MATHEMATICS SOLUTIONS
Junior Certificate Higher Level
CONTENTS

PAPER 1

PAPER 2

2015 SEC Sample Paper 1 (Phase 3)

1. **(a)** $P \cup Q = \{1, 2, 3, 4, 5, 6\}$ ← All the numbers in *P* or *Q*

$Q \cap R = \{5, 6\}$ ← Numbers common to both *Q* and *R*

$P \cup (Q \cap R) = \{1, 2, 4, 5, 6\}$ ← *P* union with the last answer

(b) Use $(P \cup Q) \cap (P \cup R)$ ← As in 4(3 + 5) = 4(3) + 4(5) in real numbers

$P \cup Q = \{1, 2, 3, 4, 5, 6\}$ and $P \cup R = \{1, 2, 4, 5, 6, 7\}$

$(P \cup Q) \cap (P \cup R) = \{1, 2, 4, 5, 6\}$ which is the same as $P \cup (Q \cap R)$

2. **(a)** $A \cup B \cup C = \{2, 3, 4, 5, 6, 8, 9, 10, 12, 14, 15, 16, 18, 20, 21, 22, 24, 25, 26, 27, 28, 30\}$

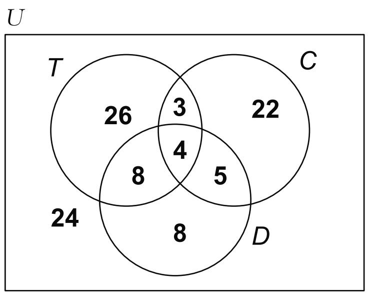
Therefore, the cardinal number of the elements not in this set is 29 – 22 = 7

i.e. {7, 11, 13, 17, 19, 23, 29}

(b) Two divisors ← Divisors: numbers which divide in evenly

(c) Prime numbers

3. **(a)**



(b) $\frac{68}{100} = \frac{17}{25}$ ← $\frac{\text{All people in Tea or Coffee ovals}}{\text{Total No. of people}}$

(c) $\frac{3}{100}$ ← Soft drink oval is excluded.

4. **(a)** 3% of €5000 = €150 less tax of 41% = 150 × 0·59 = €88·50 ← Reducing by 41% = 59% remaining i.e. multiply by 0·59

€5000 + €88·50 = €5088·50

(b) €5088·50 gives interest of 50·885*x* at *x*% interest rate.

Less the tax of 41% leaves interest of €30·02215*x*. ← 50·885*x* multiplied by 0·59

€5088·50 + €30·02215*x* = €5223·60

€30·02215*x* = €135·10

$x = \frac{135{\cdot}10}{30{\cdot}02215} = 4{\cdot}5\%$

5. **(a)** Jerry is treating €30·52 as 100% of the cost of the meal rather than as 109% of the cost of the meal.

(b) €30·52 = 109% of the cost of the meal before VAT

€0·28 = 1%

€28 = the cost of the meal before VAT

At VAT rate of 13·5% this meal would cost

€28 × 1·135 = €31·78

6. (a) If she spends €35 → $\boxed{25} \le x \le \boxed{50}$ ← All the cash she has

(b) €35 will allow the voucher to be used. → $\boxed{35} \le y \le \boxed{60}$ ← She will pay €50 and get a €10 discount.

7. (a) Let x units be the side of the square in the lower left corner.

The other square would then have a side length of $(10 - x)$ units.

The joint areas of the squares can then be given by $x^2 + (10 - x)^2$.

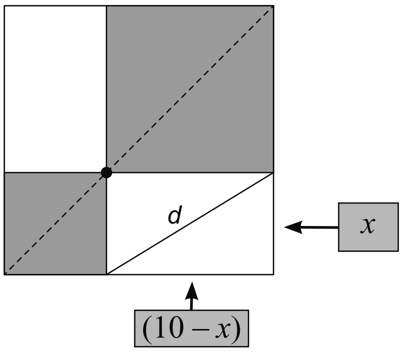
Expanding and tidying gives an area function $A = 2x^2 - 20x + 100$.

> You could also sketch the graph to see its minimum.

By completing the square this function can be written as $A = 2(x - 5)^2 + 50$.

This function has a minimum turning point at (5, 50), and hence the minimum value for area is 50 units^2.

