$a = 1,\ r = \frac{1}{3}$

$$S_\infty = \frac{a}{1-r} = \frac{1}{1-\frac{1}{3}} = \frac{3}{2}$$

Question 4 (b)

$a-d,\ a,\ a+d$

$a-d+a+a+d = 3a = 27 \Rightarrow a = 9$

$9-d,\ 9,\ 9+d$

$(9-d)^2 + 9^2 + (9+d)^2 = 293$

$81 - 18d + d^2 + 81 + 81 + 18d + d^2 = 293$

$2d^2 = 293 - 243$

$2d^2 = 50$

$d^2 = 25$

$d = \pm 5$ (Use either)

Numbers: 4, 9, 14

QUESTION 5 (25 MARKS)

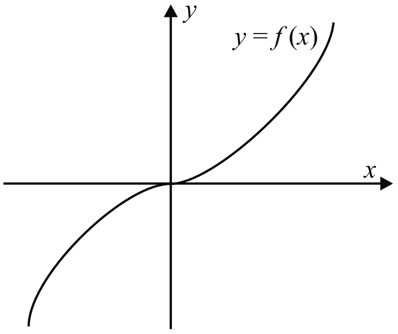
Question 5 (a)

This is a bijective function as there is a perfect one to one correspondence between the *x* and *y* values. A bijective function means it is both an injective and surjective function.

INJECTIVE FUNCTION ☑

SURJECTIVE FUNCTION ☑

BIJECTIVE FUNCTION ☑



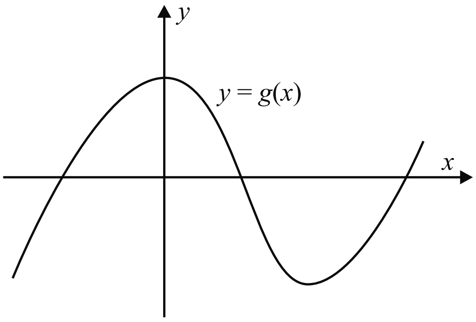
Question 5 (b)

This is a surjective function which means that every *y* value has at least one matching *x* value. It is not injective as many *y* values have more than one corresponding *x* value.

INJECTIVE FUNCTION ☒

SURJECTIVE FUNCTION ☑

BIJECTIVE FUNCTION ☒



Question 5 (c)

This is an injective function which means that every y value has its own unique matching x value.

INJECTIVE FUNCTION ☑
SURJECTIVE FUNCTION ☒
BIJECTIVE FUNCTION ☒

Domain = {0, 1, 2, 3, 4, 5,....}
Range = {0, 1, 4, 9, 16,.......}
Every element in the domain matches to a unique element in the range. There are elements in the range that do not have a matching element from the domain.

Question 5 (d)

This is a bijective function as there is a perfect one to one correspondence between the x and y values. A bijective function means it is both an injective and surjective function.

INJECTIVE FUNCTION ☑
SURJECTIVE FUNCTION ☑
BIJECTIVE FUNCTION ☑

Domain = {Positive real numbers}
Range = {Positive real numbers}
Every element in the domain matches to a unique element in the range. Every element in the range has a unique matching element from the domain.

Question 5 (e)

This is a surjective function which means that every y value has at least one matching x value. It is not injective as many y values have more than one correspoding x value.

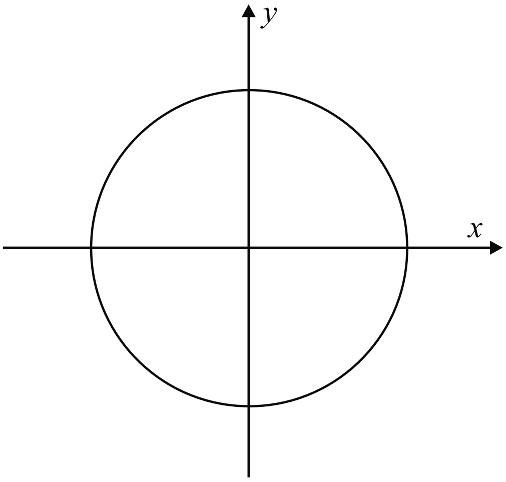
INJECTIVE FUNCTION ☒
SURJECTIVE FUNCTION ☑
BIJECTIVE FUNCTION ☒

Domain = {All real numbers}
Range = {Positive real numbers}
Every element in the domain matches to at least one element in the range. Some values in the range match to two elements in the range. For example, 2^2 and $(-2)^2$ both map on to 4.

Question 5 (f)

This is not a function because x values have two y values.

INJECTIVE FUNCTION ☒
SURJECTIVE FUNCTION ☒
BIJECTIVE FUNCTION ☒

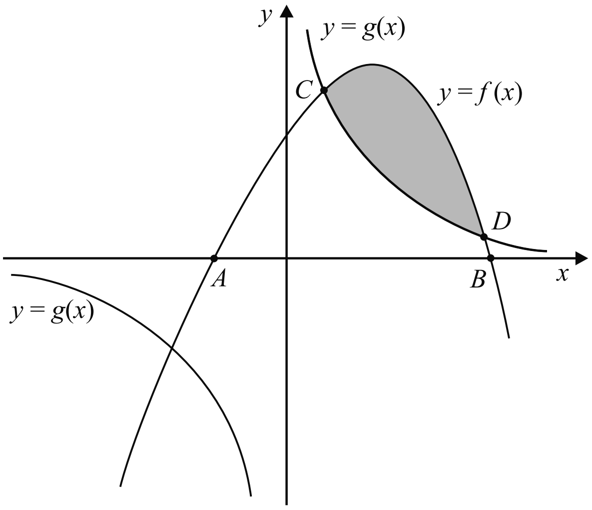


QUESTION 6 (25 MARKS)

Question 6 (a)

$f(x)$ is continuous as there are no gaps.
$g(x)$ is not continuous at there is a gap at $x = 0$.

Question 6 (b)



$$y = f(x) = -\tfrac{7}{6}x^2 + \tfrac{17}{6}x + \tfrac{31}{3}$$
$$f'(x) = -\tfrac{7}{3}x + \tfrac{17}{6}$$
$$f'(x) = 0 \Rightarrow -\tfrac{7}{3}x + \tfrac{17}{6} = 0$$
$$-14x + 17 = 0$$
$$\therefore x = \tfrac{17}{14}$$

Question 6 (c)

$$y = f(x) = -\tfrac{7}{6}x^2 + \tfrac{17}{6}x + \tfrac{31}{3}$$
$$y = 0 \Rightarrow -\tfrac{7}{6}x^2 + \tfrac{17}{6}x + \tfrac{31}{3} = 0$$
$$7x^2 - 17x - 62 = 0$$
$$(7x - 31)(x + 2) = 0$$
$$x = -2,\ \tfrac{31}{7}$$
$$\therefore A(-2,\ 0),\ B(\tfrac{31}{7},\ 0)$$

Question 6 (d)

$$(1,\ 12) \in f(x)?$$
$$f(x) = -\tfrac{7}{6}x^2 + \tfrac{17}{6}x + \tfrac{31}{3}$$
$$f(1) = -\tfrac{7}{6}(1)^2 + \tfrac{17}{6}(1) + \tfrac{31}{3}$$
$$= -\tfrac{7}{6} + \tfrac{17}{6} + \tfrac{31}{3} = \tfrac{36}{3} = 12 \text{ (True)}$$

$$(1,\ 12) \in g(x)?$$
$$g(x) = \frac{12}{x}$$
$$g(1) = \frac{12}{1} = 12 \text{ (True)}$$

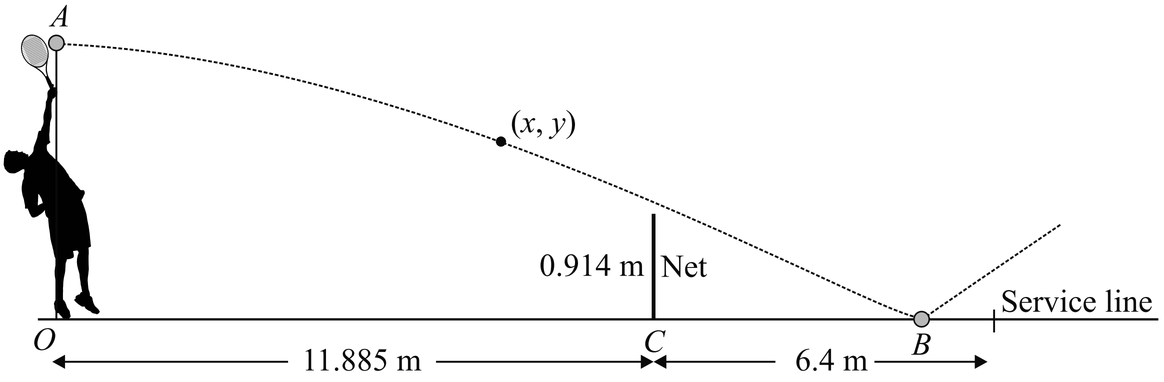
$$(4,\ 3) \in f(x)?$$
$$f(x) = -\tfrac{7}{6}x^2 + \tfrac{17}{6}x + \tfrac{31}{3}$$
$$f(4) = -\tfrac{7}{6}(4)^2 + \tfrac{17}{6}(4) + \tfrac{31}{3}$$
$$= -\tfrac{56}{3} + \tfrac{34}{3} + \tfrac{31}{3} = \tfrac{9}{3} = 3 \text{ (True)}$$

$$(4,\ 3) \in g(x)?$$
$$g(x) = \frac{12}{x}$$
$$g(4) = \frac{12}{4} = 3 \text{ (True)}$$

Question 6 (e)

$$A = \int_1^4 (f(x) - g(x))\,dx$$
$$= \int_1^4 \left(-\tfrac{7}{6}x^2 + \tfrac{17}{6}x + \tfrac{31}{3} - \frac{12}{x} \right) dx$$
$$= [-\tfrac{7}{18}x^3 + \tfrac{17}{12}x^2 + \tfrac{31}{3}x - 12\ln x]_1^4$$
$$= \{-\tfrac{7}{18}(4)^3 + \tfrac{17}{12}(4)^2 + \tfrac{31}{3}(4) - 12\ln(4)\} - \{-\tfrac{7}{18}(1)^3 + \tfrac{17}{12}(1)^2 + \tfrac{31}{3}(1) - 12\ln(1)\}$$
$$= \tfrac{111}{4} - 12\ln 4 \approx 11.1$$

QUESTION 7 (50 MARKS)



Question 7 (a)

Height of player + arm span above head = 1.85 m + 0.8 m = 2.65 m

$y = 3.3 - 13t - 4.9t^2$

$t = 0 : y = 3.3$ m

Height above base of racket = 3.3 – 2.65 = 0.65 m

Question 7 (b)

$y = 3.3 - 13t - 4.9t^2$

$y = 0 : 4.9t^2 + 13t - 3.3 = 0$

$$t = \frac{-13 \pm \sqrt{13^2 - 4(4.9)(-3.3)^2}}{2(4.9)} = 0.23 \text{ s}$$

Question 7 (c)

$x = 70t$

$t = 0.23 : x = 70(0.23) = 16.1$ m

Distance from baseline to far serve line = 11.885 m + 6.4 m = 18.285 m
Ball is in as 16.1 m < 18.285 m

Question 7 (d)

$x = 70t$

$x = 11.885 : 11.885 = 70t \Rightarrow t = 0.17$ s

Question 7 (e)

$y = 3.3 - 13t - 4.9t^2$

$t = 0.17 \text{ s} : y = 3.3 - 13(0.17) - 4.9(0.17)^2 = 0.948$ m
Yes, it will clear the net.

Question 7 (f) (i)

$y = 3.3 - 13t - 4.9t^2$

$y = 0.914 : 0.914 = 3.3 - 13t - 4.9t^2$

$4.9t^2 + 13t - 2.386 = 0$

$$t = \frac{-13 \pm \sqrt{13^2 - 4(4.9)(2.386)}}{2(4.9)} = 0.172 \text{ s}$$

Question 7 (f) (ii)

$x = bt$

$t = 0.172 : 11.885 = b(0.172) \Rightarrow b = 69.1$

QUESTION 8 (50 MARKS)

Question 8 (a)

$y = ab^t$

$\log_{10} y = \log_{10}(ab^t)$

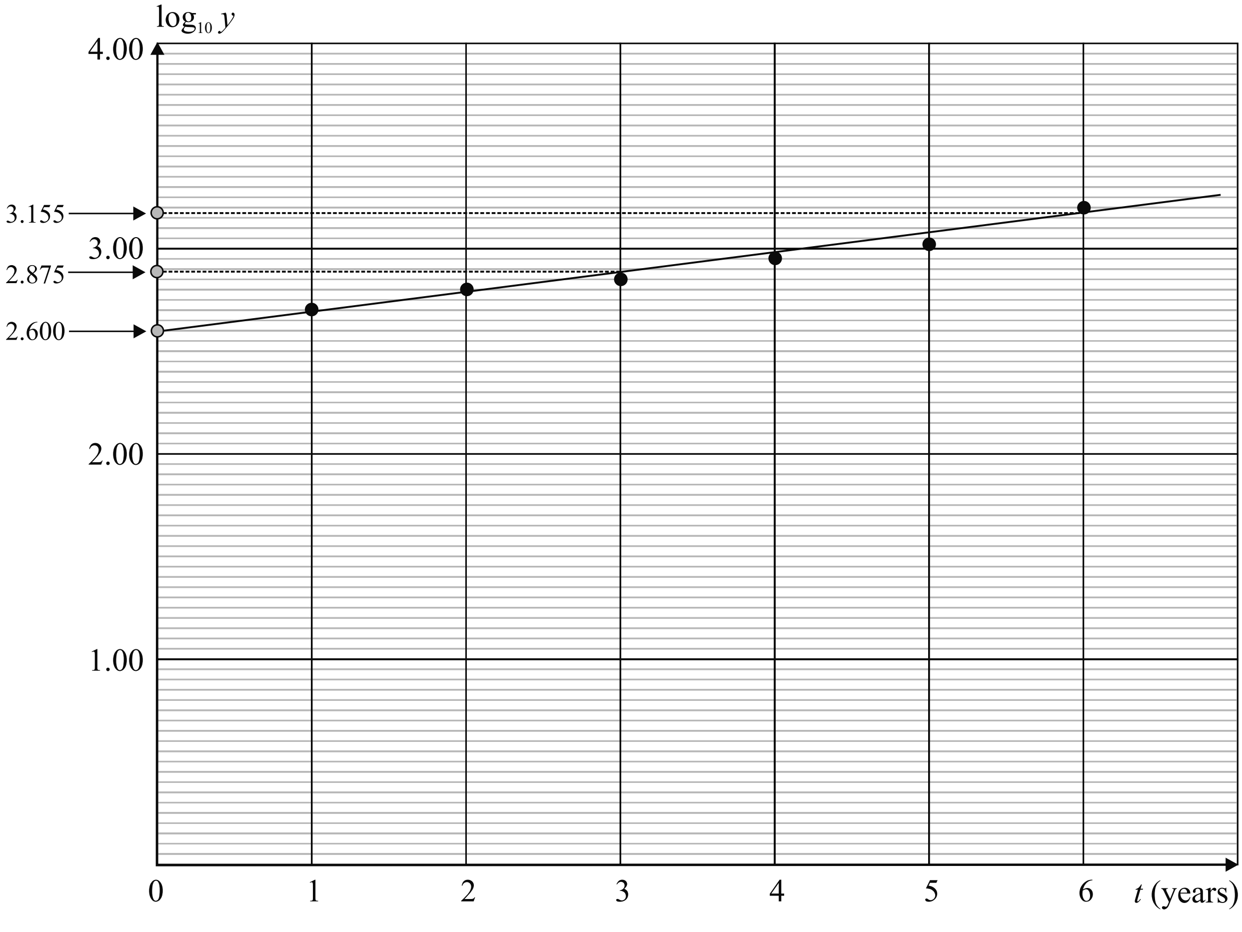
$\log_{10} y = \log_{10} a + \log_{10} b^t$

$\log_{10} y = \log_{10} a + t\log_{10} b$

Question 8 (b) (i)

t (Number of years after 2006)	1	2	3	4	5	6
y (Cost €)	500	610	720	890	1050	1540
$\log_{10} y$	2.70	2.79	2.86	2.95	3.02	3.19

Question 8 (b) (ii)



Question 8 (c)

$y = mx + c$

$\log_{10} y = t\log_{10} b + \log_{10} a$

$c = \log_{10} a$

$2.6 = \log_{10} a \Rightarrow a = 10^{2.6} = 398$

$m = \log_{10} b$

Points on graph: (3, 2.875), (6, 3.155)

$$m = \frac{3.155 - 2.875}{6 - 3} = 0.093$$

$0.093 = \log_{10} b \Rightarrow b = 10^{0.093} = 1.24$

Question 8 (d)

€398

Question 8 (e)

$y = ab^t$

$2200 = 398 \times 1.24^t$

$$\frac{2200}{398} = 1.24^t$$

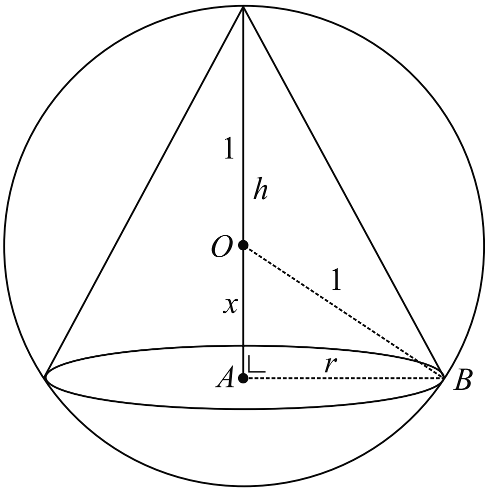
$$\log_{10}\left(\frac{2200}{398}\right) = t\log_{10} 1.24$$

$$\therefore t = \frac{\log_{10}\left(\frac{2200}{398}\right)}{\log_{10} 1.24} = 7.9 \approx 8 \text{ years}$$

QUESTION 9 (50 MARKS)

Question 9 (a) (i)

$h = 1 + x$



Question 9 (a) (ii)

$x^2 + r^2 = 1^2$

$r = \sqrt{1 - x^2}$

Question 9 (b)

$$\begin{aligned} V &= \tfrac{1}{3}\pi r^2 h \\ &= \tfrac{1}{3}\pi(1 - x^2)(1 + x) \\ &= \tfrac{1}{3}\pi(1 + x - x^2 + x^3) \end{aligned}$$

Question 9 (c)

$V = \tfrac{1}{3}\pi(1 + x - x^2 - x^3)$

$\dfrac{dV}{dx} = \tfrac{1}{3}\pi(1 - 2x - 3x^2) = 0$

$3x^2 + 2x - 1 = 0$

$(3x - 1)(x + 1) = 0$

$x = \cancel{-1},\ \tfrac{1}{3}$

$\therefore V_{\text{Max.}} = \tfrac{1}{3}\pi(1 + (\tfrac{1}{3}) - (\tfrac{1}{3})^2 - (\tfrac{1}{3})^3) = \tfrac{32}{81}\pi$

Question 9 (d)

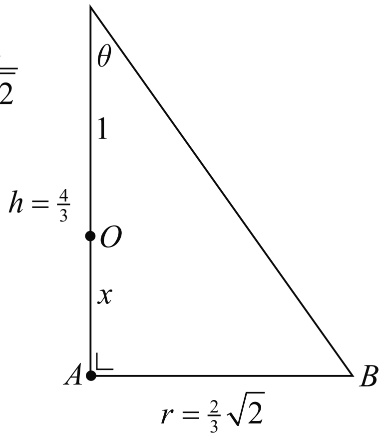
$x = \tfrac{1}{3}$

$h = 1 + x = \tfrac{4}{3}$

$r = \sqrt{1 - x^2} = \sqrt{1 - (\tfrac{1}{3})^2} = \sqrt{\tfrac{8}{9}} = \tfrac{2}{3}\sqrt{2}$

$\tan\theta = \dfrac{r}{h} = \dfrac{\frac{2}{3}\sqrt{2}}{\frac{4}{3}} = \dfrac{1}{\sqrt{2}}$

$\theta = \tan^{-1}\left(\dfrac{1}{\sqrt{2}}\right) \approx 35^\circ$



Question 9 (e)

Cone: $\dfrac{dV}{dt} = -0.2 \text{ cm}^3\text{ s}^{-1}$

Sphere: $\dfrac{dV}{dt} = +0.2 \text{ cm}^3\text{ s}^{-1}$

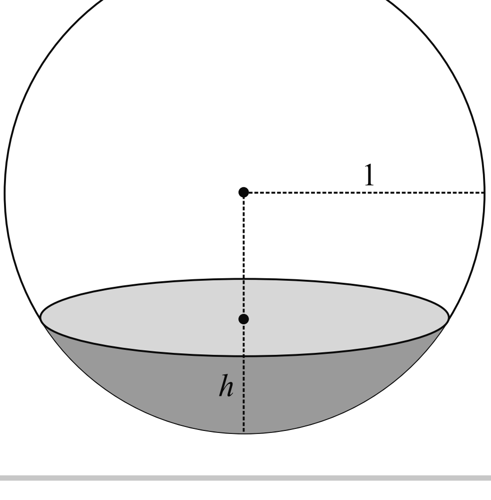
$V = \pi h^2 - \tfrac{1}{3}\pi h^3$

$\dfrac{dV}{dh} = 2\pi h - \pi h^2$

$\dfrac{dV}{dt} = \dfrac{dV}{dh} \times \dfrac{dh}{dt} = (2\pi h - \pi h^2)\dfrac{dh}{dt}$

$\left(\dfrac{dV}{dt}\right)_{h=0.3} = 0.2 = [2\pi(0.3) - \pi(0.3)^2]\dfrac{dh}{dt}$

$\therefore \dfrac{dh}{dt} = \dfrac{0.2}{[2\pi(0.3) - \pi(0.3)^2]} = 0.125 \text{ cm s}^{-1}$



SAMPLE PAPER 4: PAPER 2

QUESTION 1 (25 MARKS)

Question 1 (a)

$x + y - 3 = 0 \Rightarrow m_1 = -1$

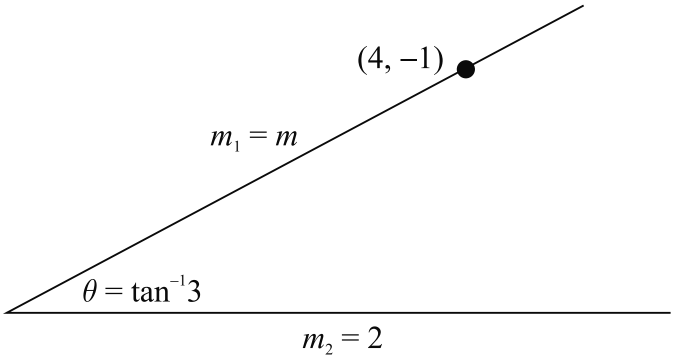
$2x + y - 1 = 0 \Rightarrow m_2 = -2$

$$\tan\theta = +\left(\frac{-1-(-2)}{1+(-1)(-2)}\right) = \left(\frac{-1+2}{1+2}\right) = \frac{1}{3}$$

$$\boxed{\tan\theta = \pm\left(\frac{m_1 - m_2}{1+m_1m_2}\right)}$$

$\therefore \theta = \tan^{-1}(\tfrac{1}{3}) = 18.4^{\circ}$

Question 1 (b)



$\theta = \tan^{-1} 3 \Rightarrow \tan\theta = 3$

$2x - y - 8 = 0 \Rightarrow m_2 = 2$

$$3 = \pm\left(\frac{m-2}{1+2m}\right)$$

$\therefore 3(1+2m) = (m-2)$	$\therefore 3(1+2m) = -(m-2)$
$3 + 6m = m - 2$	$3 + 6m = -m + 2$
$5m = -5$	$7m = -1$
$m = -1$	$m = -\frac{1}{7}$

Equations of line:

$l_1 : m = -1 \Rightarrow x + y + k = 0$

$(4,\ -1) \in l_1 \Rightarrow 4 - 1 + k = 0 \Rightarrow k = -3$

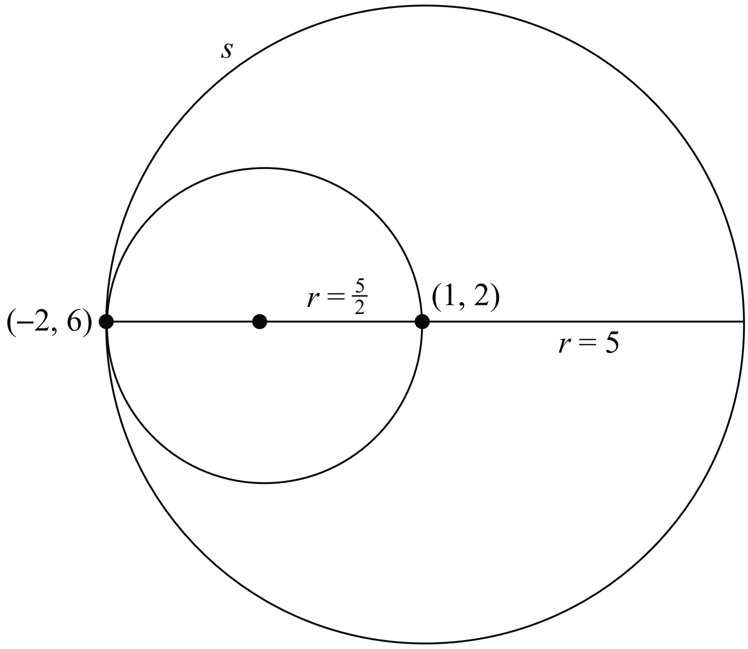
$l_1 : x + y - 3 = 0$

$l_2 : m = -\frac{1}{7} \Rightarrow x + 7y + k = 0$

$(4,\ -1) \in l_2 \Rightarrow 4 - 7 + k = 0 \Rightarrow k = 3$

$l_2 : x + 7y + 3 = 0$

QUESTION 2 (25 MARKS)



$s : x^2 + y^2 - 2x - 4y - 20 = 0$

Centre (1, 2), $r = \sqrt{1^2 + 2^2 + 20} = \sqrt{25} = 5$

Centre (h, k), Radius r
$(x - h)^2 + (y - k)^2 = r^2$

Radius of new circle: $r = \frac{5}{2}$
Centre of new circle is midpoint of (1, 2) and (–2, 6).

$$\text{Centre} = \left(\frac{-2+1}{2}, \frac{6+2}{2}\right) = (-\tfrac{1}{2}, 4)$$

Equation of new circle: $(x + \frac{1}{2})^2 + (y - 4)^2 = (\frac{5}{2})^2$

$$x^2 + x + \tfrac{1}{4} + y^2 - 8y + 16 = \tfrac{25}{4}$$

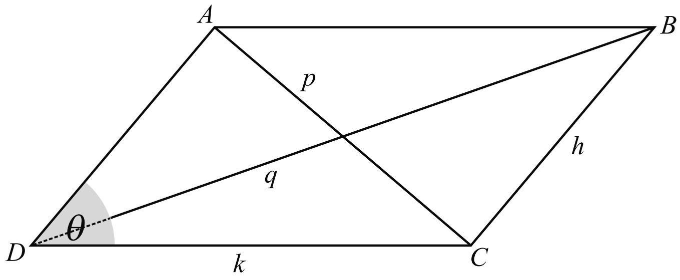
$$x^2 + y^2 + x - 8y + 10 = 0$$

QUESTION 3 (25 MARKS)

Question 3 (a)

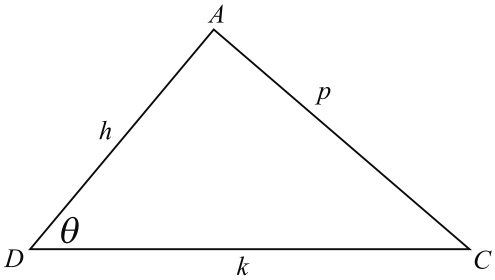
The adjacent interior angles in a parallelogram add up to 180°.

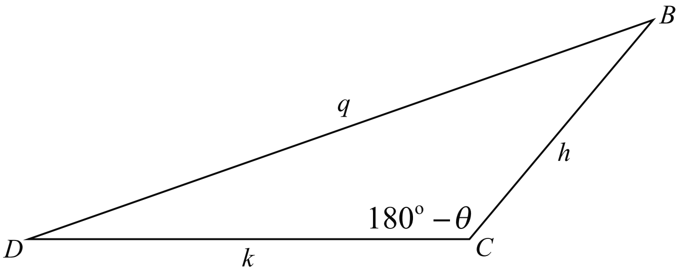
Question 3 (b)



$$\cos(180^\circ - \theta) = \cos 180^\circ \cos\theta - \sin 180^\circ \sin\theta$$
$$= (-1)\cos\theta - (0)\sin\theta$$
$$= -\cos\theta$$

Question 3 (c)





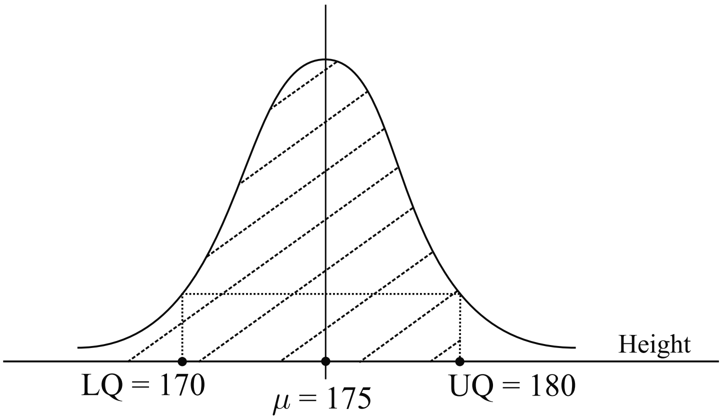
$$p^2 = h^2 + k^2 - 2kh\cos\theta ....(\mathbf{1})$$

$$q^2 = h^2 + k^2 - 2kh\cos(180^\circ - \theta)$$
$$q^2 = h^2 + k^2 + 2kh\cos\theta ....(\mathbf{2})$$

Add equations (**1**) and (**2**):

$$\therefore p^2 + q^2 = 2h^2 + 2k^2$$

QUESTION 4 (25 MARKS)



Question 4 (a)

(i) Median M = 175 cm

(ii) Lower quartile (LQ) = 170 cm

(iii) Interquartile range = 180 – 170 = 10 cm

Question 4 (b)

75% of students have a height less than 180 cm.

Question 4 (c)

$$P(x \leq 180) = 0.75$$

$$P(z \leq Z) = 0.75 \Rightarrow z = 0.675$$

$$x = 180 : z = \frac{x - \mu}{\sigma} \Rightarrow 0.675 = \frac{180 - 175}{\sigma}$$

$$\therefore \sigma = \frac{180 - 175}{0.675} = 7.4 \text{ cm}$$

Question 4 (d)

$$x = 170 : z = \frac{170 - 175}{7.4} = -0.675$$

$$x = 180 : z = \frac{180 - 175}{7.4} = 0.675$$

$$\begin{aligned} & P(170 \leq x \leq 180) \\ &= P(-0.675 \leq z \leq 0.675) \\ &= P(z \leq 0.675) - P(z \leq -0.675) \\ &= P(z \leq 0.675) - \{1 - P(z \leq 0.675)\} \\ &= 2P(z \leq 0.675) - 1 \\ &= 2(0.75) - 1 \\ &= 1.5 - 1 \\ &= 0.5 \end{aligned}$$

QUESTION 5 (25 MARKS)

x	1	2	3	4	5
$P(x)$	0.1	0.2	0.3	0.3	0.1

Question 5 (a) (i)

$$P(\text{At least } 3) = 1 - [P(1) + P(2)]$$
$$= 1 - 0.1 - 0.2 = 0.7$$

Or

$P(3) + P(4) + P(5) = 0.3 + 0.3 + 0.1 = 0.7$

Question 5 (a) (ii)

$$\bar{x} = \frac{\sum xP(x)}{\sum P} = \frac{9(0.1) + 2(0.2) + 3(0.3) + 4(0.3) + 5(0.1)}{0.1 + 0.2 + 0.3 + 0.3 + 0.1} = 3.1$$

Gender	For	Against	Total
Male	58	85	143
Female	84	73	157
Total	142	158	300

Question 5 (b) (i)

$$P(\text{Male } \textit{and } \text{Against treaty}) = \frac{85}{300} = \frac{17}{60}$$

Question 5 (b) (ii)

$$P(\text{Female } \textit{or } \text{For treaty}) = \frac{58 + 157}{300} = \frac{43}{60}$$

Question 5 (b) (iii)

$$P = \frac{58}{142} = \frac{29}{71}$$

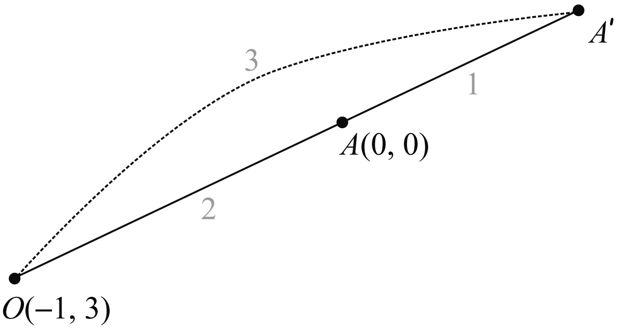
QUESTION 6 (25 MARKS)

Question 6 (a)

$A(0,\ 0),\ B(1,\ 4),\ C(4,\ 1)$

$$\begin{aligned} A &= \tfrac{1}{2}\left|x_1y_2 - x_2y_1\right| \\ &= \tfrac{1}{2}\left|(1)(1)-(4)(4)\right| \\ &= \tfrac{1}{2}\left|-15\right| \\ &= \tfrac{15}{2} \end{aligned}$$

Question 6 (b)



$$k = \frac{|\text{Image Length}|}{|\text{Object Length}|} = \frac{|OA'|}{|OA|} = \frac{3}{2}$$

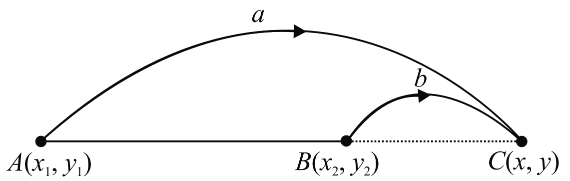
$a = 3,\ b = 1$

$O(-1,\ 3) = (x_1,\ y_1);\ A(0,\ 0) = (x_2,\ y_2)$

$$A' = \left(\frac{3(0)-1(-1)}{3-1},\ \frac{3(0)-1(3)}{3-1}\right) = \left(\frac{1}{2},\ -\frac{3}{2}\right)$$

EXTERNAL DIVISION

$$\boxed{x = \frac{ax_2 - bx_1}{a-b},\ y = \frac{ay_2 - by_1}{a-b}}$$



$a = 3,\ b = 1$

$O(-1,\ 3) = (x_1,\ y_1);\ B(1,\ 4) = (x_2,\ y_2)$

$$B' = \left(\frac{3(1)-1(-1)}{3-1},\ \frac{3(4)-1(3)}{3-1}\right) = \left(2,\ \frac{9}{2}\right)$$

$a = 3,\ b = 1$

$O(-1,\ 3) = (x_1,\ y_1);\ C(4,\ 1) = (x_2,\ y_2)$

$$C' = \left(\frac{3(4)-1(-1)}{3-1},\ \frac{3(1)-1(3)}{3-1}\right) = \left(\frac{13}{2},\ 0\right)$$

Question 6 (c)

$$A' = \left(\frac{1}{2},\ -\frac{3}{2}\right) \rightarrow \left(-6,\ -\frac{3}{2}\right)$$

$$B' = \left(2,\ \frac{9}{2}\right) \rightarrow \left(-\frac{9}{2},\ \frac{9}{2}\right)$$

$$C' = \left(\frac{13}{2},\ 0\right) \rightarrow (0,\ 0)$$

$$A = \tfrac{1}{2}\left|(-6)\left(\frac{9}{2}\right) - \left(-\frac{3}{2}\right)\left(-\frac{9}{2}\right)\right| = \tfrac{1}{2}\left|-27 - \frac{27}{4}\right| = \tfrac{1}{2}\left|-\frac{135}{4}\right| = \frac{135}{8}$$

$$\frac{\text{Area } A'B'C'}{\text{Area } ABC} = \frac{\left(\dfrac{135}{8}\right)}{\left(\dfrac{15}{2}\right)} = \frac{9}{4} = \left(\frac{3}{2}\right)^2 = k^2$$

QUESTION 7 (50 MARKS)

Question 7 (a)

(i) A random variable is a function that associates a unique numerical value with every outcome of an experiment. It may vary from trial to trial as the experiment is repeated.

(ii) DISCRETE: **Ex**. A coin is tossed five times. The random variable is the number of heads. Its values can be 0, 1, 2, 3, 4, 5.

CONTINUOUS: **Ex**. The temperature in a house during the day can take any positive or negative value within a certain range.

(iii) Expected value $\mu = \sum xP(x) =$ Mean of a probability distribution.

Question 7 (b)

(i) A prime number is a whole positive integer (excluding 1) divisible by itself and 1 only.

(ii)

		Die *A*					
		1	2	3	4	5	6
Die *B*	1	2	3	4	5	6	7
	2	3	4	5	6	7	8
	3	4	5	6	7	8	9
	4	5	6	7	8	9	10
	5	6	7	8	9	10	11
	6	7	8	9	10	11	12

(iii) Primes in table: 2, 3, 5, 7, 11

Number of primes = 15

$$P(\text{Sum that is a prime}) = \frac{\text{Number of primes}}{\text{Number of numbers}} = \frac{15}{36} = \frac{5}{12}$$

(iv) $$P(\text{Sum that is } \textbf{not} \text{ a prime}) = \frac{\text{Number of non-primes}}{\text{Number of numbers}} = \frac{21}{36} = \frac{7}{12}$$

Question 7 (c)

Outcome	Not a Prime Sum	Prime Sum
P	$\frac{7}{12}$	$\frac{5}{12}$
Net income to Bob	–3	3
$xP(x)$	$-\frac{7}{4}$	$\frac{5}{4}$

Question 7 (d)

(i) $E = \sum xP(x) = -\frac{7}{4} + \frac{5}{4} = -\frac{1}{2}$

$E = –50$ c, on average Bob loses 50 c per game.

(ii) Expected losses: $30 \times (-\frac{1}{2}) = -€15$

Question 7 (e)

$-3\times\frac{7}{12}+\frac{5}{12}x=\frac{1}{2}$

$-21+5x=6$

$5x=27$

$x=\dfrac{27}{5}=€5.40$

QUESTION 8 (25 MARKS)

Question 8 (a)

$P(4)=\frac{1}{6}$

$P(\text{Not } 4)=\frac{5}{6}$

$$P=1-\{{}^{20}C_0(\tfrac{1}{6})^0(\tfrac{5}{6})^{20}+{}^{20}C_1(\tfrac{1}{6})^1(\tfrac{5}{6})^{19}+{}^{20}C_2(\tfrac{1}{6})^2(\tfrac{5}{6})^{18}+{}^{20}C_3(\tfrac{1}{6})^3(\tfrac{5}{6})^{17}+{}^{20}C_4(\tfrac{1}{6})^4(\tfrac{5}{6})^{16}+{}^{20}C_5(\tfrac{1}{6})^5(\tfrac{5}{6})^{15}\}$$

$=1-0.898$

$=0.102$

$=10.2\%$

Question 8 (b)

A probability of 10.2% means that a fair die will give six or more 4s 10.2% of the time in 20 throws of a die. So there is nothing unusual about the die used in the casino. If the probability of at least six 4s when a fair die was tossed 20 times was less that 5% then this probability is so small that it hasn't happened by chance and so the die is biased.

QUESTION 9 (75 MARKS)

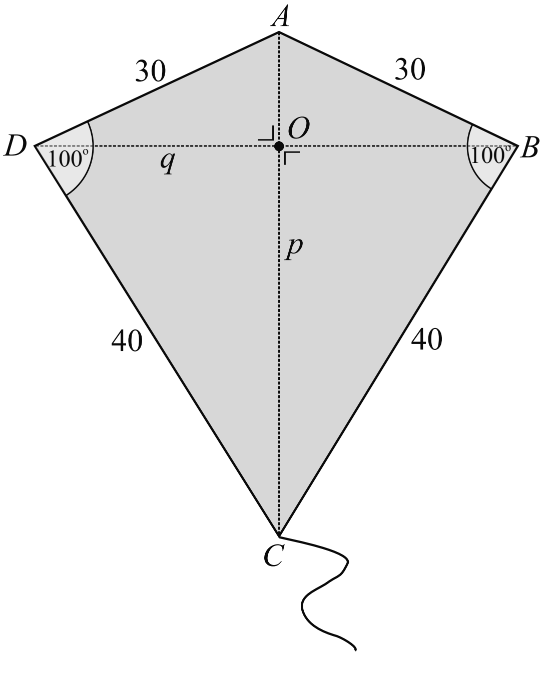
Question 9 (a)

(i) A **rhombus** is a simple quadilateral whose sides are equal.

(ii) The angles in a quadrilateral add up to 360º. If each angle is equal, each angle will be 90º and hence a **square** is the shape formed.

Question 9 (b)

(i) Perimeter $P = 2a + 2b$



(ii) Area of $\Delta ADC=\frac{1}{2}(p)(\frac{1}{2}q)=\frac{1}{4}pq$

Area of $\Delta ABC=\frac{1}{2}(p)(\frac{1}{2}q)=\frac{1}{4}pq$

Area of $ABCD=\frac{1}{4}pq+\frac{1}{4}pq=\frac{1}{2}pq$

Area of $\Delta ADC=\frac{1}{2}ab\sin\theta$

Area of $\Delta ABC=\frac{1}{2}ab\sin\theta$

Area of $ABCD=ab\sin\theta$

Question 9 (c)

(i) $|AC|^2 = 30^2 + 40^2 - 2(30)(40)\cos 100^\circ$

$\therefore |AC| = 54 \text{ cm}$

(ii) $\dfrac{\sin(|\angle CAD|)}{40} = \dfrac{\sin 100^\circ}{54}$

$\sin(|\angle CAD|) = \dfrac{40\sin 100^\circ}{54}$

$|\angle CAD| = \sin^{-1}\left(\dfrac{40\sin 100^\circ}{54}\right) = 46.8^\circ$

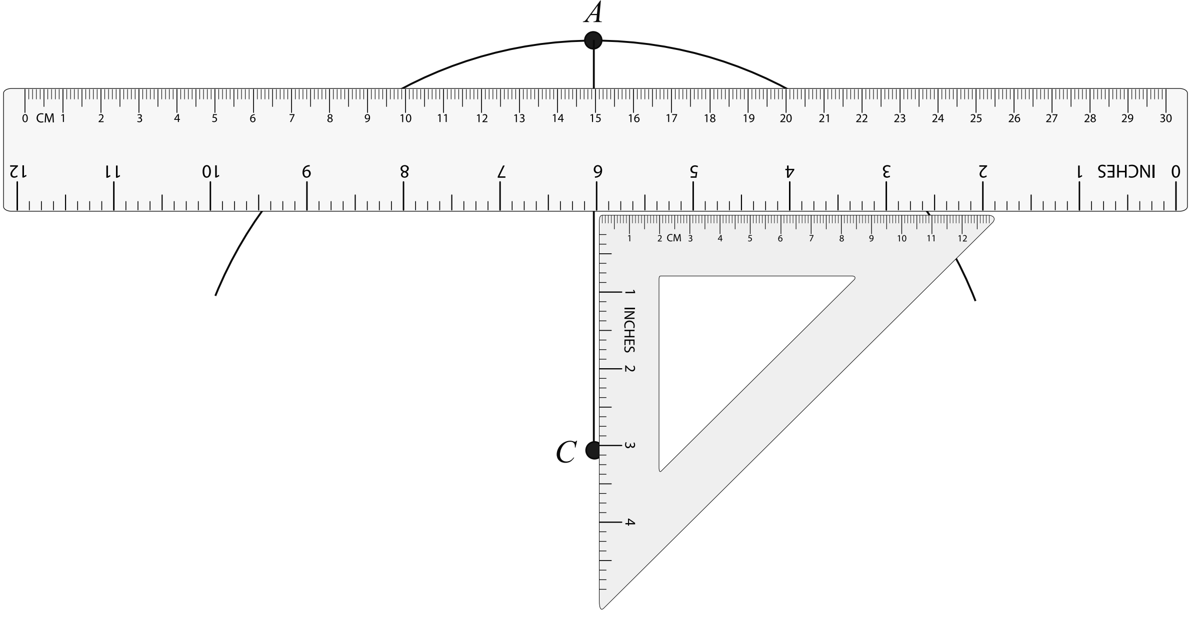
(iii) $\sin(|\angle CAD|) = \dfrac{\frac{1}{2}|BD|}{30}$

$\therefore \sin 46.8^\circ = \dfrac{|BD|}{60}$

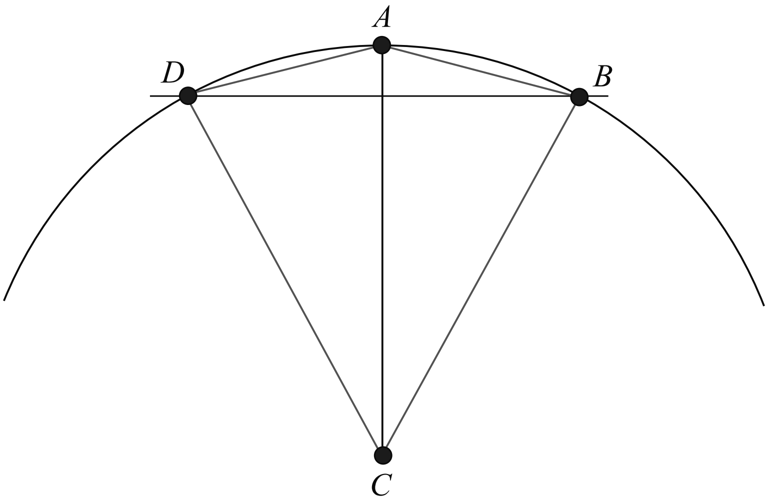
$|BD| = 60\sin 46.8^\circ = 43.7 \text{ m}$

(iv) $\text{Area} = (30)(40)\sin 100^\circ = 1182 \text{ cm}^2$

Question 9 (d)

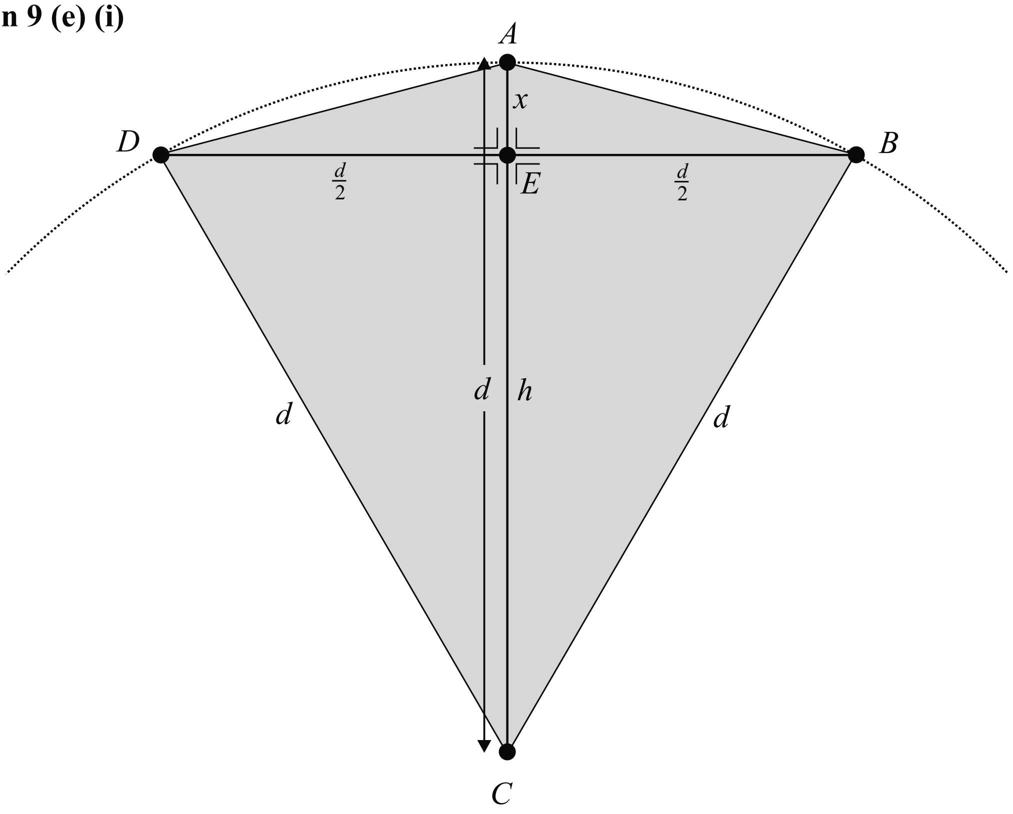


Draw a line segment [*AC*] of length 10 cm.
Using a compass draw an arc of radius 10 cm with *C* as centre.
Using a set square place a rule perpendicular to *AC* and move the ruler into position such that it measures a chord of distance of 10 cm. Draw a line along the ruler in this position.



Draw the kite *ABCD*.

Question 9 (e) (i)



Consider the right-angled triangle *BEC*:

$$d^2 = h^2 + \left(\frac{d}{2}\right)^2$$

$$d^2 = h^2 + \frac{d^2}{4}$$

$$\therefore h^2 = d^2 - \frac{d^2}{4} = \frac{3d^2}{4}$$

$$h = \sqrt{\frac{3d^2}{4}} = \frac{\sqrt{3}}{2}d$$

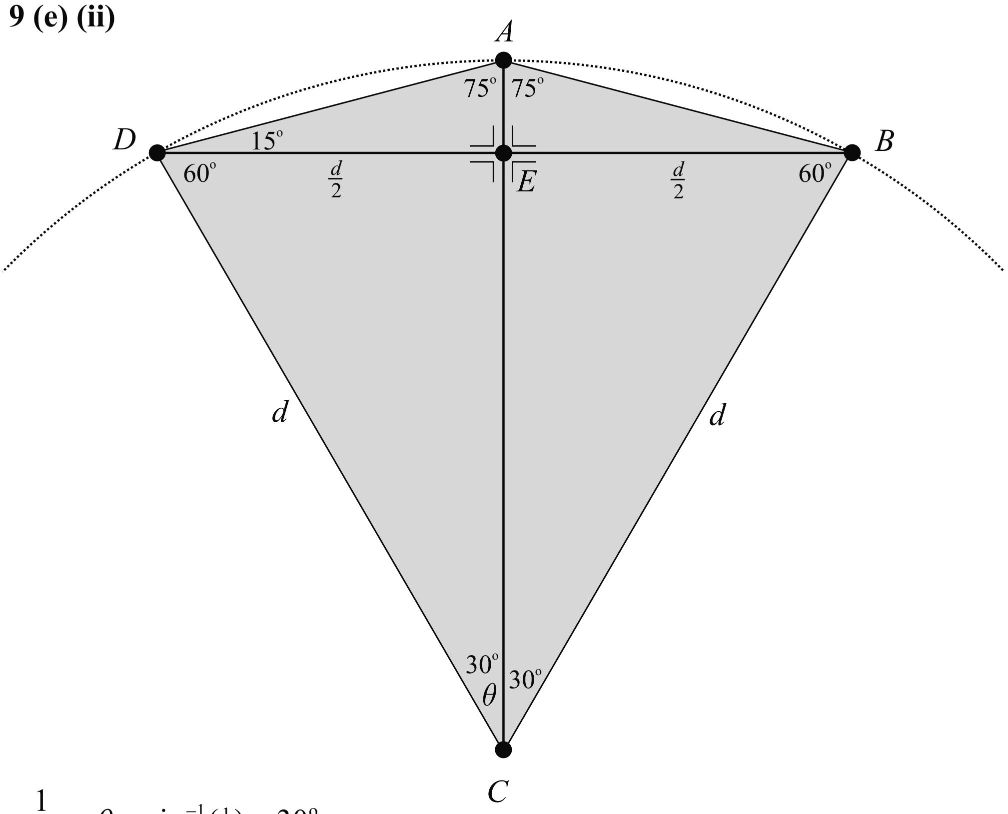
Consider the right-angled triangle *BEA*:

$$x = d - h = d - \frac{\sqrt{3}}{2}d = d\left(1 - \frac{\sqrt{3}}{2}\right) = d\left(\frac{2-\sqrt{3}}{2}\right)$$

$$\tan(|\angle ABD|) = \frac{d\left(\frac{2-\sqrt{3}}{2}\right)}{\frac{d}{2}} = (2-\sqrt{3})$$

$$\therefore |\angle ABD| = \tan^{-1}(2-\sqrt{3}) = 15^{\circ}$$

Question 9 (e) (ii)



$$\sin\theta = \frac{\frac{d}{2}}{d} = \frac{1}{2} \Rightarrow \theta = \sin^{-1}(\tfrac{1}{2}) = 30^\circ$$

$$|\angle ADC| = 75^\circ,\ |\angle ABC| = 75^\circ,\ |\angle BAD| = 150^\circ,\ |\angle BCD| = 60^\circ$$

Question 9 (f)

$$\tan 15^\circ = \tan(45^\circ - 30^\circ)$$

$$= \frac{\tan 45^\circ - \tan 30^\circ}{1 + \tan 45^\circ \tan 30^\circ}$$

$$= \frac{1 - \frac{1}{\sqrt{3}}}{1 + \frac{1}{\sqrt{3}}}$$

$$= \frac{\left(1 - \frac{1}{\sqrt{3}}\right)}{\left(1 + \frac{1}{\sqrt{3}}\right)} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3} - 1}{\sqrt{3} + 1}$$

$$= \frac{(\sqrt{3} - 1)}{(\sqrt{3} + 1)} \times \frac{(\sqrt{3} - 1)}{(\sqrt{3} - 1)} = \frac{3 - 2\sqrt{3} + 1}{3 - 1}$$

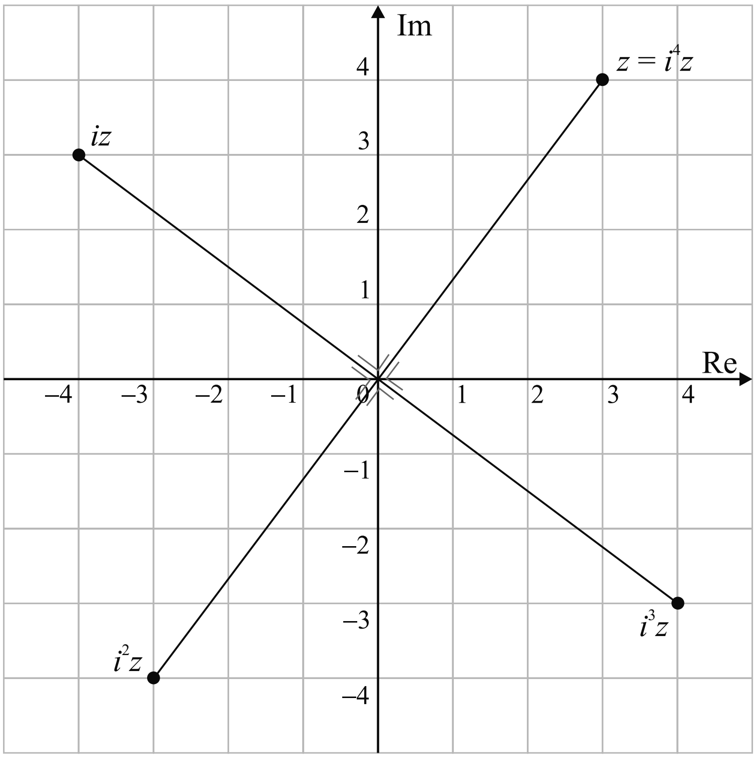
$$= \frac{4 - 2\sqrt{3}}{2}$$

$$= 2 - \sqrt{3}$$

SAMPLE PAPER 5: PAPER 1

QUESTION 1 (25 MARKS)

Question 1 (a)



Question 1 (b)

$z = 3+4i \rightarrow m = \frac{4}{3}$

$iz = i(3+4i) = 3i+4i^2 = -4+3i \rightarrow m = -\frac{3}{4}$

$i^2z = -1(3+4i) = -3-4i \rightarrow m = \frac{4}{3}$

$i^3z = -i(3+4i) = -3i-4i^2 = 4-3i \rightarrow m = -\frac{3}{4}$

$i^4z = 1(3+4i) = 3+4i \rightarrow m = \frac{4}{3}$

Question 1 (c)

Multiplying a complex number z by:	Causes:
i	an anti-clockwise rotation of 90° to z
i^2	an anti-clockwise rotation of 180° to z
i^3	an anti-clockwise rotation of 270° to z
i^4	an anti-clockwise rotation of 360° to z

QUESTION 2 (25 MARKS)

Question 2 (a)

$x^2 - 2 = 2y^2$

$3x = y + 7 \Rightarrow y = 3x - 7$

$x^2 - 2 = 2(3x - 7)^2$ [Replace y in the quadratic equation.]

$x^2 - 2 = 2(9x^2 - 42x + 49)$

$x^2 - 2 = 18x^2 - 84x + 98$

$0 = 17x^2 - 84x + 100$

$0 = (17x - 50)(x - 2)$

$\therefore x = 2, \tfrac{50}{17}$

$x = 2 : y = 3(2) - 7 = 6 - 7 = -1$

$x = \tfrac{50}{17} : y = 3(\tfrac{50}{17}) - 7 = \tfrac{150}{17} - 7 = \tfrac{31}{17}$

Ans: $(2, -1), (\tfrac{50}{17}, \tfrac{31}{17})$

Question 2 (b)

$x^3 - kx^2 - 22x + 56 = (x + 4)(x^2 + ax + 14)$ [A cubic is equal to a linear by a quadratic.]