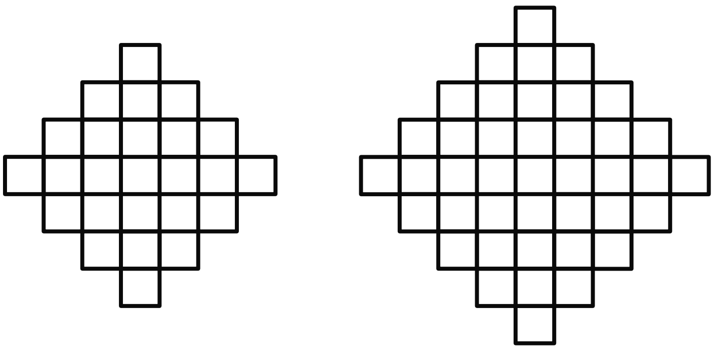
(b)



The side lengths of the right-angled triangle in the bottom right corner are d, x and $(10 - x)$.

Applying Pythagoras's theorem gives $d^2 = x^2 + (10 - x)^2$. Q.E.D.

8. (a)



(b)

Stage	Perimeter
1	4
2	12
3	20
4	28

Stage 1 = 4 + 0(8)

Stage 2 = 4 + 1(8)

Stage 3 = 4 + 2(8)

………………………..

Stage $n = 4 + (n - 1)8 = 8n - 4$

Drawing a table of values can help when searching for a formula.

(c)

Stage	Area
1	1
2	5
3	13
4	25

Stage 1 = $1^2 + 0^2$

Stage 2 = $2^2 + 1^2$

Stage 3 = $3^2 + 2^2$

………………………..

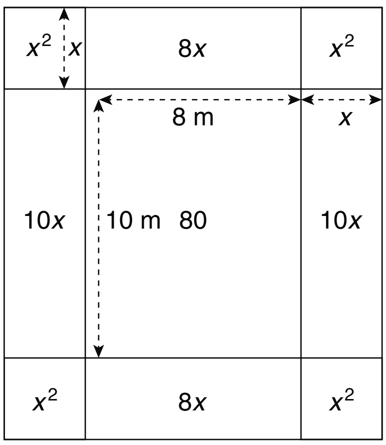
Stage $n = n^2 + (n - 1)^2 = 2n^2 - 2n + 1$

(d) Quadratic because the second differences are constant (all equal to 4)

9. (b) $x^2 + 8x + x^2 + 10x + 80 + 10x + x^2 + 8x + x^2 = 143$

$4x^2 + 36x + 80 = 143$

$4x^2 + 36x - 63 = 0$

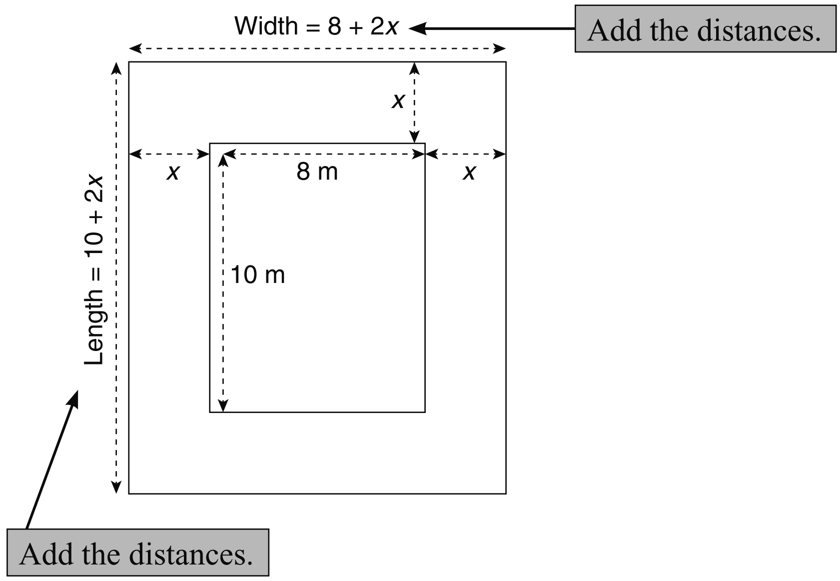


Find the area of each individual section.

(d) $(8 + 2x)(10 + 2x) = 143$

$80 + 36x + 4x^2 = 143$

$4x^2 + 36x - 63 = 0$



(e) $4x^2 + 36x - 63 = 0$

$(2x - 3)(2x + 21) = 0$

$2x - 3 = 0$ and $2x + 21 = 0$

$x = 1·5$ and $x = -10·5 \therefore x = 1·5$ m

(f) Let $x = 1$: Area is 10 by 12 = 120 m^2 … … not true (too small)

Let $x = 2$: Area is 12 by 14 = 168 m^2 … … not true (too big)

Tony would then have to try values between 1 and 2 m etc.

(g) Kevin's or Elaine's

Their algebraic method is faster and more accurate.

Tony may not have found the exact answer using his method.