$x^3 - kx^2 - 22x + 56 = x^3 + (a + 4)x^2 + (4a + 14)x + 56$

$\therefore -22 = 4a + 14 \Rightarrow 4a = -36 \Rightarrow a = -9$

$\therefore -k = a + 4 \Rightarrow -k = -9 + 4 = -5 \Rightarrow k = 5$

$x^3 - 5x^2 - 22x + 56 = (x + 4)(x^2 - 9x + 14) = 0$

$(x + 4)(x - 2)(x - 7) = 0$

$\therefore x = -4, 2, 7$

Question 3 (25 marks)

Question 3 (a) (i)

$$f(x) = \frac{b^x + b^{-x}}{b^x - b^{-x}}$$

$$f(a) = 3 \Rightarrow \frac{b^a + b^{-a}}{b^a - b^{-a}} = 3$$

$$b^a + b^{-a} = 3b^a - 3b^{-a}$$

$$4b^{-a} = 2b^a$$

$$\frac{4}{2} = \frac{b^a}{b^{-a}}$$

$$\therefore 2 = b^{2a}$$

Question 3 (a) (ii)

$$f(2a) = \frac{b^{2a} + b^{-2a}}{b^{2a} - b^{-2a}} = \frac{b^{2a} + \frac{1}{b^{2a}}}{b^{2a} - \frac{1}{b^{2a}}} = \frac{2 + \frac{1}{2}}{2 - \frac{1}{2}} = \frac{\frac{5}{2}}{\frac{3}{2}} = \frac{5}{3}$$

Question 3 (b)

$$\log_b \sqrt{x} = \sqrt{\log_b x}$$

$$\log_b x^{\frac{1}{2}} = \sqrt{\log_b x}$$

$\frac{1}{2} \log_b x = \sqrt{\log_b x}$ [Square both sides.]

$$\tfrac{1}{4}(\log_b x)^2 = \log_b x$$

$$(\log_b x)^2 - 4\log_b x = 0$$

$$\log_b x(\log_b x - 4) = 0$$

$$\therefore \log_b x = 0 \Rightarrow x = b^0 = 1$$

$$\therefore \log_b x = 4 \Rightarrow x = b^4$$

$b^4 + 1 = 17$ [The roots add to 17.]

$$b^4 = 16$$

$$b = \pm 2$$

$\therefore b = 2$ as $b > 0$

QUESTION 4 (25 MARKS)

Question 4 (a)

STEPS FOR PROOF BY INDUCTION

1. Prove result is true for some starting value of $n \in \mathbb{N}$.
2. Assume result is true for $n = k$.
3. Prove result is true for $n = (k+1)$.

1. Prove true for $n = 7$: $7! = 5040$ [Therefore, true for $n = 7$.]

$$3^7 = 2187$$

$$\therefore 7! > 3^7$$

2. Assume true for $n = k$: Assume $k! > 3^k$

3. Prove true for $n = k + 1$: Prove $(k + 1)! > 3^{k+1}$

Proof:

$$(k+1)! = (k+1)k!$$

$$> (k+1)3^k \quad \text{[From step 2 where } k! > 3^k\text{]}$$

$$> 3 \times 3^k \quad [(k + 1) > 3 \text{ because } k \geq 7 \Rightarrow (k + 1) \geq 8]$$

$$\therefore (k+1)k! > 3^{k+1}$$

Therefore, assuming true for $n = k$ means it is true for $n = k + 1$. So true for $n = 7$ and true for $n = k$ means it is true for $n = k + 1$. This implies it is true for all $n \geq 7$, $n \in \mathbb{N}$.

Question 4 (b)

$b, br^2, br^4, \dots\dots\dots\dots$

First term $= b$, Common ratio $= r^2$

$$S_n = \frac{b(1-r^{2n})}{1-r^2}$$

$a, ar, ar^2, \dots\dots\dots\dots$

First term $= a$, Common ratio $= r$

$$S_{2n} = \frac{a(1-r^{2n})}{1-r}$$

$$S_{2n} = S_n \Rightarrow \frac{a(1-r^{2n})}{1-r} = \frac{b(1-r^{2n})}{1-r^2}$$

$$\frac{a}{(1-r)} = \frac{b}{(1+r)(1-r)}$$

$$a = \frac{b}{1+r}$$

$$(1+r) = \frac{b}{a}$$

$$\therefore r = \frac{b}{a} - 1$$

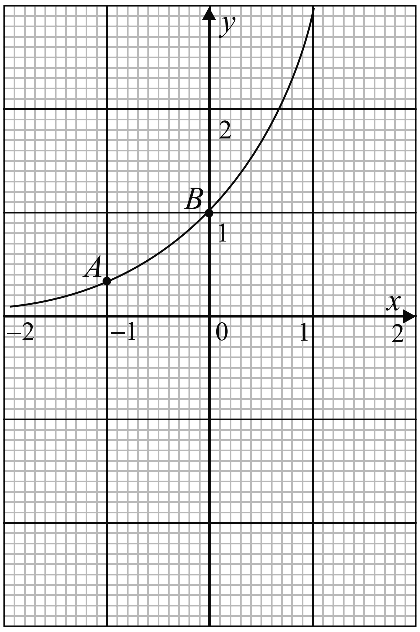
QUESTION 5 (25 MARKS)

Question 5 (a)

$f(x) = a^x$

$(-1, \tfrac{1}{3}) \in f(x) \Rightarrow f(-1) = a^{-1} = \tfrac{1}{3}$

$\dfrac{1}{a} = \dfrac{1}{3} \Rightarrow a = 3$



Question 5 (b) (i)

$f(x) = 3^x$

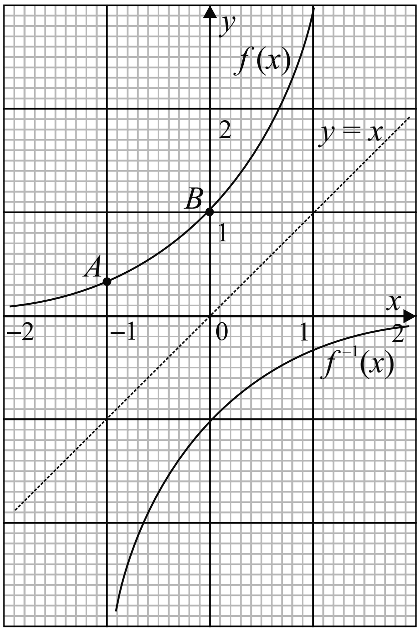
$f(0) = 3^0 = 1$

$\therefore B(0, 1)$

Question 5 (b) (ii)

The function is not bijective as every y value does not have a corresponding x value. For example, $y = -\tfrac{1}{2}$ has no x value.

Question 5 (c) [Graph to right]



Question 5 (d)

$$\boxed{y = a^x \Rightarrow \frac{dy}{dx} = a^x \times \ln a}$$

$y = 3^x \Rightarrow \dfrac{dy}{dx} = 3^x \times \ln 3 > 0$

This is always increasing as ln 3 > 0 and $3^x > 0$ for all values of x.

$\dfrac{dy}{dx} = (\ln 3) \times 3^x$

$\dfrac{d^2y}{dx^2} = (\ln 3)^2 \times 3^x$

Points of inflection: $\dfrac{d^2y}{dx^2} = 0$

$(\ln 3)^2 \times 3^x = 0$

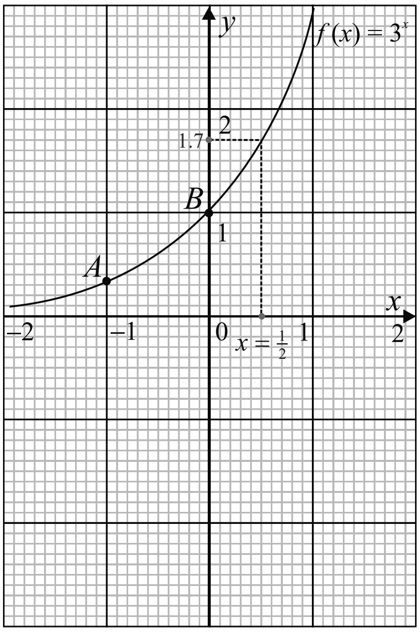
$3^x = 0$ (No solutions)

There are no points of inflection.

Question 5 (e)

$y = 3^x$

$y = \sqrt{3} = 3^{\frac{1}{2}} \approx 1.7$



QUESTION 6 (25 MARKS)

Question 6 (a)

$y = \ln(x+1)$

$\frac{dy}{dx} = \frac{1}{x+1}$

Question 6 (b)

$\frac{x+3}{x+1} = \frac{(x+1)+2}{(x+1)} = 1 + \frac{2}{x+1}$

Question 6 (c)

$\int_1^3 \frac{x+3}{x+1}dx$

$= \int_1^3 \left(1 + \frac{2}{x+1}\right)dx$

$= [x + 2\ln(x+1)]_1^3$

$= (3 + 2\ln 4) - (1 + 2\ln 2)$

$= 3 + 2\ln 4 - 1 - 2\ln 2$

$= 2 + 2(\ln 4 - \ln 2)$

$= 2 + 2\ln 2$

Average value $= \frac{2 + 2\ln 2}{3-1} = 1 + \ln 2$

Question 6 (d)

The function is not continuous. At $x = -1$ the function is not defined.

QUESTION 7 (50 MARKS)

Question 7 (a)

$$p = \frac{3000ke^{0.25t}}{1+ke^{0.25t}}$$

$$t = 0 : 500 = \frac{3000ke^{0}}{1+ke^{0}} = \frac{3000k}{1+k}$$

$$500(1+k) = 3000k$$

$$500 + 500k = 3000k$$

$$500 = 2500k$$

$$\therefore k = \frac{500}{2500} = 0.2$$

Question 7 (b)

$$p = \frac{3000ke^{0.25t}}{1+ke^{0.25t}}$$

$$1233 = \frac{3000(0.2)e^{0.25t}}{1+0.2e^{0.25t}}$$

$$1233(1+0.2e^{0.25t}) = 600e^{0.25t}$$

$$1233 + 246.6e^{0.25t} = 600e^{0.25t}$$

$$1233 = 353.4e^{0.25t}$$

$$\frac{1233}{353.4} = e^{0.25t}$$

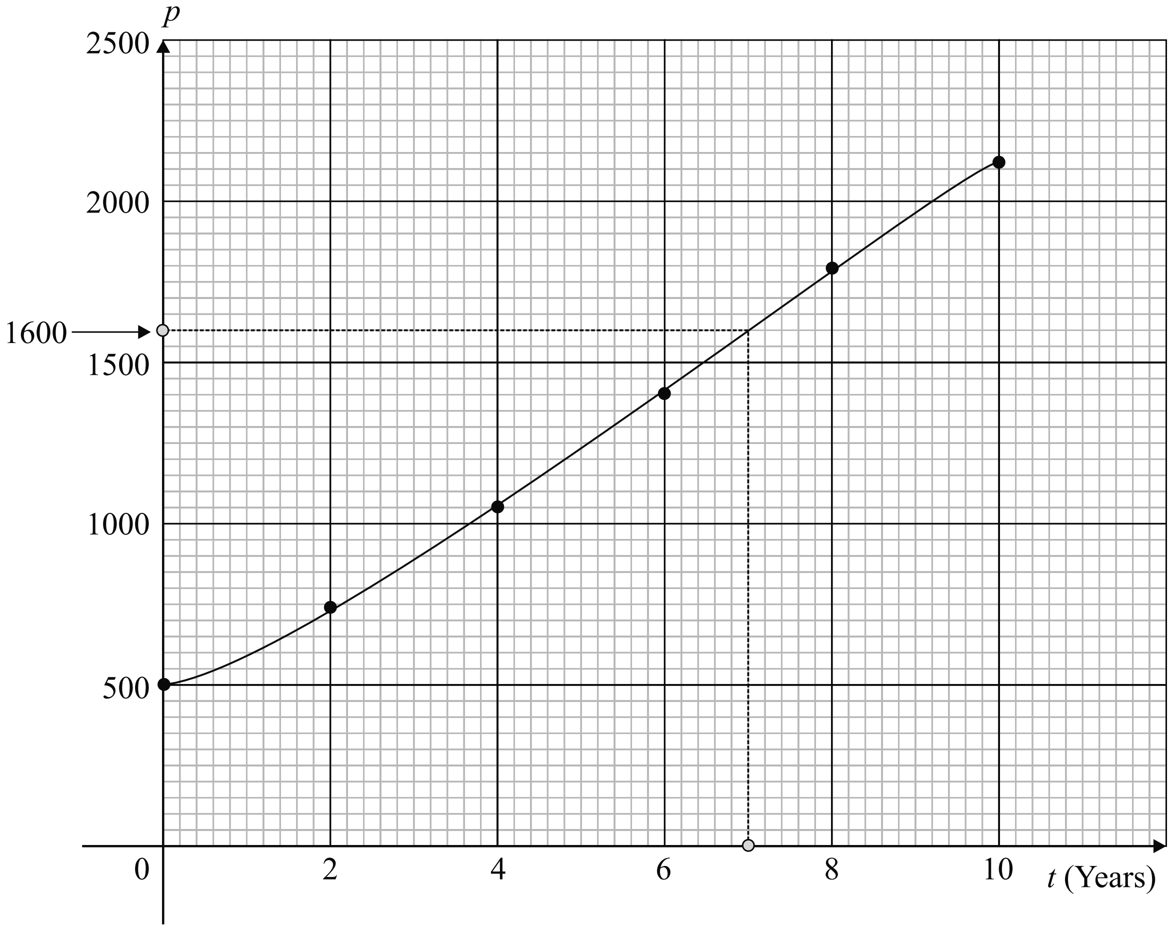
$$\ln\left(\frac{1233}{353.4}\right) = 0.25t$$

$$\therefore t = 4\ln\left(\frac{1233}{353.4}\right) \approx 5 \text{ years}$$

Question 7 (c) (i)

t (years)	0	2	4	6	8	10
p	500	744	1057	1418	1789	2127

Question 7 (c) (ii)



Question 7 (d) (i)

p = 1600 after seven years

Question 7 (d) (ii)

$$p = \frac{3000(0.2)e^{0.25(7)}}{1+(0.2)e^{0.25(7)}} \approx 1605$$

Question 7 (e)

$$p = \frac{3000(0.2)e^{0.25t}}{1+(0.2)e^{0.25t}} = \frac{600e^{0.25t}}{1+0.2e^{0.25t}}$$

$$= \frac{\frac{600e^{0.25t}}{e^{0.25t}}}{\frac{1}{e^{0.25t}}+\frac{0.2e^{0.25t}}{e^{0.25t}}} = \frac{600}{0.2+e^{-0.25t}}$$

Question 7 (f) (i)

$$\lim_{n\to\infty} r^n = \infty,\ |r|>1$$

Question 7 (f) (ii)

$$\lim_{t\to\infty} p = \lim_{t\to\infty}\left(\frac{600}{0.2+e^{-0.25t}}\right) = \lim_{t\to\infty}\left(\frac{600}{0.2+\frac{1}{e^{0.25t}}}\right) = \frac{600}{0.2+\frac{1}{\infty}} = \frac{600}{0.2+0} = 3000$$

Question 7 (g)

$$p = \frac{600}{0.2+e^{-0.25t}} = 600(0.2+e^{-0.25t})^{-1}$$

$$\frac{dp}{dt} = -600(0.2+e^{-0.25t})^{-2}(e^{-0.25t}\times(-0.25))$$

$$= \frac{150}{e^{0.25t}(0.2+e^{-0.25t})^2}$$

$$\left(\frac{dp}{dt}\right)_{t=1} = \frac{150}{e^{0.25}(0.2+e^{-0.25})^2} \approx 122 \text{ flowers/year}$$

QUESTION 8 (50 MARKS)

Question 8 (a) (i)

First repayment: $\frac{A}{1.055}$, Second repayment: $\frac{A}{(1.055)^2}$, Third repayment: $\frac{A}{(1.055)^3}$

Question 8 (a) (ii)

$$P=\frac{A}{1.055}+\frac{A}{(1.055)^2}+\frac{A}{(1.055)^3}+\frac{A}{(1.055)^4}+\frac{A}{(1.055)^5}+\frac{A}{(1.055)^6}$$

Question 8 (a) (iii)

$$P=\frac{A}{1.055}+\frac{A}{(1.055)^2}+\frac{A}{(1.055)^3}+\frac{A}{(1.055)^4}+\frac{A}{(1.055)^5}+\frac{A}{(1.055)^6}$$ [This is a geometric series.]

$$a=\frac{A}{1.055},\ r=\frac{1}{1.055}$$

$$S_6=\frac{a(1-r^6)}{1-r}=\frac{\frac{A}{1.055}\left(1-\frac{1}{1.055^6}\right)}{1-\frac{1}{1.055}}$$

$$\therefore 15\ 000=\frac{\frac{A}{1.055}\left(1-\frac{1}{1.055^6}\right)}{1-\frac{1}{1.055}}\Rightarrow\frac{15\ 000\left(1-\frac{1}{1.055}\right)}{\left(1-\frac{1}{1.055^6}\right)}=\frac{A}{1.055}$$

$$\therefore A=\frac{1.055\times 15\ 000\left(1-\frac{1}{1.055}\right)}{\left(1-\frac{1}{1.055^6}\right)}=€3002.68$$

Question 8 (b) (i)

$$P=\frac{A}{(1+i)^1}+\frac{A}{(1+i)^2}+\frac{A}{(1+i)^3}+\dots\dots\dots\dots\dots\dots+\frac{A}{(1+i)^t}$$

Question 8 (b) (ii)

$$P=\frac{A}{(1+i)^1}+\frac{A}{(1+i)^2}+\frac{A}{(1+i)^3}+\dots\dots\dots\dots\dots\dots+\frac{A}{(1+i)^t}$$ [This is a geometric series.]

$$a=\frac{A}{(1+i)},\ r=\frac{1}{(1+i)}$$

$$S_t=P=\frac{a(1-r^t)}{1-r}=\frac{\frac{A}{(1+i)}\left(1-\frac{1}{(1+i)^t}\right)}{1-\frac{1}{(1+i)}}=\frac{A\left(\frac{(1+i)^t-1}{(1+i)^t}\right)}{(1+i)\left(\frac{1+i-1}{(1+i)}\right)}=\frac{A\left(\frac{(1+i)^t-1}{(1+i)^t}\right)}{i}=\frac{A((1+i)^t-1)}{i(1+i)^t}$$

$$\therefore A=P\frac{i(1+i)^t}{((1+i)^t-1)}$$

Question 8 (b) (iii)

$$A = P\frac{i(1+i)^t}{((1+i)^t - 1)}$$

$$= 15\ 000 \times \frac{0.055(1.055)^6}{(1.055^6 - 1)} = €3002.68$$

Question 8 (c) (i)

$$(1.055)^{\frac{1}{12}} = 1.0044717 = 1 + i$$

$$i = 0.0044717 = 0.44717\%$$

Question 8 (c) (ii)

$$A = P\frac{i(1+i)^t}{((1+i)^t - 1)}$$

$$= 15\ 000 \times \frac{0.0044717(1.0044717)^{72}}{(1.0044717^{72} - 1)} = €244.13$$

Question 8 (c) (iii)

$$€3002.68 \times 6 - €244.13 \times 72 = €438.72$$

QUESTION 9 (50 MARKS)

Question 9 (a) (i)

$$\left(\frac{dN}{dt}\right)_{t=10} = 0.8e^{0.006(10)} = 0.85 \text{ billions of barrels per year}$$

Question 9 (a) (ii)

$$\left(\frac{dN}{dt}\right)_{t=20} = 0.8e^{0.006(20)} = 0.90 \text{ billions of barrels per year}$$

Question 9 (b)

$$\frac{dN}{dt} = 0.8e^{0.006t}$$

$$\int dN = \int 0.8e^{0.006t}\, dt$$

$$N = \frac{0.8}{0.006}e^{0.006t} + c = \frac{400}{3}e^{0.006t} + c$$

Boundary conditions: At $t = 0$, $N = 0$.

$$N = \frac{400}{3}e^{0.006t} + c$$

$$\therefore 0 = \frac{400}{3}e^{0} + c \Rightarrow c = -\frac{400}{3}$$

$$N = \frac{400}{3}e^{0.006t} - \frac{400}{3} = \frac{400}{3}(e^{0.006t} - 1)$$

$$t = 20 : N = \frac{400}{3}(e^{0.006(20)} - 1) = 17 \text{ billion}$$

Question 9 (c)

$$N = 55 : \frac{400}{3}(e^{0.006t} - 1) = 55$$

$$e^{0.006t} = \frac{3 \times 55}{400} + 1 = \frac{113}{80}$$

$$t = \frac{1}{0.006}\ln\left(\frac{113}{80}\right) = 57.56 \approx 58 \text{ years}$$

Question 9 (d)

$$1 \text{ barrel} = 0.159 \text{ m}^3$$

$$4.714 \times 10^{10} \text{ m}^3 = \frac{4.714 \times 10^{10}}{0.159} \text{ barrels}$$

$$= 2.965 \times 10^{11} \text{ barrels}$$

$$= 296.5 \times 10^{9} \text{ barrels}$$

$$= 296.5 \text{ billion barrels}$$

Question 9 (e)

$$N = 296.5 : \frac{400}{3}(e^{0.006t} - 1) = 296.5$$

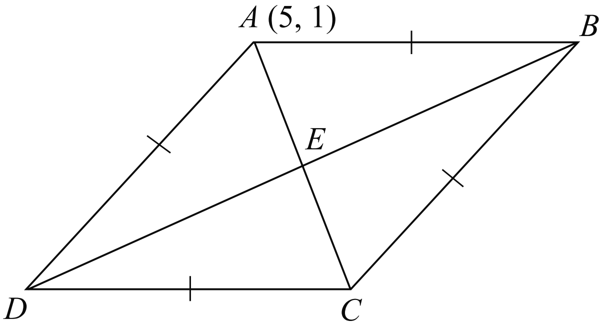
$$e^{0.006t} = \frac{3 \times 296.5}{400} + 1 = \frac{2579}{800}$$

$$t = \frac{1}{0.006}\ln\left(\frac{2579}{800}\right) \approx 195 \text{ years}$$

SAMPLE PAPER 5: PAPER 2

QUESTION 1 (25 MARKS)

Question 1 (a)



$AC \cap DC = \{C\}$

$AC : x - y - 4 = 0$

$DC : 2x + y - 5 = 0$

$3x \quad -9 = 0 \Rightarrow x = 3$

$3 - y - 4 = 0 \Rightarrow y = -1$

$\therefore C(3, -1)$

E is the midpoint of A and C

$A(5, 1), C(3, -1)$

$$E = \left(\frac{5+3}{2}, \frac{1-1}{2} \right) = (4, 0)$$

Question 1 (b)

$AC \perp BD$ [Diagonals of rhombus are perpendicular.]

$A(5, 1), C(3, -1)$

Slope of $AC = \frac{-1-1}{3-5} = \frac{-2}{-2} = 1$

Slope of $BD = -1$

$$E = \left(\frac{5+3}{2}, \frac{1-1}{2} \right) = (4, 0)$$

$DB : m = -1, (x_1, y_1) = (4, 0)$

$y - 0 = -1(x - 4)$

$y = -x + 4$

$x + y - 4 = 0$

Question 1 (c)

$DC \cap DB = \{D\}$

$DC : 2x + y - 5 = 0$

$DB : x + y - 4 = 0$

$x \quad -1 = 0 \Rightarrow x = 1$

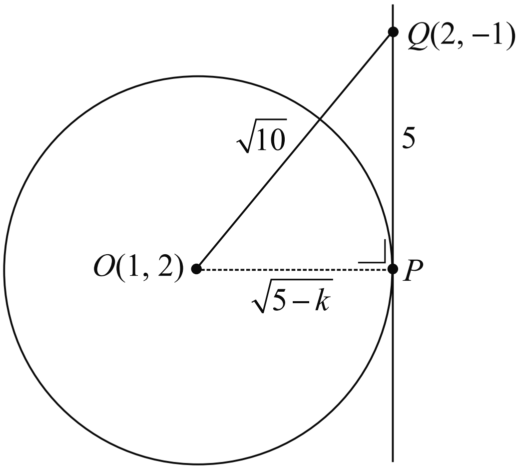
$1 + y - 4 = 0 \Rightarrow y = 3$

$\therefore D(1, 3)$

$D(1, 3) \rightarrow E(4, 0) \rightarrow B(7, -3)$

QUESTION 2 (25 MARKS)

Question 2 (a)



$s: x^2+y^2-2x-4y+k=0$

Centre $O(1, 2)$, $r=\sqrt{1^2+2^2-k}=\sqrt{5-k}$

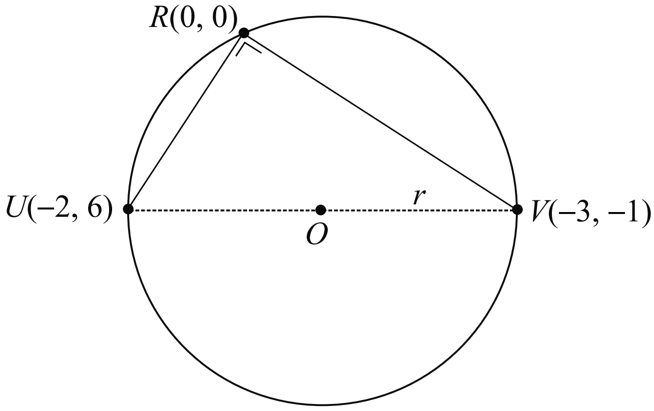
OPQ is a right-angled triangle.

$5^2+(\sqrt{5-k})^2=(\sqrt{10})^2$

$25+5-k=10$

$\therefore k=20$

Question 2 (b)



$U(-2, 6)$, $V(-3, -1)$, $R(0, 0)$

Slope of $RU=\dfrac{0-(-2)}{0-6}=-\dfrac{2}{6}=-\dfrac{1}{3}$

Slope of $RV=\dfrac{0-(-3)}{0-(-1)}=\dfrac{3}{1}=3$

$-\dfrac{1}{3}\times 3=-1 \Rightarrow RU \perp RV$

Each angle in a semicircle is a right angle. Therefore $[UV]$ is the diameter of the circle containing points U, V and R.

$U(-2, 6)$, $V(-3, -1)$

Midpoint of $UV=$ Centre $O\left(\dfrac{-2-3}{2}, \dfrac{6-1}{2}\right)$

$=(-\tfrac{5}{2}, \tfrac{5}{2})=(h, k)$

$r=\tfrac{1}{2}|UV|=\tfrac{1}{2}\sqrt{(-3-(-2))^2+(-1-6)^2}$

$=\tfrac{1}{2}\sqrt{1+49}=\tfrac{1}{2}\sqrt{50}$

Equation of circle: $(x-h)^2+(y-k)^2=r^2$

$(x+\tfrac{5}{2})^2+(y-\tfrac{5}{2})^2=(\tfrac{1}{2}\sqrt{50})^2$

$x^2+5x+\tfrac{25}{4}+y^2-5y+\tfrac{25}{4}=\tfrac{50}{4}$

$x^2+y^2+5x-5y=0$

Or

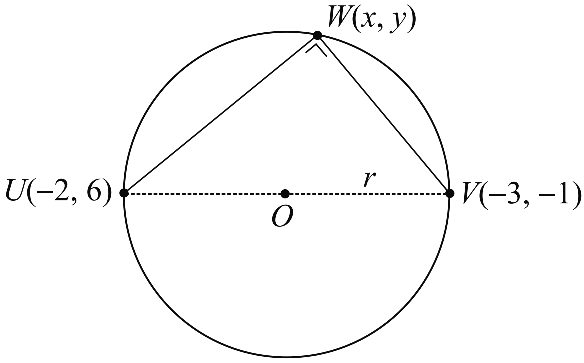
Equation of circle: $x^2+y^2+2gx+2fy+c=0$

$R(0, 0)\in x^2+y^2+2gx+2fy+c=0 \Rightarrow c=0$

$(-g, -f)=(-\tfrac{5}{2}, \tfrac{5}{2}) \Rightarrow g=\tfrac{5}{2}, f=-\tfrac{5}{2}$

$\therefore x^2+y^2+5x-5y=0$

Or



Slope of $UW \perp$ Slope of UV

$\therefore \dfrac{(y-6)}{(x+2)}\times\dfrac{(y+1)}{(x+3)}=-1$

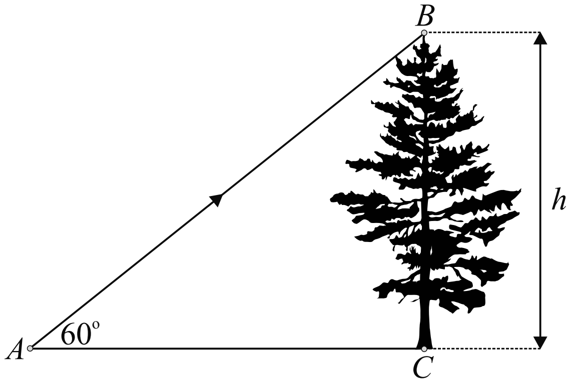
$y^2-5y-6=-(x^2+5x+6)$

$x^2+y^2+5x-5y=0$

QUESTION 3 (25 MARKS)

Question 3 (a)

$$\tan 60^\circ = \frac{h}{|AC|} \Rightarrow |AC| = \frac{h}{\tan 60^\circ} = \frac{h}{\sqrt{3}}$$



Question 3 (b)

Consider the right-angled triangle *DBC*.

$$\tan 30^\circ = \frac{h}{|DC|} \Rightarrow |DC| = \frac{h}{\tan 30^\circ} = \frac{h}{\left(\frac{1}{\sqrt{3}}\right)} = \sqrt{3}h$$

Consider the right-angled triangle *ADC*.

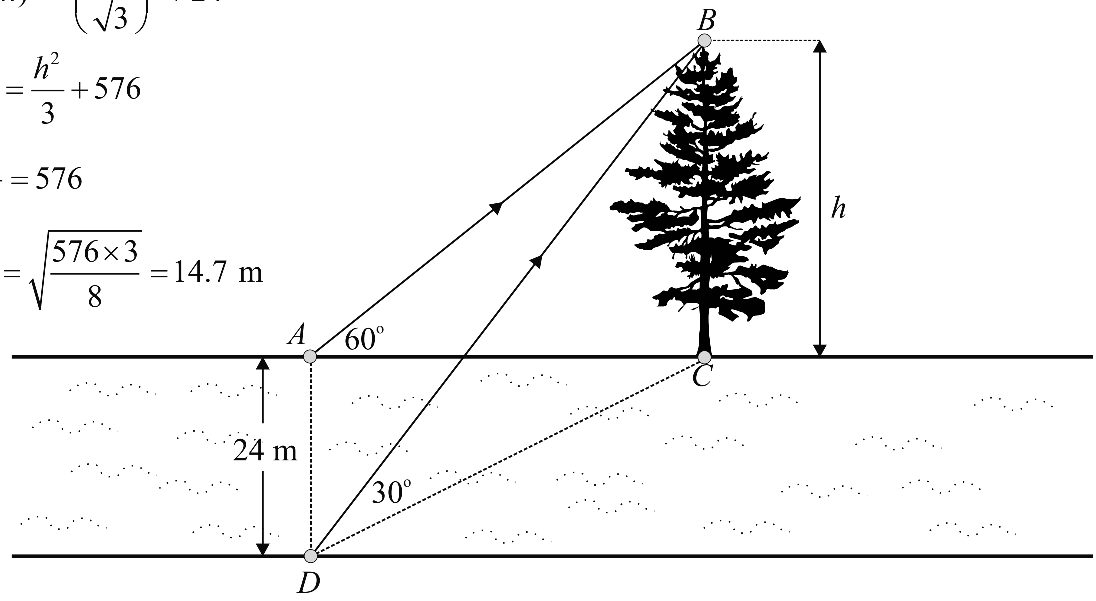
$$|DC|^2 = |AC|^2 + |AD|^2$$

$$(\sqrt{3}h)^2 = \left(\frac{h}{\sqrt{3}}\right)^2 + 24^2$$

$$3h^2 = \frac{h^2}{3} + 576$$

$$\frac{8h^2}{3} = 576$$

$$\therefore h = \sqrt{\frac{576 \times 3}{8}} = 14.7 \text{ m}$$



QUESTION 4 (25 MARKS)

Question 4 (a)

Score	f	x	fx	CF
0–10	20	5	100	20
10–20	60	15	900	80
20–30	140	25	3500	220
30–40	200	35	7000	420
40–50	400	45	18 000	820
50–60	500	55	27 500	1320
60–70	430	65	27 950	1750
70–80	260	75	19 500	2010
80–90	140	85	11 900	2150
90–100	60	95	5700	2210
	2210		122 050	

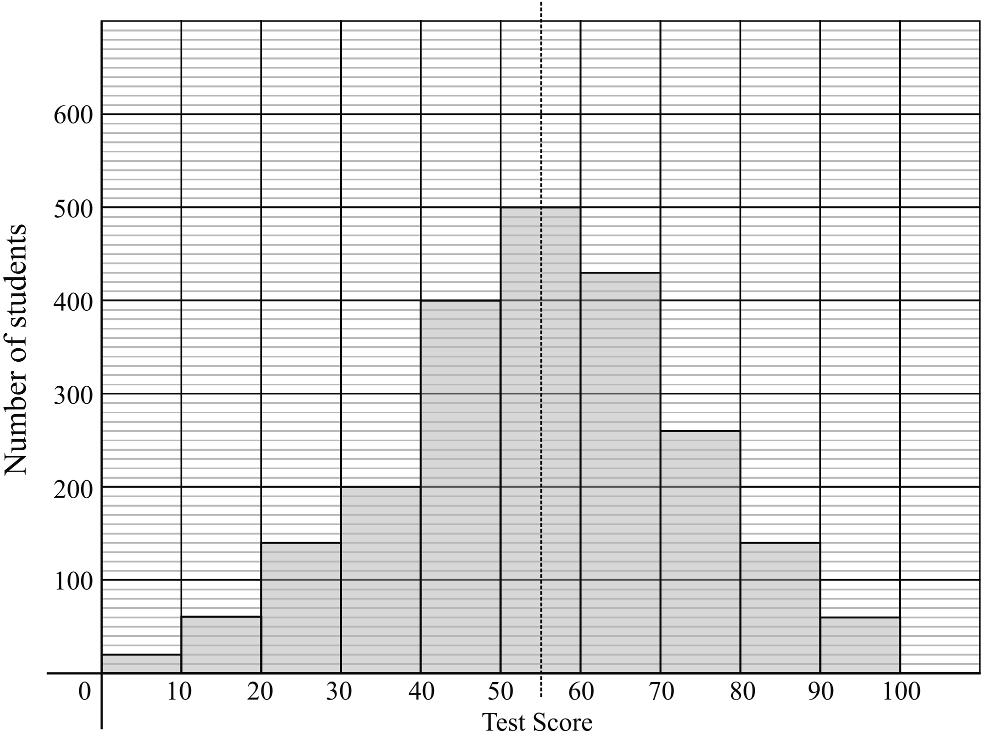
Question 4 (a) (i)

Number of students = 2210

Question 4 (a) (ii)

$$\text{Mean } \mu = \frac{122\ 050}{2210} = 55.2$$

Question 4 (b)



The 1105th. student has the median score. This student lies in the 50–60 class interval.

$$\text{Median } = \frac{(1105-820)}{500} \times 10 + 50 = 55.7$$

The median and mean are nearly the same as the distribution is close to normal.

Question 4 (c) (i)

$\sigma = 18.5,\ \mu = 55.2$

$\mu + \sigma = 55.2 + 18.5 = 73.7$

$\mu - \sigma = 55.2 - 18.5 = 36.7$

68% of students scored within 1 standard deviation of the mean.

EMPIRICAL RULE: In any normal distribution with mean $\bar{x}$ and standard deviation σ.

1. 68% of the data falls within 1σ of the mean $\bar{x}$.
2. 95% of the data falls within 2σ of the mean $\bar{x}$.
3. 99.7% of the data falls within 3σ of the mean $\bar{x}$.

Question 4 (c) (ii)

New mean = 58.2

New standard deviation = 18.5 (The spread of scores remain unchanged.)

QUESTION 5 (25 MARKS)

Question 5 (a)

(i) Two events are said to be independent if one event *does not affect* the outcome of the other.

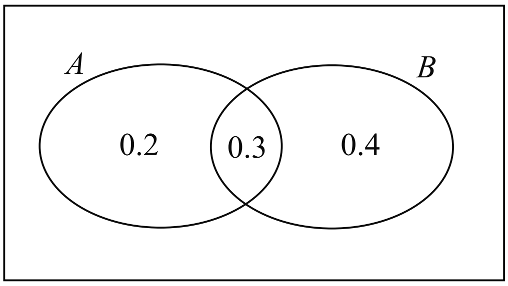
(ii) If A and B are independent events $P(A \cap B) = P(A) \times P(B)$

(iii) If A and B are independent events $P(B|A) = P(B)$

(iv) For all events $P(A \cap B) = P(A) \times P(B \mid A)$

Question 5 (b)

The events A and B are such that $P(A) = 0.5$, $P(B) = 0.7$ and $P(A \cap B) = 0.3$.



(i)
$$\begin{aligned} P(A \cup B) &= P(A) + P(B) - P(A \cap B) \\ &= 0.7 + 0.5 - 0.3 \\ &= 0.9 \end{aligned}$$

(ii) $$P(A \mid B) = \frac{P(A \cap B)}{P(B)} = \frac{0.3}{0.7} = \frac{3}{7}$$

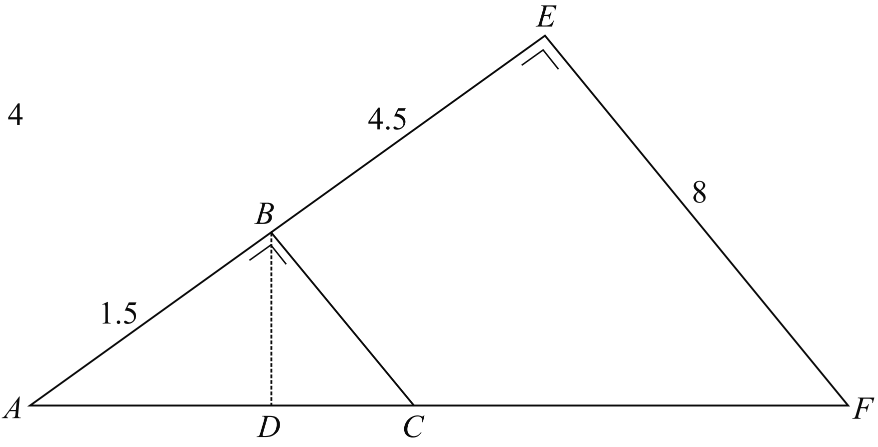
(iii) The events are not independent.

$P(A \mid B) \neq P(A)$

$\frac{3}{7} \neq 0.5$

QUESTION 6 (25 MARKS)

Question 6 (a)



$$k = \frac{|EA|}{|BA|} = \frac{1.5+4.5}{1.5} = \frac{6}{1.5} = 4$$

Question 6 (b)

$|AF|^2 = 8^2 + 6^2$

$\therefore |AF| = 10$

$k = \frac{|AF|}{|AC|} \Rightarrow 4 = \frac{10}{|AC|}$

$\therefore |AC| = \frac{10}{4} = 2.5$

$|BC| = \sqrt{2.5^2 - 1.5^2} = 2$

Area of triangle ABC: $A = \frac{1}{2} \times 1.5 \times 2 = 1.5$

Question 6 (c)

$\frac{1}{2} \times |AC||BD| = 1.5$

$\frac{1}{2} \times 2.5 \times |BD| = 1.5$

$\therefore |BD| = \frac{3}{2.5} = 1.2$

$|AD| = \sqrt{1.5^2 - 1.2^2} = 0.9$

QUESTION 7 (75 MARKS)

Question 7 (a)

Amount of claim x (€/year)	0	1000	5000	10 000
Probability	0.8	0.15	0.04	0.01

$$E = \sum xP(x) = 0(0.8) + 1000(0.15) + 5000(0.04) + 10\ 000(0.01) = €450$$

Question 7 (b)

This is the average payout per claim per year for the company.

Question 7 (c)

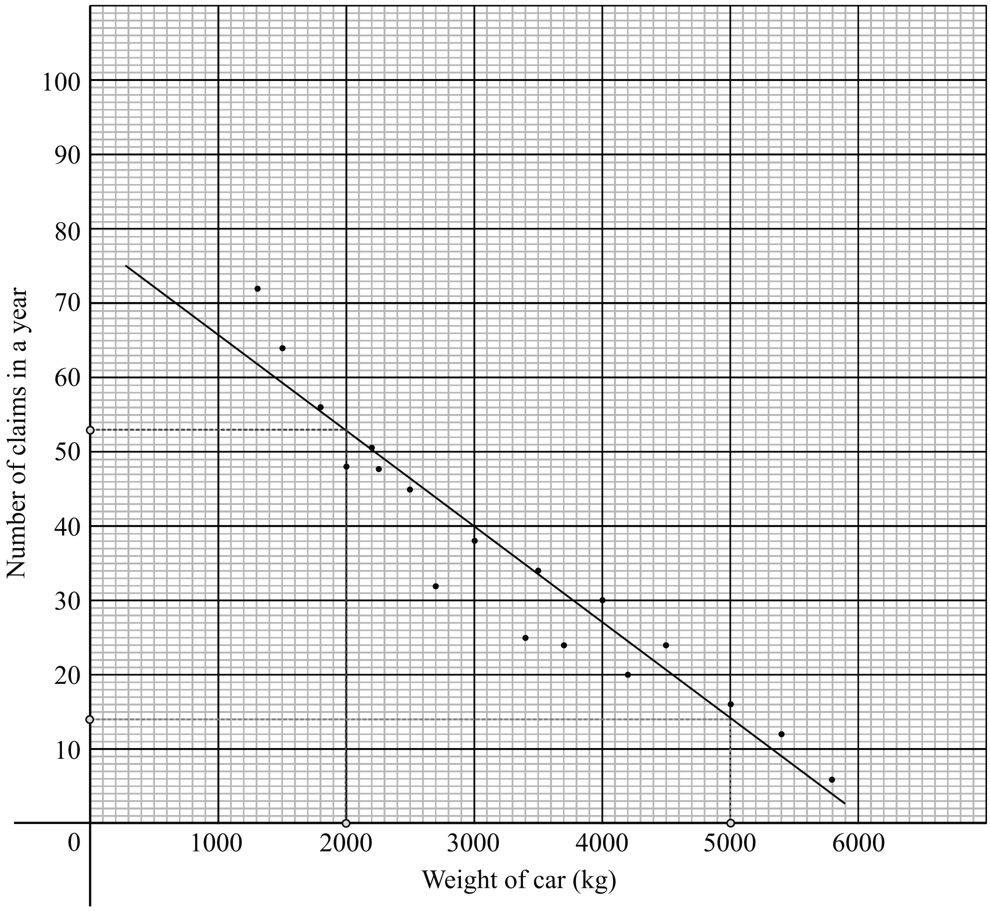
$P(x > 450) = 0.15 + 0.04 + 0.01 = 0.2$

Question 7 (d)

Profit per car (€)	M	$M - 500$	$M - 4500$	$M - 9500$
Probability	0.8	0.15	0.04	0.01

$$0.8M + 0.15(M - 500) + 0.04(M - 4500) + 0.01(M - 9500) = 200$$
$$0.8M + 0.15M - 75 + 0.04M - 180 + 0.01M - 95 = 200$$
$$M - 350 = 200$$
$$\therefore M = €550$$

Question 7 (e) (i)



There is a strong negative correlation between the number of claims per year and the weight of the car.

Question 7 (e) (ii)

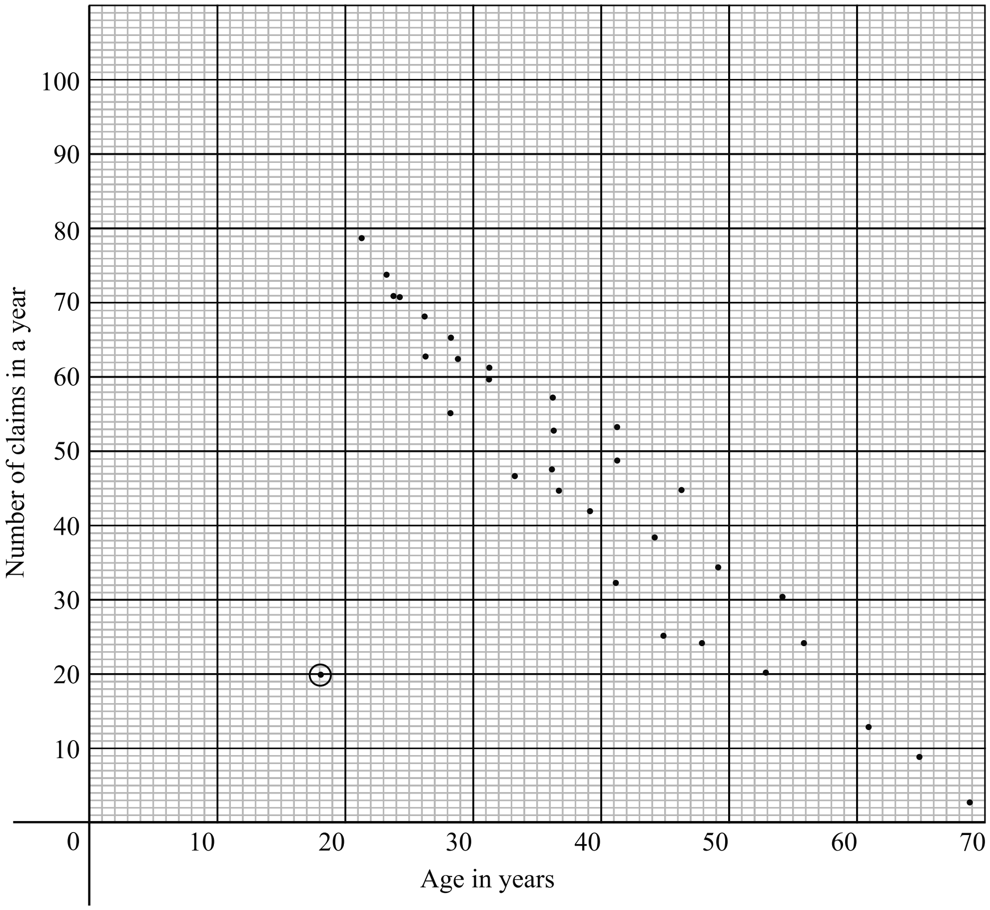
(2000, 53), (5000, 14)

$$m = \frac{14 - 53}{5000 - 2000} = -0.01$$

Question 7 (e) (iii)

No. Maybe there are fewer heavier cars insured with this company or because the premium on these vehicles is so high claims may not be made when minor damage is incurred.

Question 7 (f) (i)



Outlier (18, 20), learners usually have an experienced driver with them.

Question 7 (f) (ii)

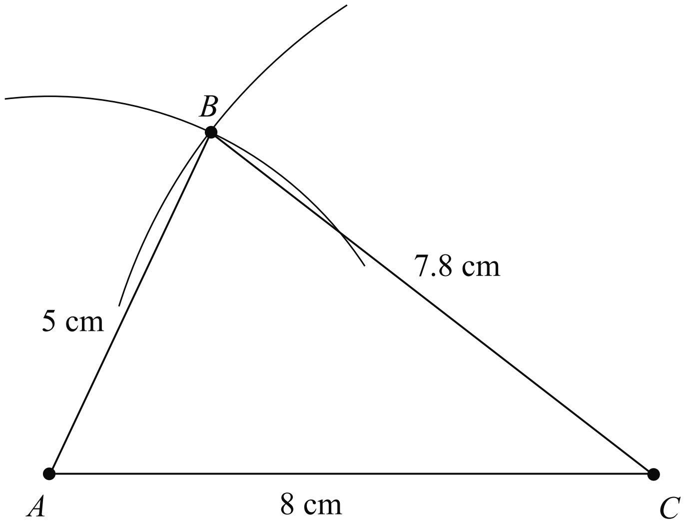
Strong negative correlation.

Question 7 (g)

According to this data, the insurance company could claim that its lowest risk is a person aged **69 years** driving a car of weight **5800 kg** and the highest risk is a person aged **21 years** driving a car of weight **1300 kg**.

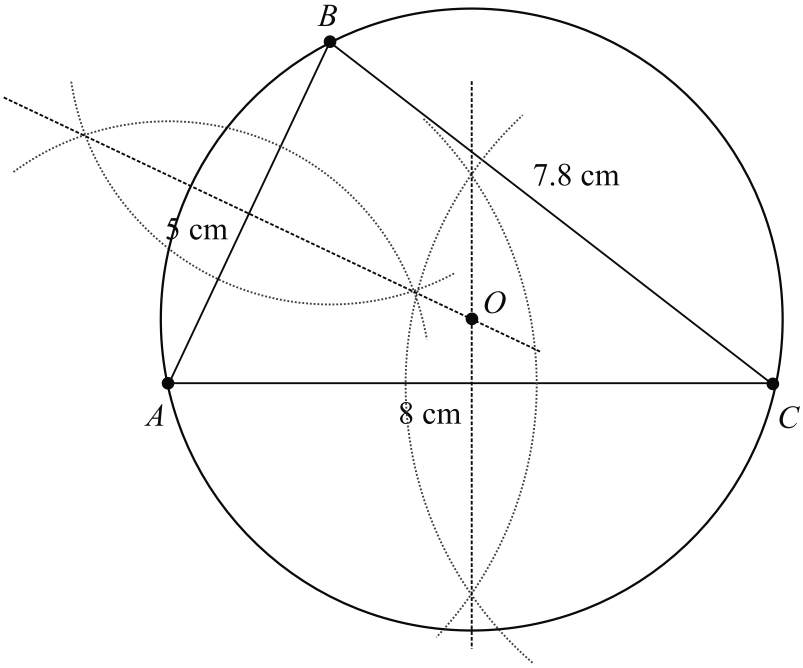
QUESTION 8 (35 MARKS)

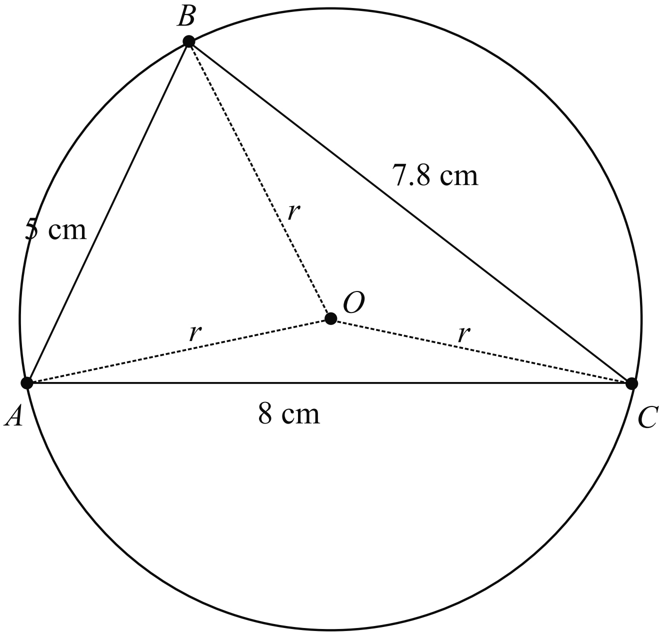
Question 8 (a)



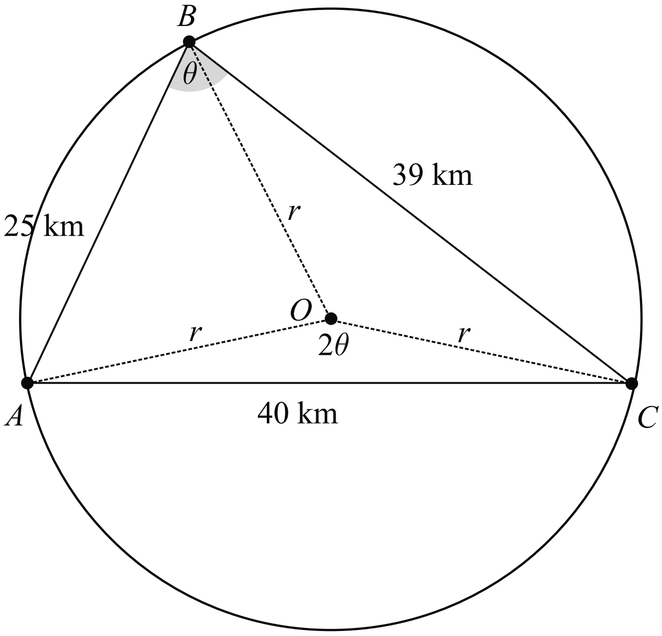
Tell students to draw the base of the triangle up near the centre of the rectangle in order to leave room below to draw the perpendicular bisectors.

Question 8 (b)





Question 8 (c)



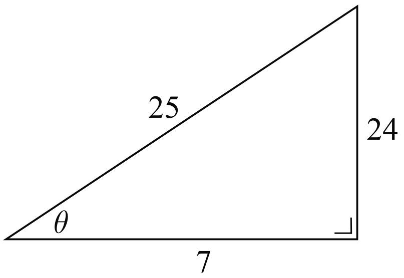
Let $|\angle ABC| = \theta$

$40^2 = 25^2 + 39^2 - 2(25)(39)\cos\theta$

$1950\cos\theta = 546$

$\therefore \cos\theta = \frac{546}{1950} = \frac{7}{25}$

Question 8 (d)



The angle *AOC* at the centre standing on the arc *AC* is twice the angle *ABC* at the circle.

Apply the Cosine rule to triangle *AOC*:

$$40^2 = r^2 + r^2 - 2(r)(r)\cos 2\theta$$
$$1600 = 2r^2 - 2r^2\cos 2\theta$$
$$1600 = 2r^2(1 - \cos 2\theta)$$
$$1600 = 2r^2(1 - \cos^2\theta + \sin^2\theta)$$
$$800 = r^2(\sin^2\theta + \sin^2\theta)$$
$$800 = 2r^2\sin^2\theta$$
$$400 = r^2\left(\frac{24}{25}\right)^2$$
$$\therefore r = \frac{20 \times 25}{24} = 20.83 \text{ km}$$

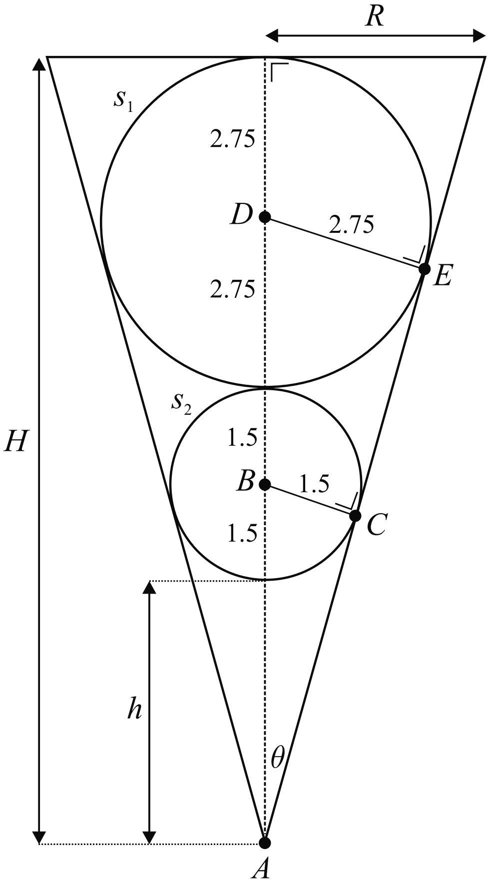
Alternative method:

Apply the Cosine rule to triangle *AOC*:

$$\boxed{\begin{array}{l}\cos^2\theta = \tfrac{1}{2}(1 + \cos 2\theta) \\ \therefore \cos 2\theta = 2\cos^2\theta - 1\end{array}}$$

$$40^2 = r^2 + r^2 - 2(r)(r)\cos 2\theta$$
$$1600 = 2r^2 - 2r^2\cos 2\theta$$
$$1600 = 2r^2(1 - \cos 2\theta)$$
$$1600 = 2r^2(1 - 2\cos^2\theta + 1)$$
$$1600 = 4r^2(1 - \cos^2\theta)$$
$$400 = r^2 \times \left(1 - \frac{49}{625}\right) = r^2\left(\frac{576}{625}\right)$$
$$\therefore r = \sqrt{\frac{400 \times 625}{576}} = 20.83 \text{ km}$$

QUESTION 9 (40 MARKS)



Question 9 (a)

$$\frac{|BC|}{|AB|}=\frac{|DE|}{|AD|}\Rightarrow\frac{1.5}{1.5+h}=\frac{2.75}{5.75+h}$$

$$1.5(5.75+h)=2.75(1.5+h)$$

$$8.625+1.5h=4.125+2.75h$$

$$4.5=1.25h$$

$$\therefore h=\frac{4.5}{1.25}=3.6\,\text{cm}$$

$$\sin\theta=\frac{1.5}{1.5+3.6}=\frac{1.5}{5.1}\Rightarrow\theta=\sin^{-1}\left(\frac{1.5}{5.1}\right)=17.1^{\circ}$$

Question 9 (b)

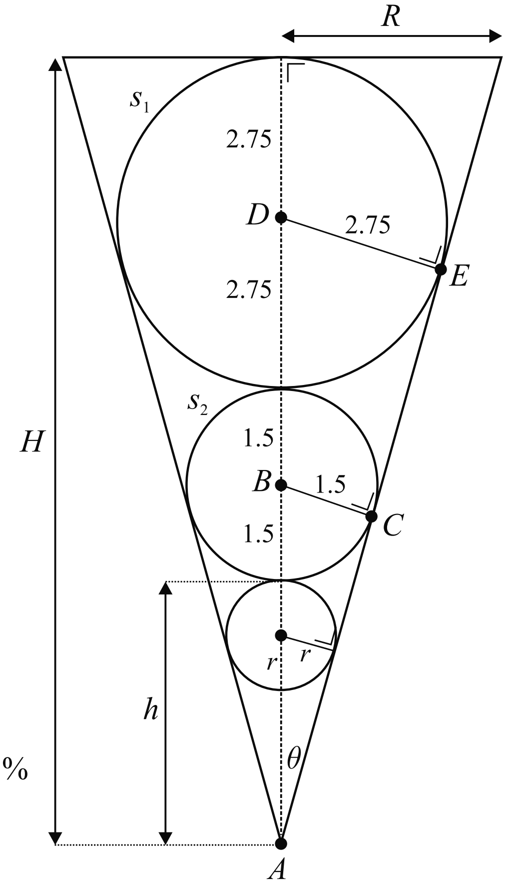
$$\text{Height } H=3.6+2(1.5)+2(2.75)=12.1\text{ cm}$$

$$\text{Radius } R:\tan\theta=\frac{R}{H}\Rightarrow R=12.1\tan17.1^{\circ}=3.72\text{ cm}$$

Question 9 (c)

$$V=\tfrac{1}{3}\pi R^2H=\tfrac{1}{3}\pi(3.72)^2(12.1)=175\text{ cm}^3$$

Question 9 (d)



$$\sin17.1^{\circ}=\frac{r}{3.6-r}$$

$$(3.6-r)\sin17.1^{\circ}=r$$

$$3.6\sin17.1^{\circ}-r\sin17.1^{\circ}=r$$

$$3.6\sin17.1^{\circ}=r+r\sin17.1^{\circ}$$

$$3.6\sin17.1^{\circ}=r(1+\sin17.1^{\circ})$$

$$\therefore r=\frac{3.6\sin17.1^{\circ}}{(1+\sin17.1^{\circ})}=0.82\text{ cm}$$

Volume of empty space with two spheres:
$V_1=175-\tfrac{4}{3}\pi(2.75)^3-\tfrac{4}{3}\pi(1.5)^3=73.75\text{ cm}^3$

Volume of empty space with three spheres:
$V_2=73.75-\tfrac{4}{3}\pi(0.82)^3=71.44\text{ cm}^3$

$$\%\text{ decrease in empty space}=\frac{73.75-71.44}{73.75}\times100\%=3.13\%$$

SAMPLE PAPER 6: PAPER 1

QUESTION 1 (25 MARKS)

Question 1 (a)

$4T_n = [1+(-1)^n][1+i^n]$

$n=1: 4T_1 = [1+(-1)^1][1+i^1] = [0][1+i] = 0 \Rightarrow T_1 = 0$

$n=2: 4T_2 = [1+(-1)^2][1+i^2] = [2][1-1] = [2][0] = 0 \Rightarrow T_2 = 0$

$n=3: 4T_3 = [1+(-1)^3][1+i^3] = [1-1][1-i] = [0][1-i] = 0 \Rightarrow T_3 = 0$

$n=4: 4T_4 = [1+(-1)^4][1+i^4] = [1+1][1+1] = [2][2] = 4 \Rightarrow T_4 = 1$

0, 0, 0, 1,........

$\therefore S_{100} = 25\times 0 + 25\times 0 + 25\times 0 + 25\times 1 = 25$

Question 1 (b)

STEPS FOR PROOF BY INDUCTION

1. Prove result is true for some starting value of $n \in \mathbb{N}$.

2. Assume result is true for $n = k$.

3. Prove result is true for $n = (k+1)$.

1. Prove true for $n=1: 7^1 - 4^1 = 7-4 = 3$ [Therefore, true for $n =$ 1.]

2. Assume true for $n=k: 7^k - 4^k = 3a,\ a \in \mathbb{N}.\ \therefore 7^k = 3a + 4^k.$

3. Prove true for $n = k+1$: Prove $7^{k+1} - 4^{k+1} = 3b,\ b \in \mathbb{N}$

Proof:

$$\begin{aligned} 7^{k+1} - 4^{k+1} &= 7(7^k) - 4^{k+1} \\ &= 7(3a + 4^k) - 4^{k+1} \\ &= 21a + 7\times 4^k - 4^{k+1} \\ &= 21a + 4^k(7 - 4^1) \\ &= 21a + 4^k(3) \\ &= 3(7a + 4^k) \\ &= 3b \end{aligned}$$

Therefore, assuming true for $n = k$ means it is true for $n = k + 1$. So true for $n = 1$ and true for $n = k$ means it is true for $n = k + 1$. This implies it is true for all $n \in \mathbb{N}$.

QUESTION 2 (25 MARKS)

Question 2 (a)

$$z = \cos\left(\frac{\pi}{3}\right) + i\sin\left(\frac{\pi}{3}\right) = \frac{1}{2} + \frac{\sqrt{3}}{2}i$$

$$|1+z| = \left|\frac{3}{2} + \frac{\sqrt{3}}{2}i\right| = \sqrt{\left(\frac{3}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2} = \sqrt{\frac{9}{4} + \frac{3}{4}} = \sqrt{\frac{12}{4}} = \sqrt{3}$$

Question 2 (b)

$|z| = 5 \Rightarrow |x+iy| = 5$

$\therefore \sqrt{x^2+y^2} = 5$

$x^2 + y^2 = 25$

This is a circle with centre (0, 0) and radius 5.

$z + \overline{z} = 8$

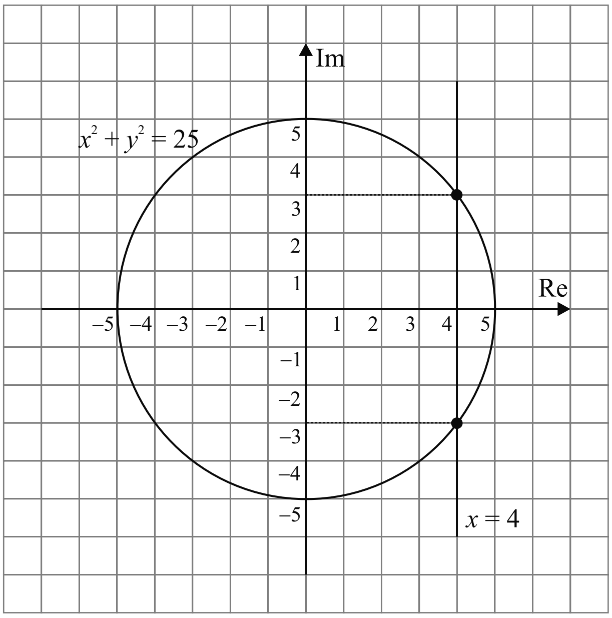
$x + iy + x - iy = 8$

$2x = 8$

$\therefore x = 4$

This is a straight line.

$A \cap B = \{4+3i,\ 4-3i\}$



QUESTION 3 (25 MARKS)

Question 3 (a)

$$S_n = 3\left(1-\left(\frac{1}{3}\right)^n\right) \qquad \boxed{T_n = S_n - S_{n-1}}$$

$$S_{n-1} = 3\left(1-\left(\frac{1}{3}\right)^{n-1}\right)$$

$$T_n = S_n - S_{n-1} = 3\left(1-\left(\frac{1}{3}\right)^n\right) - 3\left(1-\left(\frac{1}{3}\right)^{n-1}\right)$$

$$= 3 - 3\left(\frac{1}{3}\right)^n - 3 + 3\left(\frac{1}{3}\right)^{n-1}$$

$$= -3\left(\frac{1}{3}\right)^n + 3\left(\frac{1}{3}\right)^{n-1}$$

$$= 3\left(\frac{1}{3}\right)^{n-1}\left[1-\frac{1}{3}\right]$$

$$= 3\left(\frac{1}{3}\right)^{n-1}\left[\frac{2}{3}\right] = 2\left(\frac{1}{3}\right)^{n-1}$$

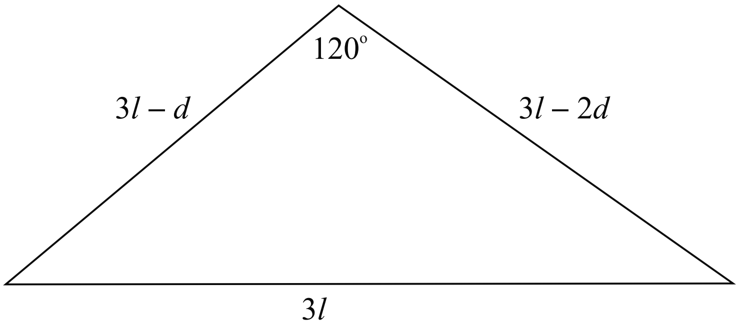
$$\boxed{\frac{T_n}{T_{n-1}} = \text{Constant}}$$

$$T_n = 2\left(\frac{1}{3}\right)^{n-1},\ T_{n-1} = 2\left(\frac{1}{3}\right)^{n-2}$$

$$\frac{T_n}{T_{n-1}} = \frac{2\left(\frac{1}{3}\right)^{n-1}}{2\left(\frac{1}{3}\right)^{n-2}} = \frac{1}{3}$$

Question 3 (b)

$$\boxed{a^2 = b^2 + c^2 - 2bc\cos A}$$



$$(3l)^2 = (3l-d)^2 + (3l-2d)^2 - 2(3l-d)(3l-2d)\cos 120^\circ$$

$$9l^2 = 9l^2 - 6ld + d^2 + 9l^2 - 12ld + 4d^2 - 2(-\tfrac{1}{2})(9l^2 - 9ld + 2d^2)$$

$$9l^2 = 9l^2 - 6ld + d^2 + 9l^2 - 12ld + 4d^2 + 9l^2 - 9ld + 2d^2$$

$$0 = 7d^2 - 27ld + 18l^2$$

$$0 = (7d - 6l)(d - 3l)$$

$$d = \frac{6l}{7},\ 3l$$

$$3l,\ 3l - \frac{6l}{7},\ 3l - \frac{12l}{7} = 3l,\ \frac{15l}{7},\ \frac{9l}{7}$$

QUESTION 4 (25 MARKS)

Question 4 (a)

$$x : y = 3 : 2 \Rightarrow \frac{x}{y} = \frac{3}{2}$$

$$\therefore x = \frac{3y}{2}$$

$$x - y = 8$$

$$\frac{3y}{2} - y = 8$$

$$3y - 2y = 16$$

$$\therefore y = 16$$

$$x = \frac{3(16)}{2} = 24$$

Question 4 (b)

V_1 = Volume of small bucket
V_2 = Volume of large bucket

$$\therefore \frac{V_1}{2} = \frac{3V_2}{8} \Rightarrow \frac{V_2}{V_1} = \frac{8}{6} = \frac{4}{3}$$

Question 4 (c)

$$(x+a)(x+b)(x+c)$$

$$= (x+a)(x^2 + bx + cx + bc)$$

$$= x^3 + bx^2 + cx^2 + bcx + ax^2 + abx + acx + abc$$

$$= x^3 + (a+b+c)x^2 + (ab+ac+bc) + abc$$

Question 4 (c) (i)

$$(x+1)(x+2)(x+3)$$

$$= x^3 + (1+2+3)x^2 + (2+3+6)x + 6$$

$$= x^3 + 6x^2 + 11x + 6$$

Question 4 (c) (ii)

$$(x-1)(x-5)(x+7)$$

$$= x^3 + (-1-5+7)x^2 + (5-7-35)x + 35$$

$$= x^3 + x^2 - 37x + 35$$

QUESTION 5 (25 MARKS)

Question 5 (a)

$$y = f(x) = \frac{2}{x} - 1 + \ln\left(\frac{x}{2}\right) = 2x^{-1} - 1 + \ln\left(\frac{x}{2}\right)$$

$$\frac{dy}{dx} = -2x^{-2} + \frac{1}{\left(\frac{x}{2}\right)} \times \frac{1}{2} = -\frac{2}{x^2} + \frac{1}{x}$$

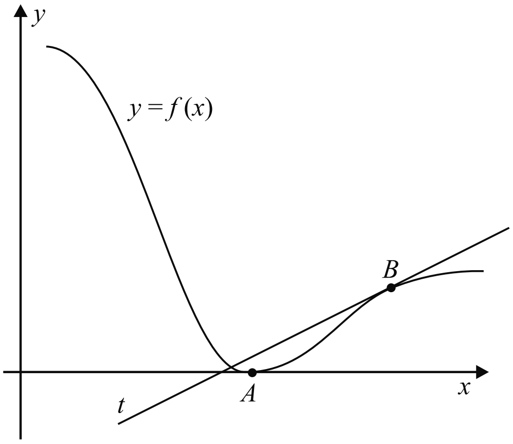
$$\frac{dy}{dx} = 0 \Rightarrow -\frac{2}{x^2} + \frac{1}{x} = 0$$

$$-2 + x = 0$$

$$\therefore x = 2$$

$$y = f(2) = \frac{2}{2} - 1 + \ln\left(\frac{2}{2}\right) = 0$$

Local minimum $A(2, 0)$



Question 5 (b)

$$\frac{dy}{dx} = -\frac{2}{x^2} + \frac{1}{x} = -2x^{-2} + x^{-1}$$

$$\frac{d^2y}{dx^2} = 4x^{-3} - 1x^{-2} = \frac{4}{x^3} - \frac{1}{x^2}$$

$$\frac{d^2y}{dx^2} = 0 \Rightarrow \frac{4}{x^3} - \frac{1}{x^2} = 0$$

$$4 - x = 0$$

$$\therefore x = 4$$

$$f(4) = \frac{2}{4} - 1 + \ln\left(\frac{4}{2}\right) = \tfrac{1}{2} - 1 + \ln 2 = \ln 2 - \tfrac{1}{2}$$

Point of inflection $B(4, \ln 2 - \frac{1}{2})$

Question 5 (c)

$$\frac{dy}{dx} = -\frac{2}{x^2} + \frac{1}{x} = \frac{x-2}{x^2}$$

$$\frac{dy}{dx} > 0 \Rightarrow \frac{x-2}{x^2} > 0$$

$$x - 2 > 0$$

$$\therefore x > 2,\ x \in \mathbb{R}$$

Question 5 (d)

$$\frac{dy}{dx} = -\frac{2}{x^2} + \frac{1}{x}$$

$$\left(\frac{dy}{dx}\right)_{x=4} = -\frac{2}{4^2} + \frac{1}{4} = -\frac{1}{8} + \frac{1}{4} = \frac{1}{8}$$

Equation of $t : m = \frac{1}{8},\ B(4,\ \ln 2 - \frac{1}{2})$

$$x - 8y + k = 0$$

$$4 - 8\ln 2 + 4 + k = 0 \Rightarrow k = 8\ln 2 - 8$$

Equation of $t : x - 8y + 8(\ln 2 - 1) = 0$

QUESTION 6 (25 MARKS)

Question 6 (a)

$$f(x) = 5 + 4x - x^2$$

$$f(x+h) = 5 + 4(x+h) - (x+h)^2$$
$$= 5 + 4x + 4h - x^2 - 2hx - h^2$$

$$f(x+h) - f(x) = 5 + 4x + 4h - x^2 - 2hx - h^2 - 5 - 4x + x^2$$
$$= 4h - 2hx - h^2$$

$$\frac{f(x+h) - f(x)}{h} = \frac{4h - 2hx - h^2}{h} = 4 - 2x - h$$

$$\frac{dy}{dx} = \lim_{h \to 0}\left[\frac{f(x+h) - f(x)}{h}\right] = 4 - 2x$$

Question 6 (b)

$$y = 5 + 4x - x^2$$
$$= -(x^2 - 4x - 5)$$
$$= -(x^2 - 4x + 4 - 9)$$
$$= -((x-2)^2 - 9)$$
$$= 9 - (x-2)^2$$

Local maximum: $y = 9,\ x = 2$

Local maximum: (2, 9)

Question 6 (c)

$$y = 5 + 4x - x^2$$

$$\frac{dy}{dx} = 4 - 2x$$

Question 6 (d)

It is not injective because it is not strictly increasing or decreasing for all $x \in \mathbb{R}$. This means that there are y values that correspond to more than one value of x.

QUESTION 7 (50 MARKS)

Question 7 (a)

$F = 71.6^\circ \text{F}$

$C = \tfrac{5}{9}(F - 32) = \tfrac{5}{9}(71.6 - 32) = 22^\circ \text{C}$

Question 7 (c)

$\theta = T_a + (T_0 - T_a)e^{-kt}$

$\theta - T_a = (T_0 - T_a)e^{-kt}$

$$\frac{1}{e^{-kt}} = \frac{(T_0 - T_a)}{(\theta - T_a)}$$

$$e^{kt} = \frac{(T_0 - T_a)}{(\theta - T_a)}$$

$$kt = \ln\left(\frac{T_0 - T_a}{\theta - T_a}\right)$$

Question 7 (d) (i)

t (hours)	$\theta(^\circ\text{C})$
0	37
t	27
$t + 1$	26

$T_a = 20^\circ\text{C}$, $T_0 = 37^\circ\text{C}$

Question 7 (d) (iii)

$$0.154t = \ln\left(\frac{17}{7}\right)$$

$$\therefore t = \frac{1}{0.154}\ln\left(\frac{17}{7}\right) = 5.76 \text{ h} = 5 \text{ h } 46 \text{ mins}$$

Time of death: 4:44 pm

Question 7 (b)

$k = 1.8\,\text{h}^{-1}, t = 0.5 \text{ h}, T_0 = 96^\circ\text{C}, T_a = 22^\circ\text{C}$

$\theta = T_a + (T_0 - T_a)e^{-kt}$

$\quad = 22 + (96 - 22)e^{-1.8\times0.5} \approx 52^\circ\text{C}$

Question 7 (d) (ii)

$$kt = \ln\left(\frac{37-20}{27-20}\right) = \ln\left(\frac{17}{7}\right)$$

$$k(t+1) = \ln\left(\frac{37-20}{26-20}\right) = \ln\left(\frac{17}{6}\right)$$

$$k(t+1) - kt = \ln\left(\frac{17}{6}\right) - \ln\left(\frac{17}{7}\right)$$

$$kt + k - kt = \ln\left(\frac{17}{6}\times\frac{7}{17}\right)$$

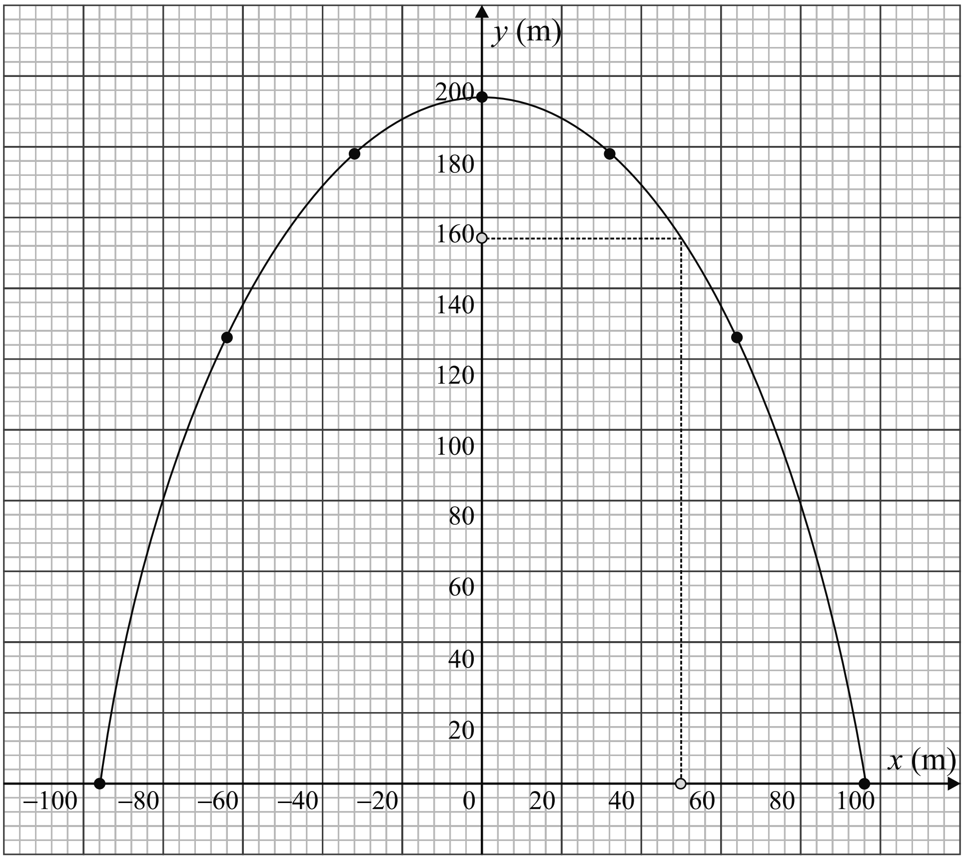
$$k = \ln\left(\frac{7}{6}\right) = 0.154 \text{ h}^{-1}$$

QUESTION 8 (50 MARKS)

Question 8 (a) (i)

x (m)	–96	–64	–32	0	32	64	96
y (m)	0	126	178	192	178	126	0

Question 8 (a) (ii)



Question 8 (b) (i)

Maximum height = 192 m

$x = 50$ m, $y = 154$ m

Question 8 (b) (ii)

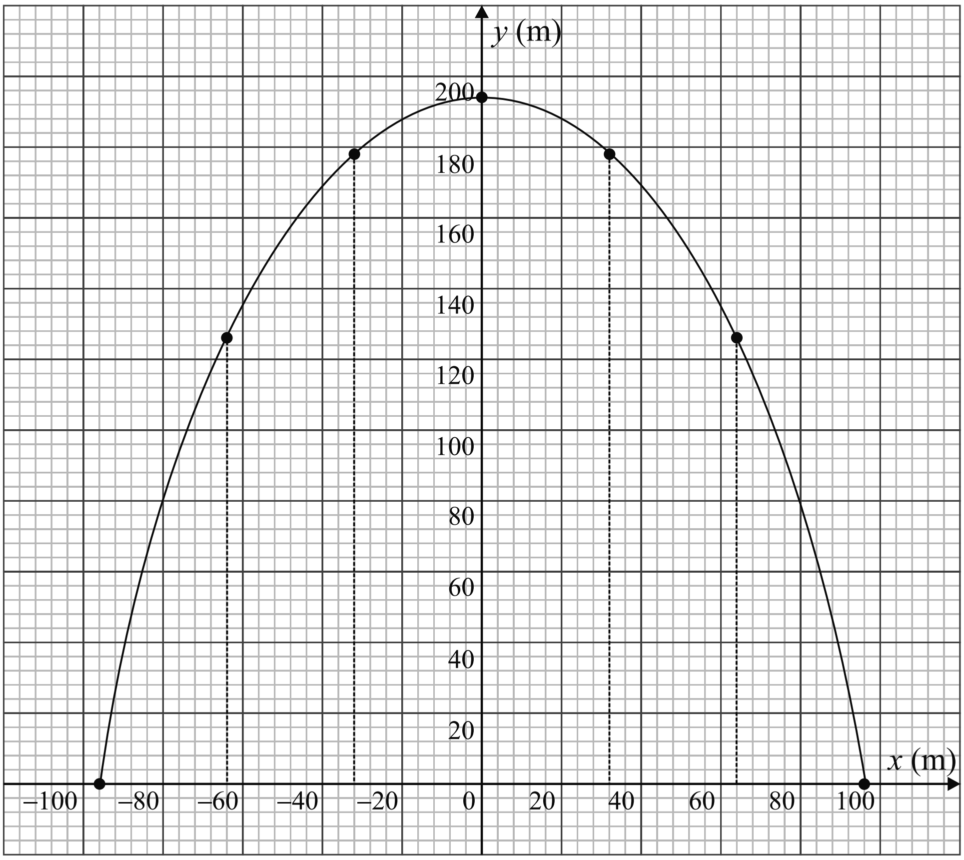
$$1\text{ ft} = 0.3048\text{ m}$$

$$\frac{1}{0.3048}\text{ ft} = 1\text{ m}$$

$$\frac{1}{0.3048} \times 192\,\text{ft} = 192\text{ m} \Rightarrow 192\text{ m} = 629.92126\text{ ft}$$

$$\%\text{ error} = \left(\frac{630 - 629.92126}{630}\right) \times 100\% = 0.0125\%$$

Question 8 (c)



h =32 m, y_1 = 0 m, y_2 = 126 m, y_3 = 178 m, y_4 = 192 m, y_5 = 178 m, y_6 = 126 m, y_7 = 0 m

$$A=\frac{h}{2}[y_1+y_n+2(y_2+y_3+y_4+................+y_{n-1})]$$

$$=\frac{32}{2}[0+0+2(126+178+192+178+126)]$$

$$=25\,600\text{ m}^2$$

Question 8 (d) (i)

$$e^x=1+x+\frac{x^2}{2}$$

$$e^{-x}=1+(-x)+\frac{(-x)^2}{2}=1-x+\frac{x^2}{2}$$

$$\therefore e^x+e^{-x}=2+x^2$$

Question 8 (d) (ii)

$$e^x+e^{-x}=2+x^2$$

$$\therefore e^{\frac{1}{39}x}+e^{-\frac{1}{39}x}=2+(\tfrac{1}{39}x)^2=2+\frac{x^2}{1521}$$

$$y=231-19.57\{e^{\frac{1}{39}x}+e^{-\frac{1}{39}x}\}$$

$$=231-19.57\left\{2+\frac{x^2}{1521}\right\}$$

$$=231-39.14-0.013x^2$$

$$=191.86-0.013x^2$$

Question 8 (e)

$$y = 191.86 - 0.013x^2$$

$$y = 0 \Rightarrow 191.86 - 0.013x^2 = 0$$

$$191.86 = 0.013x^2$$

$$x = \sqrt{\frac{191.86}{0.013}} \approx 243 \text{ m}$$

Question 8 (f)

$$y = 191.86 - ax^2$$

$$y = 0 : x = \sqrt{\frac{191.86}{a}} = 96$$

$$\frac{191.86}{a} = 9216$$

$$\therefore a = \frac{191.86}{9216} = 0.0208$$

QUESTION 9 (50 MARKS)

Question 9 (a)

$V = 340\sin(100\pi t)$

Period $P = \frac{2\pi}{100\pi} = \frac{1}{50} \text{ s} = 0.02 \text{ s}$

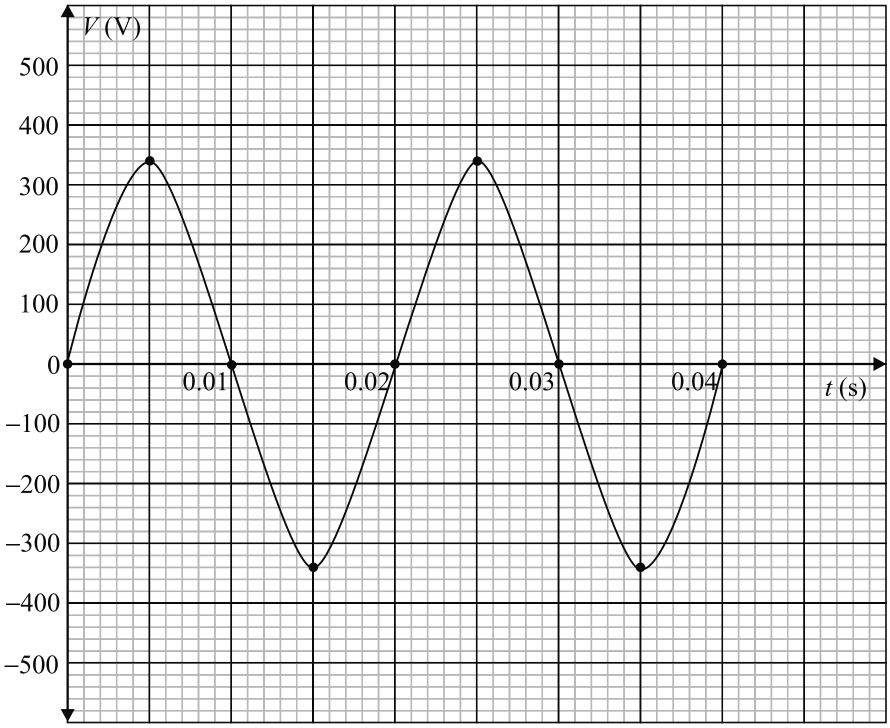
Range $R = [-340, 340]$

Question 9 (b)

Maximum voltage is 340 V.

Question 9 (c)

t	0	0.005	0.01	0.015	0.02	0.025	0.03	0.035	0.04
$100\pi t$	0	$\frac{1}{2}\pi$	π	$\frac{3}{2}\pi$	2π	$\frac{5}{2}\pi$	3π	$\frac{7}{2}\pi$	4π
$\sin(100\pi t)$	0	1	0	−1	0	1	0	−1	0
$340\sin(100\pi t)$	0	340	0	−340	0	340	0	−340	0



Question 9 (d)

$$\bar{V} = \frac{340}{0.02}\int_0^{0.02} \sin 100\pi t \, dt$$

$$= \frac{340}{0.02}\left[\frac{-\cos(100\pi t)}{100\pi}\right]_0^{0.02}$$

$$= -\frac{340}{2\pi}(\cos(100\pi(0.02) - \cos(100\pi(0))$$

$$= -\frac{340}{2\pi}(\cos(2\pi) - \cos(0)) = 0$$

Question 9 (e)

$$V = 340\sin(100\pi t)$$

$$\frac{dV}{dt} = 340 \times 100\pi\cos(100\pi t) = 34\ 000\pi\cos(100\pi t)$$

$$\left(\frac{dV}{dt}\right)_{t=0} = 34\ 000\pi\cos(100\pi(0)) = 34\ 000\pi \ \text{V s}^{-1}$$

$$\left(\frac{dV}{dt}\right)_{t=0.005} = 34\ 000\pi\cos(100\pi(0.005)) = 34\ 000\pi\cos(\tfrac{1}{2}\pi) = 0 \ \text{V s}^{-1}$$

$$\left(\frac{dV}{dt}\right)_{t=0.01} = 34\ 000\pi\cos(100\pi(0.01)) = 34\ 000\pi\cos(\pi) = -34\ 000\pi \ \text{V s}^{-1}$$

Question 9 (f)

$$V = 340\sin(100\pi t)$$

$$V^2 = 340^2\sin^2(100\pi t)$$

$$= 115\ 600 \times \tfrac{1}{2}(1 - \cos 200\pi t)$$

$$= 57\ 800(1 - \cos 200\pi t)$$

$$\text{Period } V^2 = \frac{2\pi}{200\pi} = \frac{1}{100} = 0.01 \text{ s}$$

$$\bar{V}^2 = \frac{1}{0.01}\int_0^{0.01} 57\ 800(1 - \cos 200\pi t)\, dt$$

$$= 5\ 780\ 000\left[t - \frac{\sin(200\pi t)}{200\pi}\right]_0^{0.01}$$

$$= 5\ 780\ 000\left\{0.01 - \frac{\sin(200\pi(0.01))}{200\pi}\right\}$$

$$= 5\ 780\ 000\left\{0.01 - \frac{\sin 2\pi}{200\pi}\right\}$$

$$= 57\ 800 \ \text{V}^2$$

SAMPLE PAPER 6: PAPER 2

QUESTION 1 (25 MARKS)

Question 1 (a)

(x, 0) is a point on the x-axis.

Distance d_1 of (x, 0) to line k: $d_1 = \dfrac{|3x-4(0)+5|}{\sqrt{3^2+4^2}} = \dfrac{|3x+5|}{5}$

$$\boxed{d = \frac{|ax_1+by_1+c|}{\sqrt{a^2+b^2}}}$$

Distance d_2 of (x, 0) to line l: $d_2 = \dfrac{|5x-12(0)-1|}{\sqrt{5^2+12^2}} = \dfrac{|5x-1|}{13}$

$$d_1 = d_2 \Rightarrow \frac{|3x+5|}{5} = \frac{|5x-1|}{13}$$

$$13(3x+5) = \pm 5(5x-1)$$

$$39x+65 = 25x-5 \Rightarrow 14x = -70$$

$$\therefore x = -5$$

or

$$39x+65 = -25x+5 \Rightarrow 64x = -60$$

$$\therefore x = -\tfrac{15}{16}$$

ANSWERS: $(-5, 0)$, $(-\tfrac{15}{16}, 0)$

Question 1 (b)

$$\tan\theta = \left|\frac{\frac{t}{10}-(-\frac{3}{4})}{1+\frac{t}{10}(-\frac{3}{4})}\right| = \left|\frac{4t+30}{40-3t}\right|$$

$$\boxed{\tan\theta = \left|\frac{m_1-m_2}{1+m_1m_2}\right|}$$

$$\tan\theta = \left|\frac{-2-(-\frac{3}{4})}{1+(-2)(-\frac{3}{4})}\right| = \left|\frac{-8+3}{4+6}\right| = \left|\frac{-5}{10}\right| = \left|\frac{1}{2}\right|$$

$$\therefore \frac{4t+30}{40-3t} = \pm\frac{1}{2}$$

$$2(4t+30) = 1(40-3t)$$

$$8t+60 = 40-3t$$

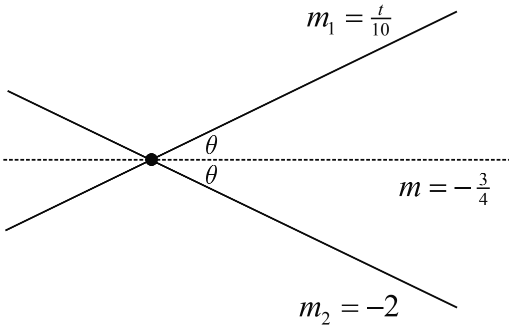
$$11t = -20 \Rightarrow t = -\tfrac{20}{11}$$

or

$$2(4t+30) = -1(40-3t)$$

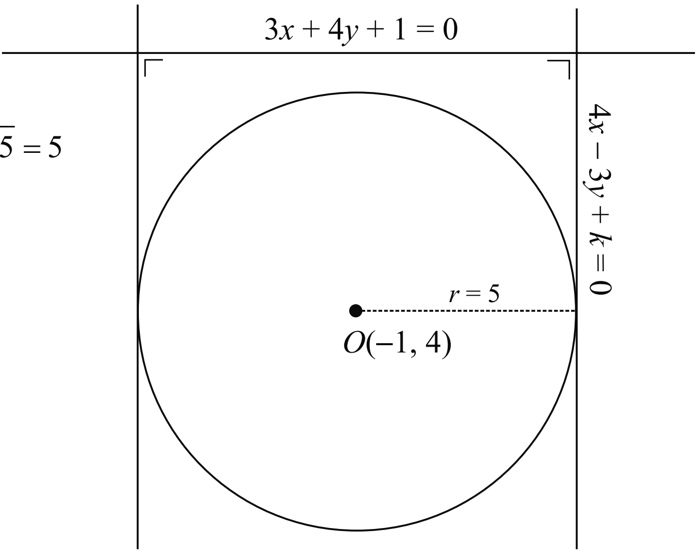
$$8t+60 = -40+3t$$

$$5t = -100 \Rightarrow t = -20$$



QUESTION 2 (25 MARKS)

Question 2 (a)



$s: x^2 + y^2 + 2x - 8y - 8 = 0$

Centre $O(-1, 4)$, $r = \sqrt{(-1)^2 + 4^2 + 8} = \sqrt{25} = 5$

Equation of tangents: $4x - 3y + k = 0$

$$\frac{|4(-1) - 3(4) + k|}{\sqrt{4^2 + (-3)^2}} = 5 \qquad \boxed{d = \frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}}$$

$$\frac{|-4 - 12 + k|}{\sqrt{25}} = 5$$

$$|-16 + k| = 25$$

$$-16 + k = \pm 25$$

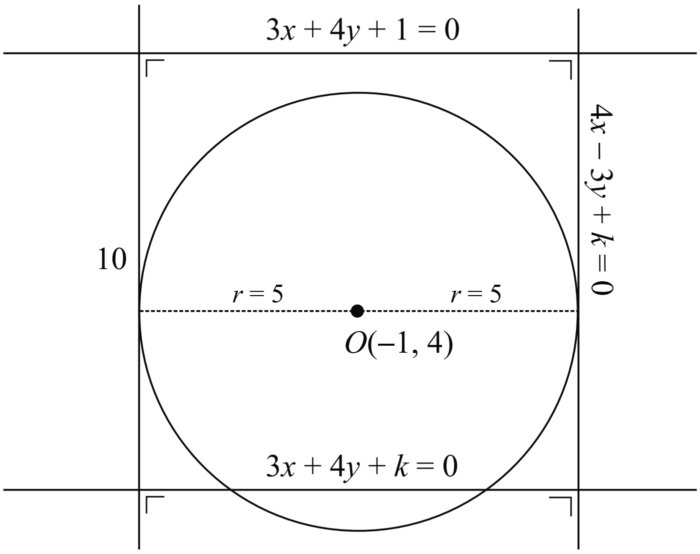
$$\therefore k = -9, 41$$

t_1: $4x - 3y + 41 = 0$, t_2: $4x - 3y - 9 = 0$

Question 2 (b)

The length of a side of the square is 10 units. A line parallel to $3x + 4y + 1 = 0$ has the form $3x + 4y + k = 0$.

Pick a point on the line $3x + 4y + 1 = 0$.
$(1, -1)$ is on this line.



$$\boxed{d = \frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}}$$

$d = 10$, $(x_1, y_1) = (1, -1)$

$$10 = \frac{|3(1) + 4(-1) + k|}{\sqrt{3^2 + 4^2}}$$

$$10 = \frac{|3 - 4 + k|}{5}$$

$$50 = |k - 1|$$

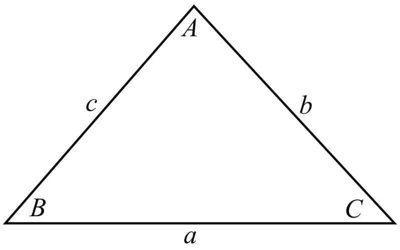
$$\pm 50 = k - 1$$

$$\therefore k = -49, 51$$

Equations: $3x + 4y - 49 = 0$, $3x + 4y + 51 = 0$

QUESTION 3 (25 MARKS)

Question 3 (a)



$a^2 = b^2 + c^2 - 2bc\cos A,$

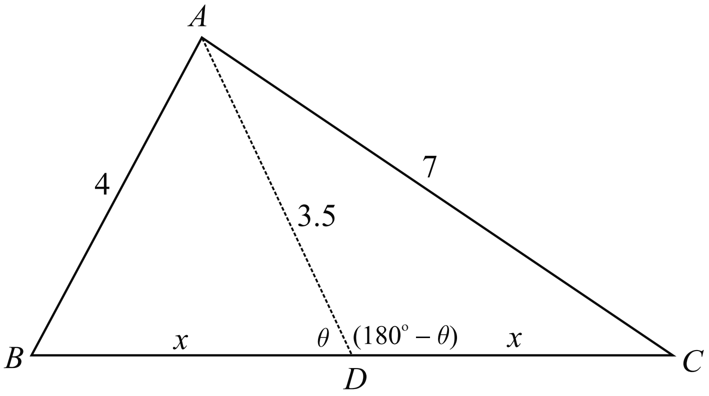
$b^2 = a^2 + c^2 - 2ac\cos B,$

$c^2 = a^2 + b^2 - 2ab\cos C$

Question 3 (b)

$|BD| = |DC|$

Question 3 (c)



Apply the Cosine rule to triangle ABD: $4^2 = 3.5^2 + x^2 - 2(3.5)(x)\cos\theta$

Apply the Cosine rule to triangle ADC: $7^2 = 3.5^2 + x^2 - 2(3.5)(x)\cos(180^\circ - \theta)$

$16 = 3.5^2 + x^2 - 7x\cos\theta$

$49 = 3.5^2 + x^2 + 7x\cos\theta$

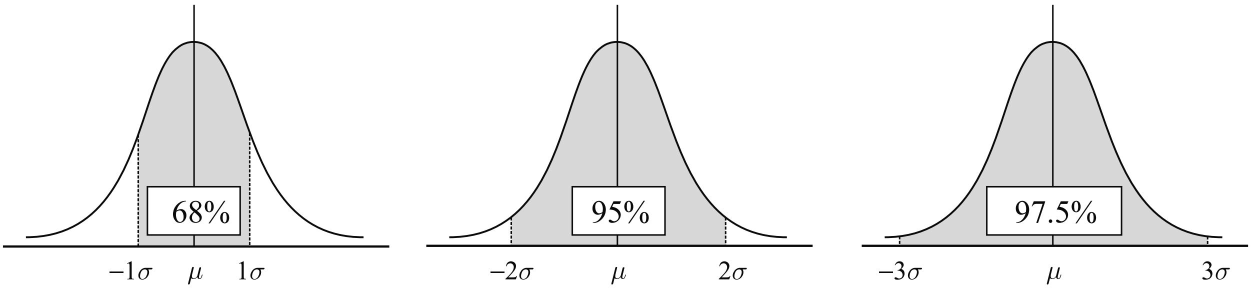
$65 = 2(3.5^2) + 2x^2$

$x^2 = \frac{81}{4} \Rightarrow x = \frac{9}{2}$

$\therefore |BC| = 9$

QUESTION 4 (25 MARKS)

Question 4 (a)



In any normal distribution with mean $\overline{x}$ and standard deviation σ.

1. 68% of the data falls within 1σ of the mean $\overline{x}$.
2. 95% of the data falls within 2σ of the mean $\overline{x}$.
3. 99.7% of the data falls within 3σ of the mean $\overline{x}$.

Question 4 (b)

$$\mu - 2\sigma = 24.8$$
$$\underline{\mu + 2\sigma = 36.4}$$
$$2\mu \quad = 61.2 \Rightarrow \mu = 30.6 \text{ years}$$

$$30.6 - 2\sigma = 24.8$$
$$5.8 = 2\sigma$$
$$\therefore \sigma = 2.9 \text{ years}$$

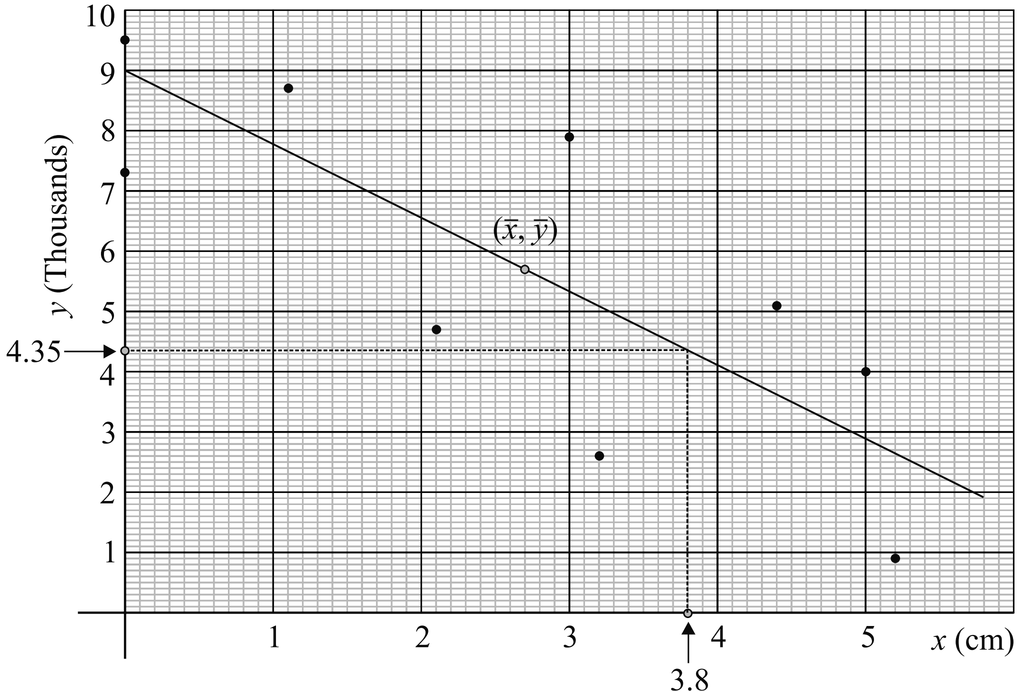
Question 4 (c)

$$\mu = 30.6 \text{ years}$$
$$\sigma = 2.9 \text{ years}$$
$$P(x \geq 30) = ?$$
$$x = 30 : z = \frac{x-\mu}{\sigma} = \frac{30-30.6}{2.9} = -0.21$$
$$P(x \geq 30) = P(z \geq -0.21)$$
$$= 1 - P(z \leq -0.21)$$
$$= 1 - P(z \geq 0.21)$$
$$= 1 - \{1 - P(z \leq 0.21)\}$$
$$= P(z \leq 0.21)\}$$
$$= 0.5832$$
$$= 58.32\%$$

QUESTION 5 (25 MARKS)

Question 5 (a)

Rainfall (x cm)	4.4	3.0	5.2	5.0	2.1	0	0	1.1	3.2
Number of tourists (y thousands)	5.1	7.9	0.9	4.0	4.7	7.3	9.5	8.7	2.6



Question 5 (b)

$$\bar{x} = \frac{4.4+3.0+5.2+5.0+2.1+0+0+1.1+3.2}{9} = 2.7$$

$$\bar{y} = \frac{5100+7900+900+4000+4700+7300+9500+8700+2600}{9} = 5633$$

Question 5 (c) [See graph]

Question 5 (d)

(2.7, 5633), (0, 9000)

$$m = \frac{9000-5633}{0-2.7} = -1247$$

Question 5 (e)

For every increase of 1 cm in rainfall, there is a decrease in 1247 tourists.

Question 5 (f)

$m = -1247$, $(x_1, y_1) = (0, 9000)$

$y - 9000 = -1247(x - 0)$

$1247x + y - 9000 = 0$

Question 5 (g)

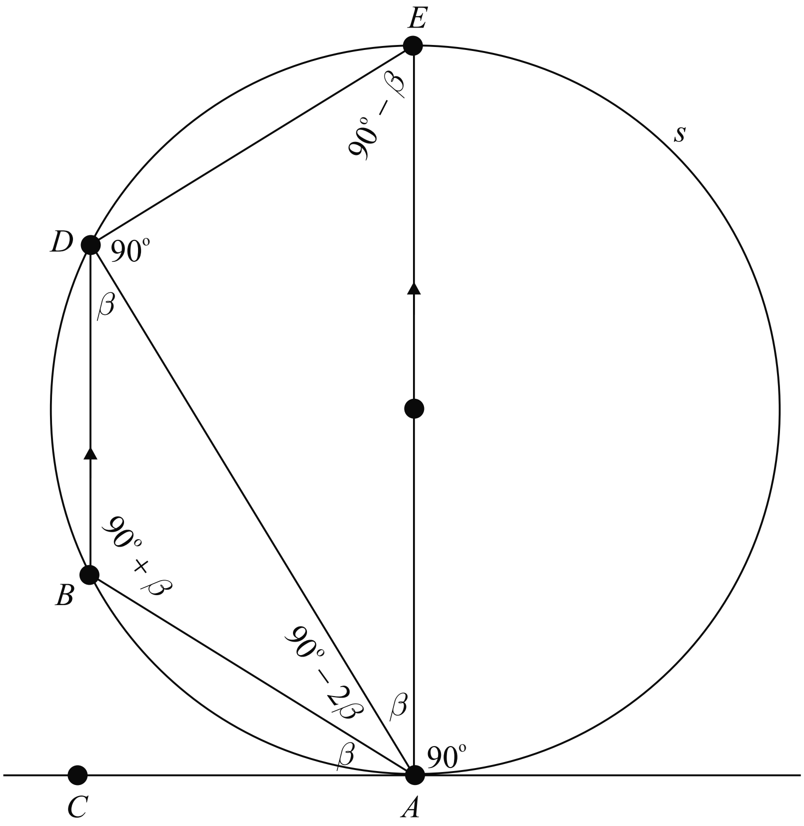
(i) Number of tourists = 4350

(ii) $x = 3.8 : 1247(3.8) + y - 9000 = 0$

$\therefore y = 4261$

The student's result is smaller by 89 or is about 2% smaller $(\frac{89}{4350} \times 100\% \approx 2\%)$.

Question 6 (25 marks)



Consider triangle ADE. Let $|\angle DAE| = \beta$.

$[AE]$ is a diameter. Therefore, $|\angle ADE| = 90^\circ$. (The angle at the circle standing on a diameter is a right angle).

$\therefore |\angle DEA| = 180^\circ - 90^\circ - \beta = 90^\circ - \beta$

$|\angle AED| + |\angle ABD| = 180^\circ$ (Opposite angles of a cyclic quadrilateral add up to 180°.)

$\therefore |\angle ABD| = 180^\circ - (90^\circ - \beta) = 90^\circ + \beta$

$|\angle DAE| = |\angle ADB| = \beta$ (Alternate angles)

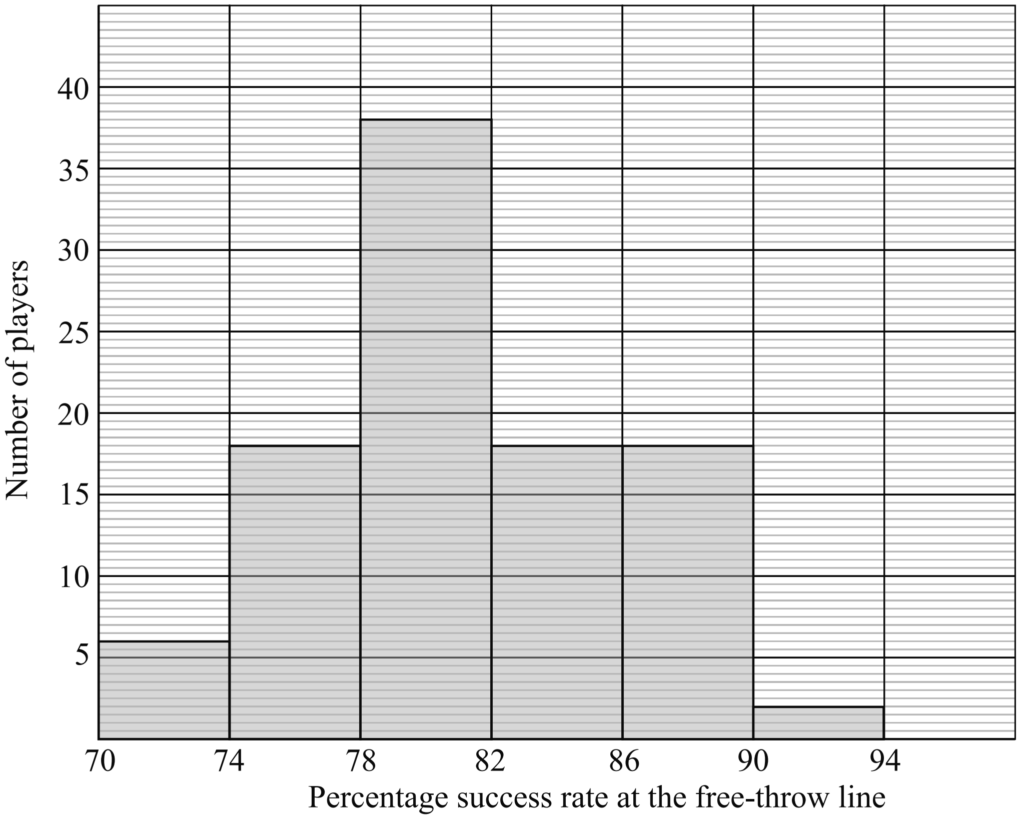
$\therefore |\angle DAB| = 180^\circ - \beta - (90^\circ + \beta) = 90^\circ - 2\beta$

$\therefore |\angle BAC| = 90^\circ - \beta - (90^\circ - 2\beta) = \beta$

$\therefore |\angle BAC| = |\angle DAE| = |\angle ADB| = \beta$

QUESTION 7 (50 MARKS)

Question 7 (a)



Percentage success	Frequency f	Mid-interval Value x	fx
70–74	6	72	432
74–78	18	76	1368
78–82	38	80	3040
82–86	18	84	1512
86–90	18	88	1584
90–94	2	92	184
	100		8120

$$\mu = \frac{\sum fx}{\sum f} = \frac{8120}{100} = 81.2$$

Mean percentage success rate = 81.2%

Question 7 (b)

Percentage success	Cumulative frequency	
<74	6	
<78	24	
<82	62	Median
<86	80	
<90	98	
<94	100	

$6\times1+18\times1+38x=38(1-x)+18\times1+18\times1+2\times1$

$24+38x=38-38x+38$

$76x=52$

$x=\frac{52}{76}=\frac{13}{19}$

$\therefore$ Median $=77.5+\frac{13}{19}\times4=80.2$

Question 7 (c)

(i) Mean $\mu=81.2$

Median $=80.2$

$\%$ difference $=\frac{81.2-80.2}{81.2}\times100\%=1.23\%$

(ii) It is approximately normal because the mean is approximately equal to the median.

(iii) $\sigma=4.75$

Question 7 (d)

(i) $\mu=81.2,\ \sigma=4.75$

$P(z\le Z)=0.9\Rightarrow z=1.28$

$z=\frac{x-\mu}{\sigma}\Rightarrow1.28=\frac{x-81.2}{4.75}$

$\therefore x=87.3\%$

(ii) $\mu=81.2,\ \sigma=4.75$

$P(x<85)=?$

$x=85: z=\frac{x-\mu}{\sigma}=\frac{85-81.2}{4.75}=0.8$

$P(z<0.8)=0.7881=78.8\%$

Question 7 (e)

Conditions for a Bernoulli Trial:
Condition: There are only two possible outcomes (success or failure) in each trial.
Condition: There is a fixed number of trials n.
Condition: The probability of success p is fixed from trial to trial.
Condition: The trials are independent.
Condition: The binomial random variable is the number of successes in n trials.

Question 7 (f)

(i) $P(\text{Success})=0.927$

$P(\text{Failure})=0.073$

$P(\text{Scores all five})={}^5C_5(0.927)^5(0.073)^0=0.685=68.5\%$

(ii) $P(\text{Scores three out of five})={}^5C_3(0.927)^3(0.073)^2=0.042=4.2\%$

(iii) He has two successes and two failures on the first four throws and he scores on the last.

$P(\text{Scores two out of first four and scores last})={}^4C_2(0.927)^2(0.073)^2(0.927)=0.025=2.5\%$

QUESTION 8 (30 MARKS)

Question 8 (a)

True population proportion = Sample proportion ± 1.96(Standard error of the proportion)

$$\text{Standard error of the proportion} = \sqrt{\frac{p(1-p)}{n}}$$

Confidence interval
= [sample parameter – 1.96(standard error), sample parameter + 1.96(standard error)]

For about 95% of all samples the confidence interval covers the population values of the parameter and for the other 5% it does not.
For a 99% confidence level, the confidence interval must be wider so that more samples cover the population parameter.

Confidence interval
= [sample parameter – 2.576(standard error), sample parameter + 2.576(standard error)]

Question 8 (b)

$$\text{Sample proportion } p = \frac{200}{300} = \frac{2}{3}$$

$n = 300$

$$\text{Population proportion} = \frac{2}{3} \pm 1.96\sqrt{\frac{\frac{2}{3}(1-\frac{2}{3})}{300}} = 0.613,\ 0.72$$

Confidence interval = [0.613, 0.72]
For about 95% of all samples, the population proportion of all fifth years who believe the Junior Certificate should be scrapped is between 61.3% and 72%.

Question 8 (c)

Standard deviation of population σ = 10 cm
Mean of the sample $\bar{x} = 173$ cm
Number of the sample n = 50
Population mean μ = ?

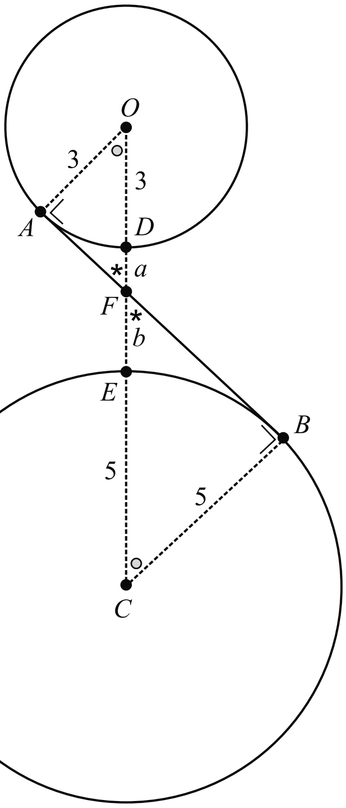
$$\mu = \bar{x} \pm 1.96 \times \frac{\sigma}{\sqrt{n}}$$

$$\mu = 173 \pm 1.96 \times \frac{10}{\sqrt{50}} = 170.23,\ 175.77$$

For about 95% of all samples, the population mean height of university students is between 170.23 cm and 175.77 cm.

QUESTION 9 (70 MARKS)

Question 9 (a)



(i) $|OC| = 10$ cm

$|OC| = |OD| + |DF| + |FE| + |EC|$

$\therefore 10 = 3 + a + b + 5$

$a + b = 2$

(ii) Triangles *OAF* and *CBF* are similar because:

$|\angle OAF| = |\angle FBC|$ (90° angles)

$|\angle OFA| = |\angle BFC|$ (Vertically opposite)

$\therefore |\angle AOF| = |\angle BCF|$

(iii) $\dfrac{|BC|}{|OA|} = \dfrac{|FC|}{|OF|} \Rightarrow \dfrac{5}{3} = \dfrac{b+5}{a+3}$

$5a + 15 = 3b + 15$

$5a = 3b$

$\therefore b = \dfrac{5a}{3}$

$a + b = 2$

$a + \dfrac{5a}{3} = 2 \Rightarrow 3a + 5a = 6$

$8a = 6$

$\therefore a = 0.75$ cm

$\therefore b = \dfrac{5(0.75)}{3} = 1.25$ cm

(iv) $|AB| = |AF| + |FB|$

$\Delta AOF : |OF|^2 = |OA|^2 + |AF|^2$

$\therefore 3.75^2 = 3^2 + |AF|^2$

$|AF| = \sqrt{3.75^2 - 3^2} = 2.25$ cm

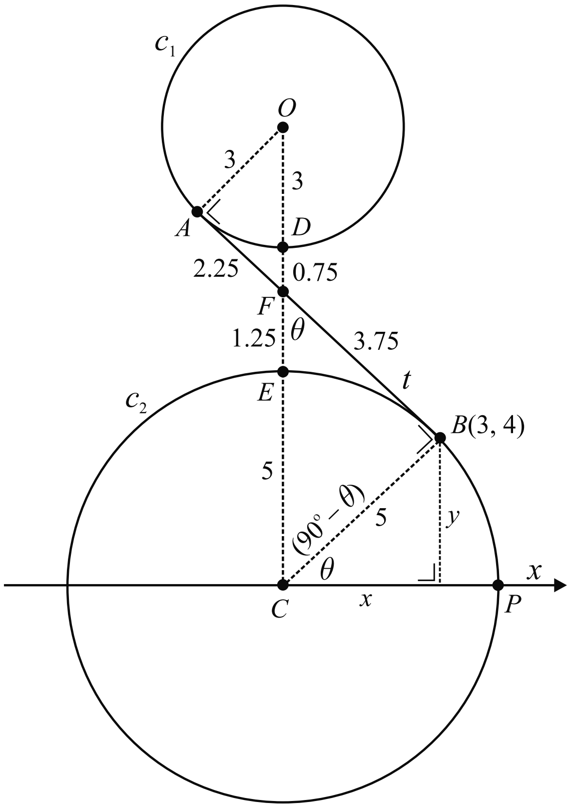
$\Delta FBC : |FC|^2 = |BC|^2 + |FB|^2$

$\therefore 6.25^2 = 5^2 + |FB|^2$

$|FB| = \sqrt{6.25^2 - 5^2} = 3.75$ cm

$\therefore |AB| = 2.25 + 3.75 = 6$ cm

Question 9 (b)

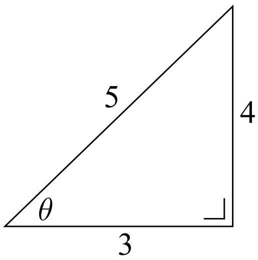


(i) $E(0, 5)$
$F(0, 6.25)$
$D(0, 7)$
$O(0, 10)$

(ii) $c_1 : x^2 + (y-10)^2 = 9$

$c_2 : x^2 + y^2 = 25$

(iii) $\tan\theta = \dfrac{5}{3.75} = \dfrac{4}{3}$



(iv) $|\angle FCB| = 180^\circ - 90^\circ - \theta = (90^\circ - \theta)$

$\therefore |\angle BCP| = 90^\circ - (90^\circ - \theta) = \theta$

$\sin\theta = \dfrac{y}{5} = \dfrac{4}{5} \Rightarrow y = 4$

$\cos\theta = \dfrac{x}{5} = \dfrac{3}{5} \Rightarrow x = 3$

$\therefore B(3, 4)$

(v) Slope of $BC = \frac{4}{3}$

Slope of tangent $t = -\frac{3}{4}$

Equation of t: $m = -\frac{3}{4}$, $(x_1, y_1) = (3, 4)$

$y - 4 = -\frac{3}{4}(x-3)$

$4y - 16 = -3x + 9$

$3x + 4y - 25 = 0$

(vi) $c_1 : x^2 + (y-10)^2 = 9$

$t : 3x + 4y - 25 = 0 \Rightarrow x = \dfrac{25-4y}{3}$

$\therefore \left(\dfrac{25-4y}{3}\right)^2 + (y-10)^2 = 9$

$\dfrac{625 - 200y + 16y^2}{9} + y^2 - 20y + 100 - 9 = 0$

$625 - 200y + 16y^2 + 9y^2 - 180y + 900 - 81 = 0$

$25y^2 - 380y + 1444 = 0$

$(5y - 38)(5y - 38) = 0$

$\therefore y = \frac{38}{5}$

$x = \dfrac{25 - 4(\frac{38}{5})}{3} = -\dfrac{9}{5}$

$\therefore A(-\frac{9}{5}, \frac{38}{5})$

(vii) $A(-\frac{9}{5}, \frac{38}{5})$, $B(3, 4)$

$|AB| = \sqrt{(3 + \frac{9}{5})^2 + (4 - \frac{38}{5})^2} = 6$

SAMPLE PAPER 7: PAPER 1

QUESTION 1 (25 MARKS)

Question 1 (a) (i)

First 10 natural squares: 1, 4, 9, 16, 25, 36, 49, 64, 81, 100

Question 1 (a) (ii)

Consecutive numbers: $n - 1, n$
Consecutive squares: $n^2, (n+1)^2$

$$n^2 - (n-1)^2 = n^2 - (n^2 - 2n + 1) = n^2 - n^2 + 2n - 1 = 2n - 1$$

$2n$ is always an even number
$2n - 1$ is always an odd number

Question 1 (b) (i)

$$1^2 + 2^2 = \frac{2 \times 3 \times 5}{6}$$

$$1 + 4 = \frac{30}{6}$$

$$5 = 5$$

Question 1 (b) (ii)

$$1^2 + 2^2 + 3^2 = \frac{3 \times 4 \times 7}{6}$$

$$1 + 4 + 9 = 2 \times 7$$

$$14 = 14$$

Question 1 (b) (iii)

$$1^2 + 2^2 + 3^2 + 4^2 = \frac{4 \times 5 \times 9}{6}$$

$$1 + 4 + 9 + 16 = \frac{180}{6}$$

$$30 = 30$$

Question 1 (b) (iv)

$$1^2 + 2^2 + 3^2 + 4^2 + \ldots\ldots + 10^2 = \frac{10 \times 11 \times 21}{6} = 5 \times 11 \times 7 = 385$$

Question 1 (c) (i)

$$1^2 + 2^2 + 3^2 + \ldots\ldots + n^2 = \frac{n(n+1)(2n+1)}{6}$$

Question 1 (c) (ii)

$$S_{100} = \frac{100(101)(201)}{6} = 338\ 350$$

QUESTION 2 (25 MARKS)

Question 2 (a)

$S_n = pn^2 + qn$

$S_{n-1} = p(n-1)^2 + q(n-1) = pn^2 - 2pn + p + qn - q$

$S_n - S_{n-1} = T_n \Rightarrow T_n = pn^2 + qn - pn^2 + 2pn - p - qn + q$

$= 2pn - p + q$

$T_{n+1} = 2p(n+1) - p + q$

$T_{n+1} - T_n = 2p(n+1) - p + q - 2pn + p - q$

$= 2pn + 2p - p + q - 2pn + p - q$

$= 2p =$ Common Difference

$a = T_1 \Rightarrow a = 2p - p + q = p + q$

$d = T_{n+1} - T_n = 2p$

Question 2 (b) (i)

$a,\ ar,\ ar^2$

$\log a,\ \log ar,\ \log ar^2$

$= \log a,\ \log a + \log r,\ \log a + 2\log r$

$d = \log a + \log r - \log a = \log r$

$a = \log a$

$S_n = \frac{n}{2}[2\log a + (n-1)\log r]$

Question 2 (b) (ii)

$S_{50} = 2450 \Rightarrow 25[2\log_3 1 + 49\log_3 r] = 2450$

$0 + 49\log_3 r = 98$

$\log_3 r = 2$

$\therefore r = 3^2 = 9$

QUESTION 3 (25 MARKS)

Question 3 (a)

$$z^3 = 1$$

$$z^3 - 1 = 0$$

$$(z-1)(z^2 + z + 1) = 0$$

$$z = \frac{-1 \pm \sqrt{1^2 - 4(1)(1)}}{2(1)} = \frac{-1 \pm \sqrt{-3}}{2} = \frac{-1 \pm \sqrt{3}i}{2}$$

$$z = 1,\ -\frac{1}{2} + \frac{\sqrt{3}}{2}i,\ -\frac{1}{2} - \frac{\sqrt{3}}{2}i$$

Question 3 (b)

$$w = -\frac{1}{2} + \frac{\sqrt{3}}{2}i$$

$$w^2 = \frac{1}{4} - \frac{\sqrt{3}i}{2} + \frac{3i^2}{4} = -\frac{1}{2} - \frac{\sqrt{3}}{2}i$$

$$\frac{1}{w} = \frac{2}{-1 + \sqrt{3}i}$$

$$= \frac{2}{(-1 + \sqrt{3}i)} \times \frac{(-1 - \sqrt{3}i)}{(-1 - \sqrt{3}i)}$$

$$= \frac{-2 - 2\sqrt{3}i}{1 - 3i^2}$$

$$= \frac{-2 - 2\sqrt{3}i}{4}$$

$$= \frac{-1 - \sqrt{3}i}{2} = w^2$$

Roots: $w,\ \frac{1}{w} = -\frac{1}{2} + \frac{\sqrt{3}}{2}i,\ -\frac{1}{2} - \frac{\sqrt{3}}{2}i$

Sum S: $\frac{-1 + \sqrt{3}i}{2} + \frac{-1 - \sqrt{3}i}{2} = -1$

Product P: $w \times \frac{1}{w} = 1$

$$z^2 - Sz + P = 0$$

$$z^2 - (-1)z + 1 = 0$$

$$\therefore z^2 + z + 1 = 0$$

QUESTION 4 (25 MARKS)

Using the lining up method, a cubic equals a quadratic by a linear.

$$x^3+3px^2+3qx+r=(x^2-px+q)(x+t)$$
$$x^3+3px^2+3qx+r=x^3+tx^2-px^2-ptx+qx+qt$$
$$x^3+3px^2+3qx+r=x^3+(t-p)x^2+(q-pt)x+qt$$

Question 4 (a)

Lining up the coefficients gives you three equations. Replace *t* from equation (**1**) in the other equations.

$3p=t-p$.....(**1**)	$3q=q-pt$.....(**2**)	$r=qt$.....(**3**)
$4p=t$	$3q=q-p(4p)$	$r=q(4p)$
	$2q=-4p^2$	$r=4pq$
	$q=-2p^2$..(i)	

Question 4 (b)

Result (i) is proved under equation (**2**).
To prove result (ii) replace *q* under equation (**3**).

$$r=4pq=4p(-2p^2)=-8p^3\text{...(ii)}$$

Question 4 (c)

$$x^3+3px^2+3qx+r=(x^2-px+q)(x+4p)$$
$$\Rightarrow x^3+3px^2+3qx+r=(x^2-px-2p^2)(x+4p)=(x-2p)(x+p)(x+4p)=0$$
$$\therefore x=-4p,\ -p,\ 2p$$

QUESTION 5 (25 MARKS)

Question 5 (a)

$f'(x) = y = ax^2 + bx + c$

$(0, 6) \in f'(x) \Rightarrow a(0)^2 + b(0) + c = 6$

$\therefore c = 6$

$(1, 0) \in f'(x) \Rightarrow a(1)^2 + b(1) + 6 = 0$

$\therefore a + b = -6....(\mathbf{1})$

$(3, 0) \in f'(x) \Rightarrow a(3)^2 + b(3) + 6 = 0$

$\therefore 9a + 3b = -6$

$3a + b = -2....(\mathbf{2})$

$(\mathbf{2}) - (\mathbf{1}) : 2a = 4 \Rightarrow a = 2$

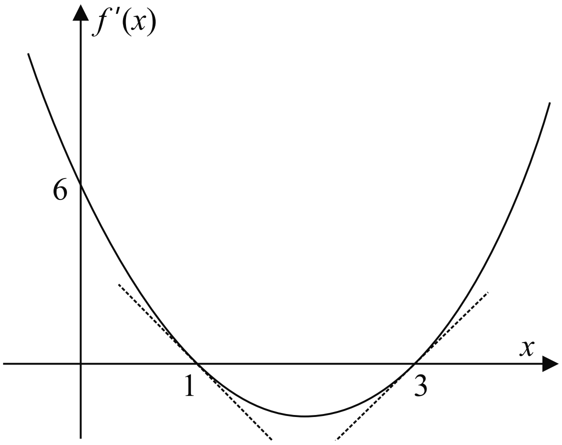
$(2) + b = -6....(\mathbf{1}) \Rightarrow b = -8$

$\therefore f'(x) = y = 2x^2 - 8x + 6$

Question 5 (b)

Local minimum at $x = 3$ as $\frac{dy}{dx} = 0$ and the slope is positive.

Local maximum at $x = 1$ as $\frac{dy}{dx} = 0$ and the slope is negative.



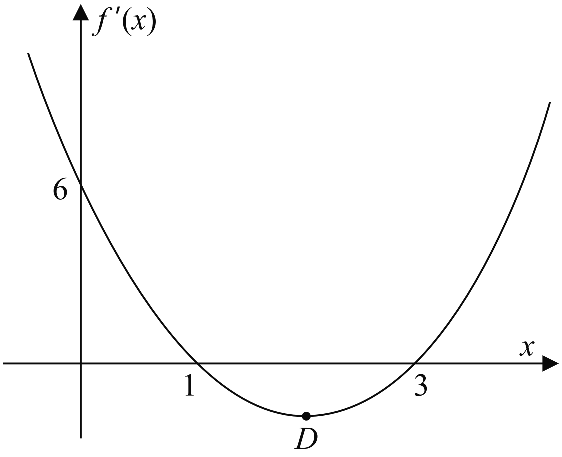
Question 5 (c)

$f'(x) = y = 2x^2 - 8x + 6$

$f''(x) = 4x - 8$

$f''(x) = 0 \Rightarrow 4x - 8 = 0$

$\therefore x = 2$



Question 5 (d)

$\frac{dy}{dx} = 2x^2 - 8x + 6$

$y = \frac{2}{3}x^3 - 4x^2 + 6x + c$

$x = 0,\ y = 0 \Rightarrow c = 0$

$\therefore y = f(x) = \frac{2}{3}x^3 - 4x^2 + 6x$

Question 5 (e)

$f(x) = \frac{2}{3}x^3 - 4x^2 + 6x$

$f(3) = \frac{2}{3}(3)^3 - 4(3)^2 + 6(3) = 18 - 36 + 18 = 0$

Local minimum: (3, 0)

$f(1) = \frac{2}{3}(1)^3 - 4(1)^2 + 6(1) = \frac{2}{3} - 4 + 6 = \frac{8}{3}$

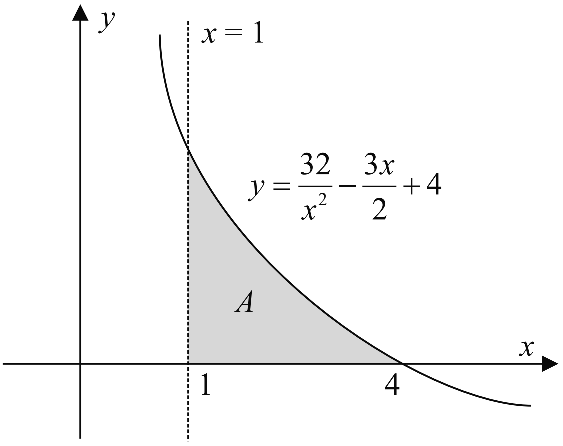
Local maximum: $(1, \frac{8}{3})$

$f(2) = \frac{2}{3}(2)^3 - 4(2)^2 + 6(2) = \frac{16}{3} - 16 + 12 = \frac{4}{3}$

Point of inflection: $(2, \frac{4}{3})$

QUESTION 6 (25 MARKS)

Question 6 (a)



$$y = \frac{32}{x^2} - \frac{3x}{2} + 4$$

$$x = 4 : \frac{32}{4^2} - \frac{3(4)}{2} + 4 = 2 - 6 + 4 = 0$$

Question 6 (b)

x	1	1.5	2	2.5	3	3.5	4
y	34.5	15.97	9.00	5.37	3.06	1.36	0

$$A = \frac{0.5}{2}\{34.5 + 0 + 2(15.97 + 9 + 5.37 + 3.06 + 1.36)\} = 26.005$$

Question 6 (c)

$$A = \int_1^4 (32x^{-2} - \tfrac{3}{2}x + 4)\,dx$$

$$= \left[\frac{32x^{-1}}{-1} - \frac{3x^2}{4} + 4x\right]_1^4$$

$$= \left[-\frac{32}{x} - \frac{3x^2}{4} + 4x\right]_1^4$$

$$= \left(-\frac{32}{4} - \frac{3(4)^2}{4} + 4(4)\right) - \left(-\frac{32}{1} - \frac{3(1)^2}{4} + 4(1)\right)$$

$$= -8 - 12 + 16 + 32 + \tfrac{3}{4} - 4$$

$$= \tfrac{99}{4} = 24.75$$

Question 6 (d)

$$\%\text{ error} = \left(\frac{26.005 - 24.75}{24.75}\right) \times 100\% \approx 5\%$$

Question 7 (50 marks)

Question 7 (a)

$P = €60\ 000,\ t = 5 \text{ years},\ i = 0.065$

$$A = 60\ 000\frac{0.065(1.065)^5}{(1.065)^5 - 1} = €14\ 438.07$$

$$\boxed{A = P\frac{i(1+i)^t}{(1+i)^t - 1}}$$

Question 7 (b)

Payment #	Fixed Payment	Interest	Debt Payment	Balance
0				€60 000
1	€14 438.07	€3900	€10 538.07	€49 461.93
2	€14 438.07	€3215.03	€11 223.04	€38 238.89
3	€14 438.07	€2485.53	€11 952.54	€26 286.35
4	€14 438.07	€1708.61	€12 729.46	€13 556.89
5	€14 438.07	€881.20	€13 556.87	0

Calculation for Year 1

Payment Number 1: €14 438.07

Interest: $€60\ 000 \times 0.065 = €3900$

Debt Payment: $€14\ 438.07 - €3900.00 = €10\ 538.07$

Balance: $€60000 - €10538.07 = €49461.93$

Question 7 (c) (i)

$$P = \frac{5000}{(1.045)^8} = €3515.93$$

$$\boxed{P = \frac{F}{(1+i)^t}}$$

Question 7 (c) (ii)

$$P = 250 + \frac{250}{1.045^1} + \frac{250}{1.045^2} + \frac{250}{1.045^3} + \frac{250}{1.045^4} + \frac{250}{1.045^5} + \frac{250}{1.045^6} + \frac{250}{1.045^7}$$

$$P = 250\left(1 + \frac{1}{1.045^1} + \frac{1}{1.045^2} + \frac{1}{1.045^3} + \frac{1}{1.045^4} + \frac{1}{1.045^5} + \frac{1}{1.045^6} + \frac{1}{1.045^7}\right)$$

$$a = 1,\ r = \frac{1}{1.045},\ n = 8$$

$$\therefore P = 250\left(\frac{\left(1 - \left(\frac{1}{1.045}\right)^8\right)}{1 - \frac{1}{1.045}}\right) = €1723.18$$

Question 7 (c) (iii)

Minimum price = €3515.93 + €1723.18 = €5239.11
Minimum price bonds can be offered is €5239 to the nearest euro.

QUESTION 8 (50 MARKS)

Question 8 (a) (i)

$x+y=15....(\mathbf{1})(\times 0.4)$

$0.4x+0.25y=0.3\times 15 \Rightarrow 0.4x+0.25y=4.5...(\mathbf{2})$

$$\begin{array}{r} 0.4x+0.4y\ \ =6 \\ 0.4x+0.25y=4.5 \\ \hline 0.15y=1.5 \Rightarrow y=10\ 1 \end{array}$$

$x+10=15 \Rightarrow x=5\ 1$

Question 8 (a) (ii)

There is 4.5 l of acid in 15 l of the solution $(15\times 0.3=4.5)$.
When 1 l of water is added, there is now 16 l of solution.

Concentration of acid in solution $=\dfrac{4.5}{16}\times 100\% = 28.125\%$

Question 8 (b)

	pH value	$[H^+]$
Acidic	<7	$>10^{-7}$
Alkaline	>7	$<10^{-7}$
Neutral	$=7$	$=10^{-7}$

$\text{pH}=-\log_{10}[H^+]$

$\text{pH}=7$

$\therefore 7=-\log_{10}[H^+]$

$-7=\log_{10}[H^+]$

$\therefore 10^{-7}=[H^+]$

Question 8 (c) (i)

Apple juice: $[H^+]=0.00028$ moles per litre

$\text{pH}=-\log_{10}[H^+]=-\log_{10}(0.00028)=3.55<7$ (Acidic)

Ammonia: $[H^+]=1.32\times 10^{-9}$ moles per litre

$\text{pH}=-\log_{10}[H^+]=-\log_{10}(1.32\times 10^{-9})=8.88>7$ (Alkaline)

Question 8 (c) (ii)

Distilled water: $\text{pH} = 7 \Rightarrow [H^+]=10^{-7}$ moles per litre

$\text{pH}=3.22$

$\therefore 3.22=-\log_{10}[H^+]$

$-3.22=\log_{10}[H^+]$

$10^{-3.22}=[H^+]$

$\therefore [H^+]=6\times 10^{-4}$ moles/litre

Question 8 (d) (i)

pH values: 2.0, 2.5, 3.0, 3.5, 4.0, 4.5.
After five hours the pH value is 4.5.

Or

pH values: 2.0, 2.5, 3.0,.......

$a = 2,\ d = 0.5,\ n = 6$

$T_n = a + (n-1)d = 2 + 5(0.5) = 4.5$

Question 8 (d) (ii)

Arithmetic sequence: 2, 2.5, 3, 3.5,....

Geometric sequence: $10^{-2}, 10^{-2.5}, 10^{-3}$,......

$$a = 10^{-2},\ r = \frac{10^{-2.5}}{10^{-2}} = 10^{-0.5}$$

$$\therefore 10^{-8.7} = 10^{-2}(10^{-0.5})^{n-1}$$

$$10^{-6.7} = 10^{-0.5n+0.5}$$

$$-6.7 = -0.5n + 0.5$$

$$0.5n = 0.5 + 6.7 = 7.2$$

$$\therefore n = \frac{7.2}{0.5} = 14.4 \text{ hours}$$

Therefore, the pH value is 8.7 after 13.4 hours.

QUESTION 9 (50 MARKS)

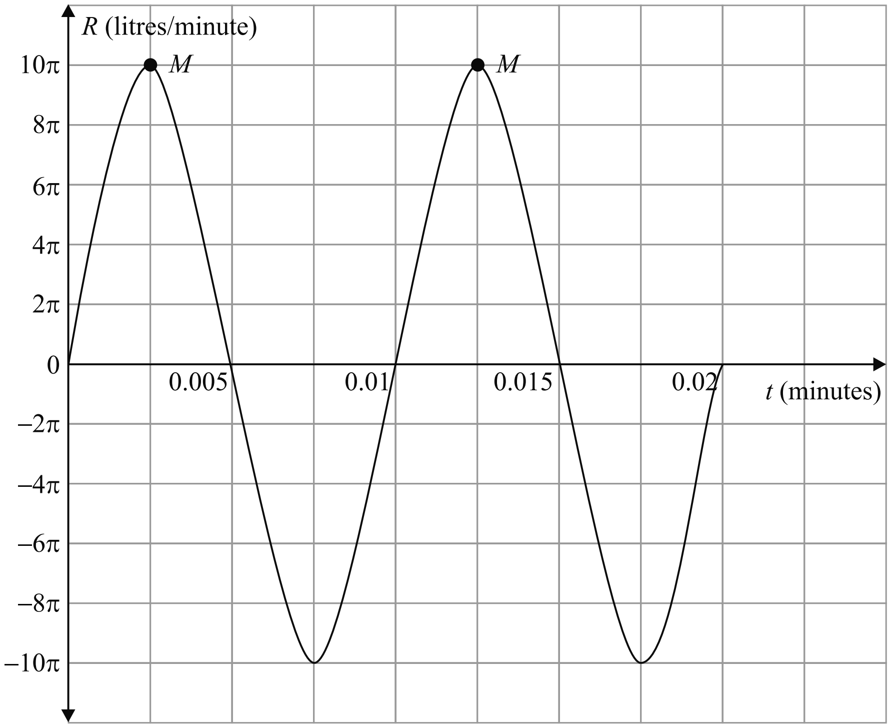
Question 9 (a)

$$\text{Period} = \frac{2\pi}{200\pi} = \frac{1}{100}\text{min} = 0.01\,\text{min}$$

$$\text{Range} = [-10\pi, 10\pi]$$

Question 9 (b)

t (minutes)	0	0.0025	0.005	0.0075	0.01	0.0125	0.015	0.0175	0.02
$200\pi t$	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π	$\frac{5\pi}{2}$	3π	$\frac{7\pi}{2}$	4π
$\sin(200\pi t)$	0	1	0	-1	0	1	0	-1	0
$10\pi\sin(200\pi t)$	0	10π	0	-10π	0	10π	0	-10π	0



Question 9 (c)

(i) Breathing in: 0 s – 0.0025 s, 0.0075 s – 0.0125 s, 0.0175 s – 0.02 s

(ii) Breathing out: 0.0025 s – 0.0075 s, 0.0125 s – 0.0175 s

(iii) Maximum values: 0.0025 s, 0.0125 s

Question 9 (d)

$$\frac{dV}{dt} = 10\pi \sin(200\pi t)$$

$$\int dV = 10\pi \int \sin(200\pi t)\, dt$$

$$V = -\frac{10\pi}{200\pi}\cos(200\pi t) + c$$

$$V = -0.05\cos(200\pi t) + c$$

$$t = 0,\ V = 2.95 : 2.95 = -0.05\cos(200\pi(0)) + c$$

$$2.95 = -0.05 + c$$

$$\therefore c = 3$$

$$V = -0.05\cos(200\pi t) + 3$$

Question 9 (e)

$$V = -0.05\cos(200\pi t) + 3$$

$$V_{\text{Ave.}} = \frac{1}{0.01 - 0}\int_0^{0.01} (-0.05\cos(200\pi t) + 3)\, dt$$

$$= 100\left[3t - \frac{0.5}{200\pi}\sin(200\pi t)\right]_0^{0.01}$$

$$= 100\left\{3(0.01) - \frac{0.05}{200\pi}\sin(200\pi(0.01)) - 0\right\}$$

$$= 3 - \frac{0.05}{200\pi}\sin(2\pi)$$

$$= 3 \text{ litres}$$

SAMPLE PAPER 7: PAPER 2

QUESTION 1 (25 MARKS)

Question 1 (a)

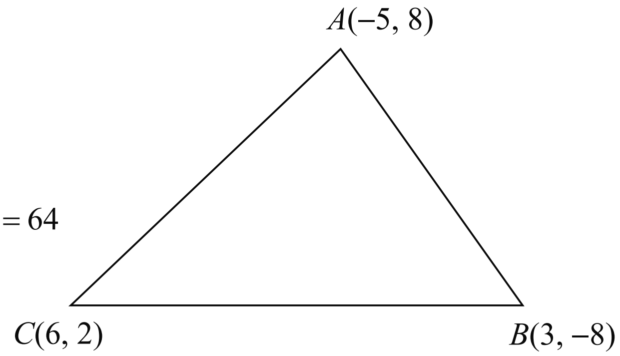
$A(-5, 8) \to (0, 0)$

$\boxed{\text{Area} = \tfrac{1}{2}\left|x_1y_2 - x_2y_1\right|}$

$B(3, -8) \to (8, -16)$

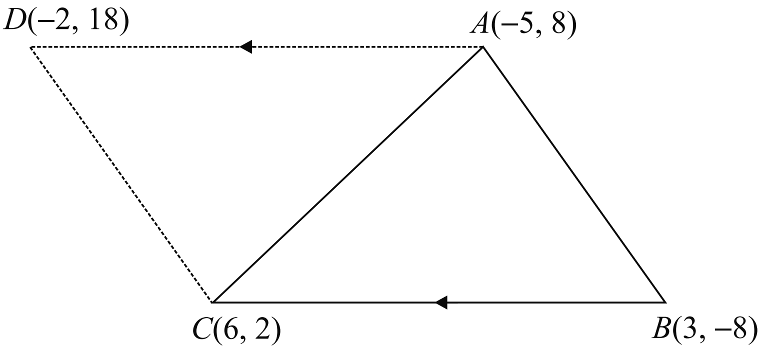
$C(6, 2) \to (11, -6)$

$\text{Area} = \tfrac{1}{2}\left|8(-6) - 11(-16)\right| = \tfrac{1}{2}\left|-48 + 176\right| = \tfrac{1}{2}\left|128\right| = 64$



$B(3, -8) \to C(6, 2)$

$A(-5, 8) \to D(-2, 18)$



Question 1 (b)

Equation of BC: $B(3, -8), C(6, 2)$

$$m = \frac{2-(-8)}{6-3} = \frac{10}{3},\ (x_1, y_1) = (3, -8) \quad \boxed{y - y_1 = m(x - x_1)}$$

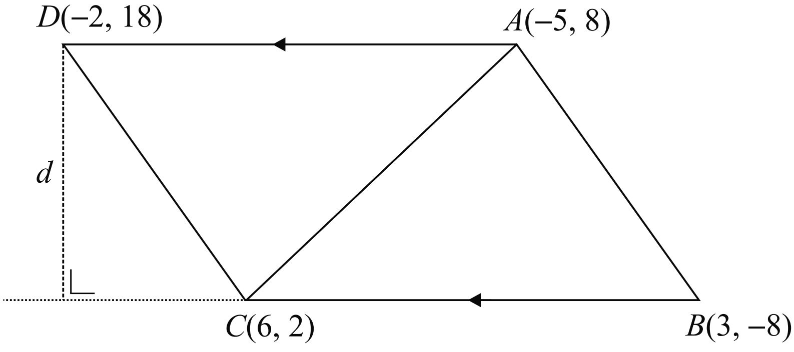
$$y - (-8) = \tfrac{10}{3}(x - 3)$$

$$3y + 24 = 10x - 30$$

$$10x - 3y - 54 = 0$$

$$\boxed{d = \frac{\left|ax_1 + by_1 + c\right|}{\sqrt{a^2 + b^2}}}$$

$$d = \frac{\left|10(-2) - 3(18) - 54\right|}{\sqrt{10^2 + (-3)^2}} = \frac{128}{\sqrt{109}}$$



$B(3, -8), C(6, 2)$

$\text{Area} = \text{Base} \times \text{Height}$

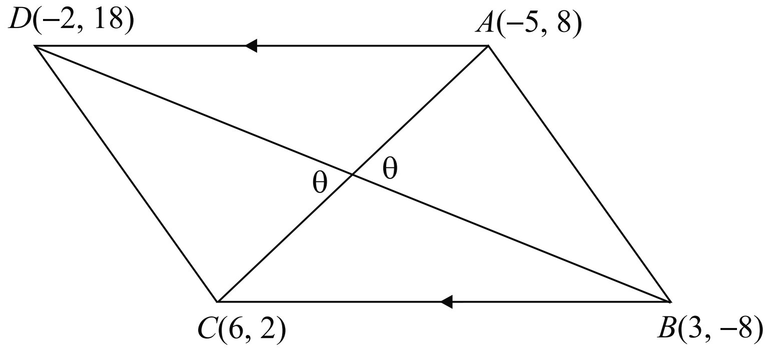
$\text{Base:} \left|CB\right| = \sqrt{(6-3)^2 + (2-(-8))^2} = \sqrt{109}$

$$\text{Area} = \sqrt{109} \times \frac{128}{\sqrt{109}} = 128$$

OR

Area of parallelogram $ABCD$ = 2(Area of triangle ABC)

Question 1 (c)



Slope of AC: $m_1 = \frac{8-2}{-5-6} = -\frac{6}{11}$

Slope of BD: $m_2 = \frac{18+8}{-2-3} = -\frac{26}{5}$

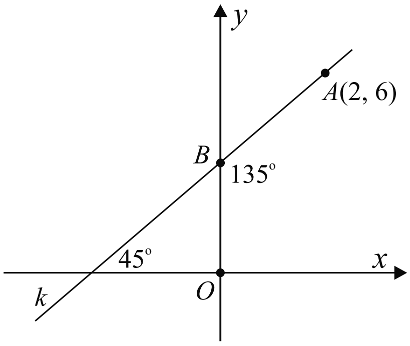
$$\tan\theta = +\left(\frac{m_1 - m_2}{1+m_1m_2}\right) = \frac{-\frac{6}{11}+\frac{26}{5}}{1+(-\frac{6}{11})(\frac{26}{5})}$$

$$\therefore \theta = \tan^{-1}\left(\frac{-\frac{6}{11}+\frac{26}{5}}{1+(-\frac{6}{11})(\frac{26}{5})}\right) = 50.5^{\circ}$$

QUESTION 2 (25 MARKS)

Question 2 (a)

Line k makes an angle of 45° with the positive x-axis. The slope of k is the tan of the angle the line makes with the positive x-axis.



$\tan 45^\circ = 1 = m,\ (x_1,\ y_1) = A(2,\ 6)$ $\boxed{y - y_1 = m(x - x_1)}$

$y - 6 = 1(x - 2)$

$y - 6 = x - 2$

$k : x - y + 4 = 0$

Question 2 (b)

$x + y - 1 = 0 \Rightarrow y = 1 - x$

$Q(x,\ y) = (x,\ 1 - x)$

$P(-1,\ 2)$

$\text{Area} = \frac{1}{2}\left|2x - (1 - x)(-1)\right| = 7$ $\boxed{\text{Area} = \frac{1}{2}\left|x_1y_2 - x_2y_1\right|}$

$\therefore \left|2x + 1 - x\right| = 14$

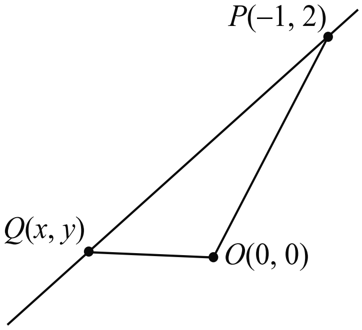
$\left|x + 1\right| = 14$

$x + 1 = \pm 14$

$\therefore x = 13,\ \cancel{-15}\ (x > 0)$

$y = 1 - x = -12$

$\therefore Q(13,\ -12)$



Question 2 (c)

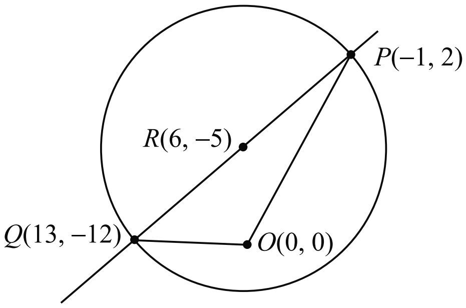
The midpoint of $[PQ]$ is the centre of the circle. Call it R.

$P(-1,\ 2),\ Q(13,\ -12)$

$$R = \left(\frac{-1 + 13}{2},\ \frac{2 - 12}{2}\right) = (6,\ -5)$$

$P(-1,\ 2),\ R(6,\ -5)$

$r = \left|PR\right| = \sqrt{(6 + 1)^2 + (-5 - 2)^2} = \sqrt{49 + 49} = 7\sqrt{2}$



Equation of circle:

$(x - h)^2 + (y - k)^2 = r^2$

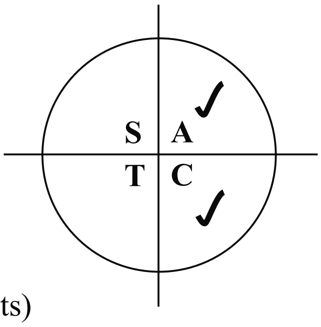
$(x - 6)^2 + (y + 5)^2 = 98$

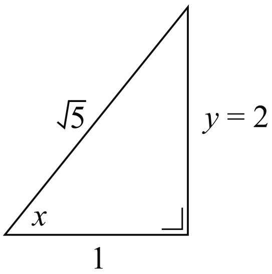
QUESTION 3 (25 MARKS)

Question 3 (a)

$$\cos x = +\frac{1}{\sqrt{5}} = \frac{\text{Adjacent}}{\text{Hypotenuse}}$$

The angle x is located in the first and fourth quadrants.





$$1^2 + y^2 = (\sqrt{5})^2$$

$$y^2 = 5 - 1 = 4$$

$$\therefore y = 2$$

$\tan x = \pm 2$ (First and fourth quadrants)

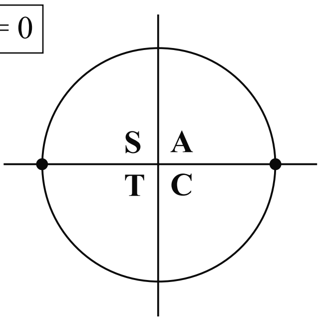
$$\tan 2x = \frac{2\tan x}{1 - \tan^2 x}$$

$$\tan x = 2 : \tan 2x = \frac{2(2)}{1 - 2^2} = -\frac{4}{3}$$

$$\tan x = -2 : \tan 2x = \frac{2(-2)}{1 - (-2)^2} = \frac{4}{3}$$

Question 3 (b) (i)

$\boxed{\sin 3x = 0}$

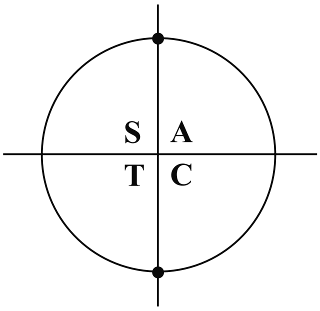


$$3x = 0 + 2n\pi,\ \pi + 2n\pi,\ n \in \mathbb{Z}$$

$$x = \frac{2n\pi}{3},\ \frac{\pi}{3} + \frac{2n\pi}{3},\ n \in \mathbb{Z}$$

$$x = 0,\ \frac{\pi}{3},\ \frac{2\pi}{3},\ \pi,\ \frac{4\pi}{3},\ \frac{5\pi}{3}$$

$\boxed{\cos 2x = 0}$



$$2x = \frac{\pi}{2} + 2n\pi,\ \frac{3\pi}{2} + 2n\pi,\ n \in \mathbb{Z}$$

$$x = \frac{\pi}{4} + n\pi,\ \frac{3\pi}{4} + n\pi,\ n \in \mathbb{Z}$$

$$x = \frac{\pi}{4},\ \frac{3\pi}{4},\ \frac{5\pi}{4},\ \frac{7\pi}{4}$$

Question 3 (b) (ii)

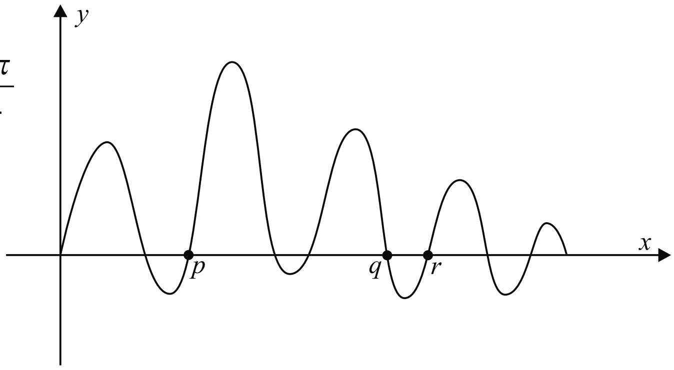
$$y = \sin 3x \cos 2x$$

$$x = 0,\ \frac{\pi}{4},\ \frac{\pi}{3},\ \frac{2\pi}{3},\ \frac{3\pi}{4},\ \pi,\ \frac{5\pi}{4},\ \frac{4\pi}{3},\ \frac{5\pi}{3},\ \frac{7\pi}{4}$$

$p = \dfrac{\pi}{3}$ (Third root)

$q = \pi$ (Sixth root)

$r = \dfrac{5\pi}{4}$ (Seventh root)



QUESTION 4 (25 MARKS)

A spinner has nine equal segments numbered 1, 2, 3, 4, 5, 6, 7, 8 and 9 (Nine numbers)
Blue: 2, 3, 6, 8, 9 (Five numbers)
Red: 1, 4, 5, 7 (Four numbers)
E is the event that the pointer lands on an even number.
E: 2, 4, 6, 8 (Four numbers)
R is the event that the pointer lands on a red colour.

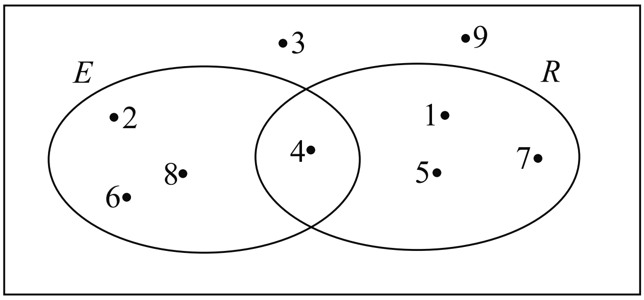
Question 4 (a)

$$P(E)=\frac{\text{Number of even numbers}}{\text{Number of numbers}}=\frac{4}{9}$$

Question 4 (b)

$$P(R)=\frac{\text{Number of red colours}}{\text{Number of numbers}}=\frac{4}{9}$$

Question 4 (c)



$$P(E\cup R)=P(E)+P(R)-P(E\cap R)$$
$$E\cap R=\{4\}$$
$$P(E\cup R)=\frac{4}{9}+\frac{4}{9}-\frac{1}{9}=\frac{7}{9}$$

Question 4 (d)

$$P(R\,|\,E)=\frac{P(R\cap E)}{P(E)}=\frac{\frac{1}{9}}{\frac{4}{9}}=\frac{1}{9}\times\frac{9}{4}=\frac{1}{4}$$

Question 4 (e)

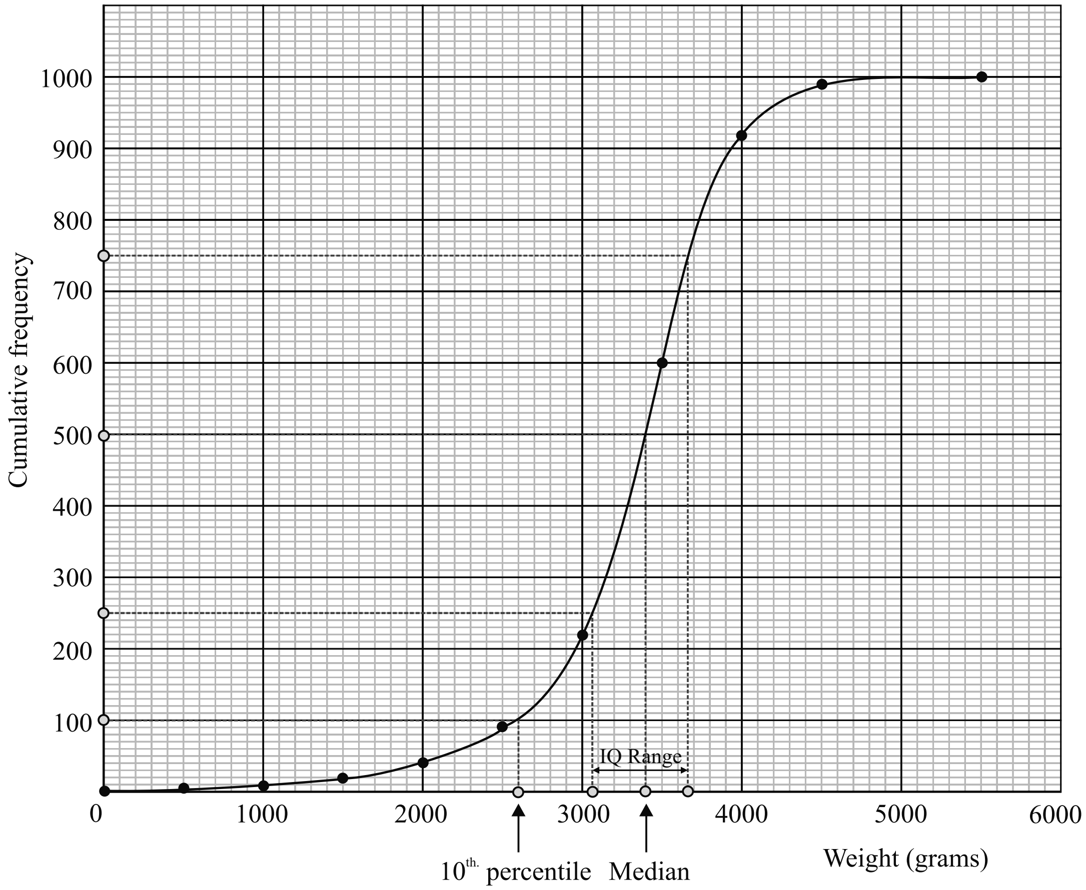
$$P(E\,|\,R)=\frac{P(E\cap R)}{P(R)}=\frac{\frac{1}{9}}{\frac{4}{9}}=\frac{1}{9}\times\frac{9}{4}=\frac{1}{4}$$

Question 4 (f)

(i) Yes, because $P(E) = P(R)$,
(ii) No: $\frac{1}{4}\neq\frac{4}{9}$

QUESTION 5 (25 MARKS)

Question 5 (a)



Median = 3400 g

Interquartile range = 3670 – 3060 = 610 g

Question 5 (b)

Range of weights below the 10th. percentile = 0–2600 g

Question 5 (c) (i)

A. $n = 12,\ P(\text{Need special care}) = 0.15,\ P(\text{Do not need special care}) = 0.85,$

$P(\text{2 need special care}) = {}^{12}C_2(0.15)^2(0.85)^{10} = 0.2924$

B. $P(\text{More than two need special care}) = 1 - P(\text{Two or one or none need special care})$

$= 1 - \{0.2924 + {}^{12}C_1(0.15)^1(0.85)^{11} + {}^{12}C_0(0.15)^0(0.85)^{12}\}$

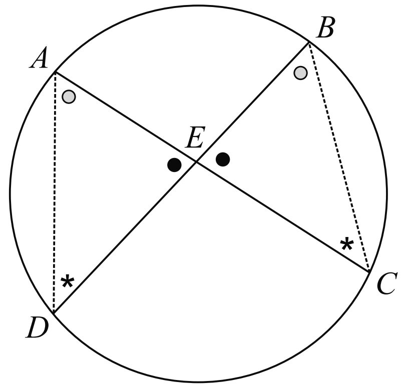
$= 0.2641$

Question 5 (c) (ii)

$E = 100 \times 0.2641 = 26.41$ occasions

QUESTION 6 (25 MARKS)

Question 6 (a) (i)



$|\angle DEA| = |\angle BEC|$ (Vertically opposite)

$|\angle EAD| = |\angle CBE|$ (Standing on same arc)

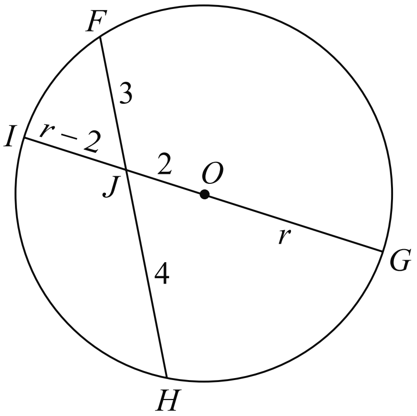
$\therefore |\angle ADE| = |\angle BCE|$

Question 6 (a) (ii)

$$\frac{|AE|}{|DE|} = \frac{|BE|}{|EC|}$$

$\therefore |AE||EC| = |BE||DE|$

Question 6A (b)



$|FJ||JH| = |IJ||JG|$

$3 \times 4 = (r-2)(r+2)$

$12 = r^2 - 4$

$16 = r^2$

$\therefore r = 4$

QUESTION 7 (45 MARKS)

Question 7 (a)

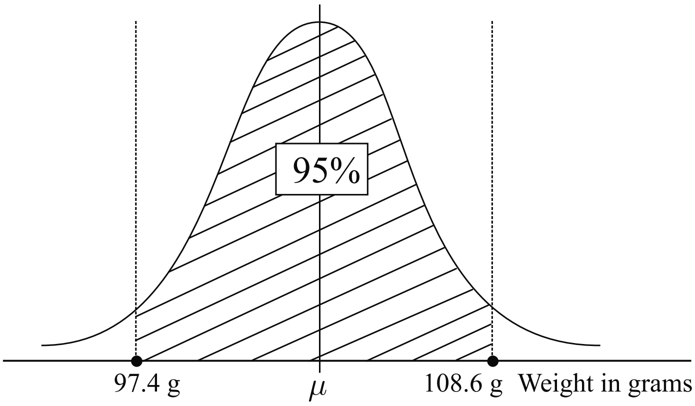
(i) The distribution is normal.

(ii) $\mu + 2\sigma = 108.6$

$\mu - 2\sigma = 97.4$

$2\mu \quad = 206 \Rightarrow \mu = 103 \text{ g}$

$103 + 2\sigma = 108.6 \Rightarrow \sigma = 2.8 \text{ g}$



Question 7 (b)

$\mu = 103 \text{ g}, \sigma = 2.8 \text{ g}$

$P(x < 100) = ?$

$$x = 100 : z = \frac{x-\mu}{\sigma} = \frac{100-103}{2.8} = -1.07$$

$P(x < 100) = P(z < -1.07)$

$= P(z > 1.07)$

$= 1 - P(z < 1.07)$

$= 1 - 0.8577$

$= 0.1423$

$= 14.23\%$

Question 7 (c)

$P(x < 100) = 0.001$

$P(x < -Z) = 0.001$

$P(z > Z) = 0.001$

$P(z < Z) = 0.999$

$\therefore z = 3.08 \Rightarrow -z = -30.8$

$$-3.08 = \frac{100 - \overline{x}}{2.8}$$

$\therefore \overline{x} = 108.624 \text{ g}$

Question 7 (d)

$$
\begin{aligned}
&P(\text{Less than advertised weight}) = 0.1423\\
&P(\text{Not less than advertised weight}) = 0.8577\\
&P(\text{At least one weighs less than advertised weight})\\
&= P(\text{One or more weighs less than advertised weight})\\
&= 1 - P(\text{None with less than advertised weight})\\
&= 1 - {}^5C_0(0.1423)^0(0.8577)^5\\
&= 0.536\\
&= 53.6\%
\end{aligned}
$$

QUESTION 8 (40 MARKS)

Question 8 (a)

The parameter in this question is the mean glass thickness, μ.
Null hypothesis H_0: $\mu = 0.954$ cm
Alternative hypothesis H_A: $\mu \neq 0.954$ cm
(This is called a two-tailed test as the glass should not be too thick or too thin.)

Question 8 (b)

Mean $\mu = 0.954$ cm
Mean of sample $\bar{x} = 0.96$ cm
Standard deviation $\sigma = 0.13$ cm
Number of sample $n = 100$
$z = ?$

$$z = \frac{\bar{x} - \mu}{\left(\frac{\sigma}{\sqrt{n}}\right)}$$

$$z = \frac{0.96 - 0.954}{\left(\frac{0.13}{\sqrt{100}}\right)} = 0.46$$

Question 8 (c)

$$
\begin{aligned}
p &= 2P(z > 0.46)\\
&= 2\{0.5 - (0.6772 - 0.5)\}\\
&= 2\{1 - 0.6772\}\\
&= 0.6456 \gg 0.05 \quad [0.05 = 5\% \text{ level of significance}]
\end{aligned}
$$

Therefore, the probability of the null hypothesis is very, very strong. We accept the null hypothesis.

Question 8 (d)

The glass company does not have sufficient evidence to conclude it is not meeting the specifications. The difference between the sample mean and the actual mean is not large enough to attribute to anything but sampling error.

QUESTION 9 (65 MARKS)

Question 9 (a) (i)

$$d = \frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$$

Question 9 (a) (ii)

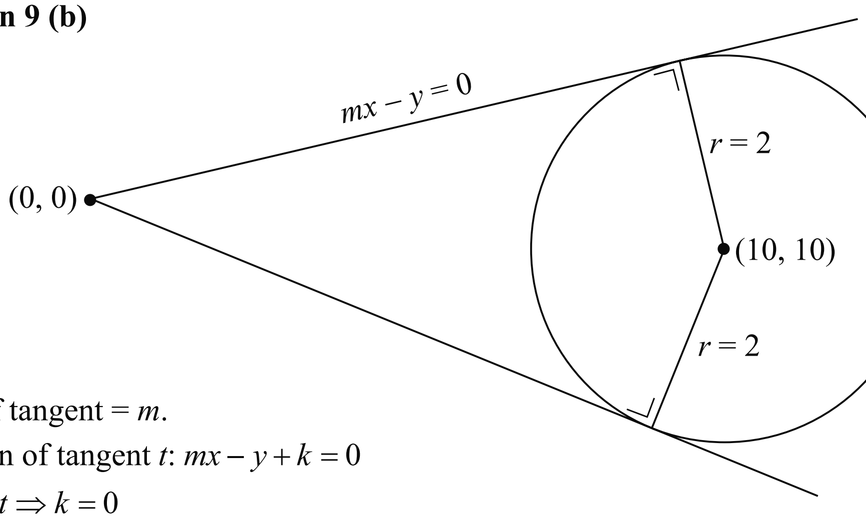
Equation of circle:

$x^2 + y^2 - 20x - 20y + 196 = 0$

Centre $= (-g, -f) = (10, 10)$

$r = \sqrt{g^2 + f^2 - c} = \sqrt{100 + 100 - 196} = \sqrt{4} = 2$

Question 9 (b)



Slope of tangent = m.

Equation of tangent t: $mx - y + k = 0$

$(0, 0) \in t \Rightarrow k = 0$

Equation of tangent t: $mx - y = 0$

$mx - y = 0,\ (x_1, y_1) = (10, 10),\ d = 2$

$$2 = \frac{|m(10) - (10)|}{\sqrt{m^2 + 1}}$$

$2\sqrt{m^2 + 1} = 10|m - 1|$

$\sqrt{m^2 + 1} = 5|m - 1|$

$m^2 + 1 = 25(m - 1)^2$

$m^2 + 1 = 25m^2 - 50m + 25$

$24m^2 - 50m + 24 = 0$

$12m^2 - 25m + 12 = 0$

$(4m - 3)(3m - 4) = 0$

$\therefore m = \frac{3}{4}, \frac{4}{3}$

Question 9 (c)

$x = 10\sqrt{2}\cos 45^\circ = 10\sqrt{2} \times \frac{1}{\sqrt{2}} = 10$ km

$y = 10\sqrt{2}\sin 45^\circ = 10\sqrt{2} \times \frac{1}{\sqrt{2}} = 10$ km

$\therefore M(10, 10)$

Question 9 (d)

The ships travelling from O are moving along tangents to the circle with centre $M(10, 10)$ of radius 2 km. This is analogous to the previous circle. You know the slopes of these two tangents.

Bearing of Ships A and B:

$\tan\alpha = m = \frac{3}{4}$

$\therefore \alpha = \tan^{-1}(\frac{3}{4}) = 36.87^\circ$ N of E

$\tan\beta = m = \frac{4}{3}$

$\therefore \beta = \tan^{-1}(\frac{4}{3}) = 53.13^\circ$ N of E

Question 9 (e)

Distances travelled by *A* and *B*:

$$v = \frac{s}{t} \Rightarrow s = v \times t$$

$v_A = 40 \text{ km/h},\ t = 2 \text{ h}$ $\qquad$ $v_B = 30 \text{ km/h},\ t = 2 \text{ h}$

$s = 40 \times 2 = 80 \text{ km}$ $\qquad$ $s = 30 \times 2 = 60 \text{ km}$

Question 9 (f)

$m_1 = \frac{4}{3},\ m_2 = \frac{3}{4}$

$$\therefore \tan\theta = +\left(\frac{\frac{4}{3} - \frac{3}{4}}{1 + (\frac{4}{3})(\frac{3}{4})}\right) = +\left(\frac{\frac{4}{3} - \frac{3}{4}}{1+1}\right) = +\left(\frac{\frac{4}{3} - \frac{3}{4}}{2}\right)$$

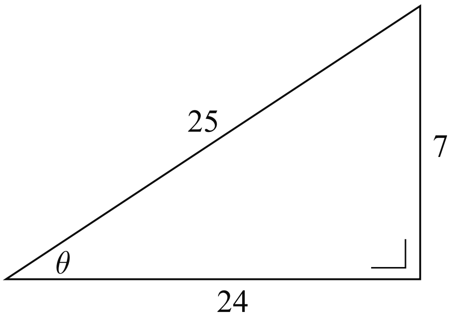
$$= \left(\frac{\frac{4}{3} - \frac{3}{4}}{2}\right) \times \frac{12}{12}$$

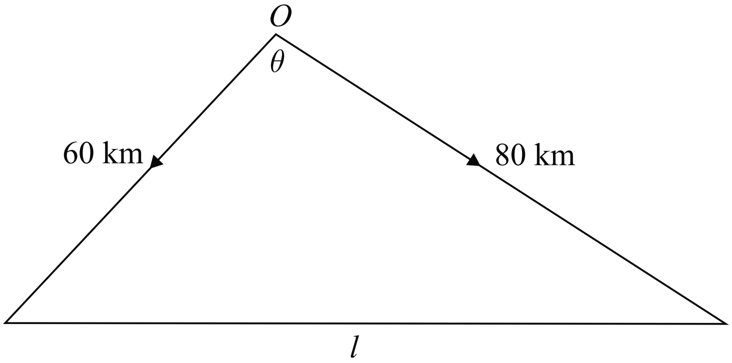
$$= \left(\frac{16-9}{24}\right)$$

$$= \frac{7}{24}$$

Question 9 (g)

$$\tan\theta = \frac{7}{24} \Rightarrow \cos\theta = \frac{24}{25}$$





$$l^2 = 60^2 + 80^2 - 2(60)(80)\cos\theta$$

$$l = \sqrt{60^2 + 80^2 - 2(60)(80)(\tfrac{24}{25})} = 28 \text{ km}$$