10. (a) $R(2, 3)$: $3 = (2)^2 + a(2) + b$

$3 = 4 + 2a + b$

$2a + b = -1$ ← The points are on the curve so substitute them into the equation.

$S(-5, -4)$: $-4 = (-5)^2 + a(-5) + b$

$-4 = 25 - 5a + b$

$-5a + b = -29$

$2a + b = -1$

(b) $-5a + b = -29$

$-2a - b = 1$ ← Solve simultaneous equations.

$-7a = -28$

$a = 4$

$b = -9$

(c) (0, –9)

(d) $x^2 + 4x - 9 = 0$

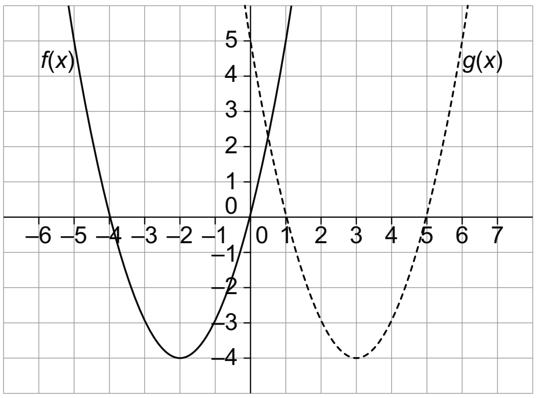
$$x = \frac{-4 \pm \sqrt{(4)^2 - 4(1)(-9)}}{2(1)}$$

Quadratic formula
See page 20 of *Formulae and Tables.*

$$x = \frac{-4 \pm \sqrt{52}}{2}$$

$x = 1{\cdot}6$ or $x = -5{\cdot}6$

11.

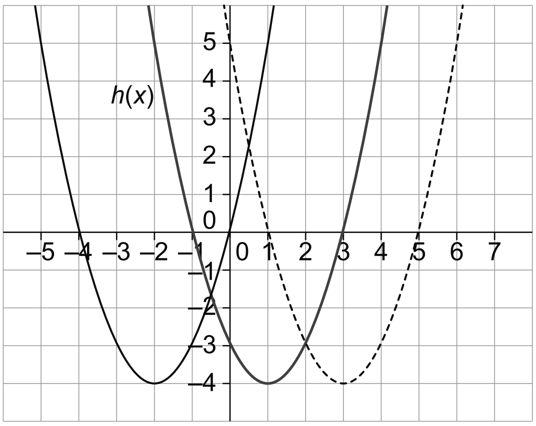


(b) Roots of f: 0 and –4

Roots of g: 1 and 5

Roots: points where the curve crosses the x-axis

(c)



(d) Complete the square on the RHS.

$x^2 - 10x + 25 - 25 + 23$

$(x - 5)^2 - 2$

Comparing with the LHS gives $p = 5$

(e) $x = 5$

Mirror line which would allow the graph to "fold onto itself"

12. Some x values have two y outputs.

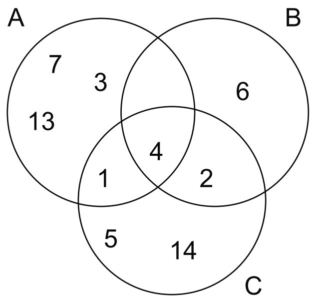
OR

It fails the Vertical Line Test.

Educate.ie Sample 1

Paper 1

1. **(a)** $\{1, 3, 4, 2, 6, 7, 13\}$



(b) $\{3, 7, 13\}$

(c) $\{1, 4\}$

(d) $\{6\}$ ← This can also be written as $(A \cup C)^c$.

(e) $\{3, 7, 13, 2, 5, 14\}$

2. **(a)** $75x^2 - 3$ ← 3 is a common factor.

$3(25x^2 - 1)$

$3(5x + 1)(5x - 1)$ ← Difference of two squares

(b) $14x^2 - 33x - 5$ ← Quadratic factors

$(7x + 1)(2x - 5)$

(c) $4pq^2 - 21pq + 5p$ ← p is a common factor.

$p(4q^2 - 21q + 5)$ ← Quadratic factors

$p(4q - 1)(q - 5)$

3. **(a)**

Even number >2	Sum of primes
4	2 + 2
6	3 + 3
8	3 + 5
10	5 + 5
12	5 + 7
14	7 + 7
16	5 + 11
18	7 + 11
20	7 + 13

(b) 10 = 7 + 3 also: 14 = 11 + 3 also:

(c) 47 + 53 = 100, 3 + 97 = 100 are two possible ways. ← Are there any more?

↑ Other ones are 83 + 17, 71 + 29, 59 + 41.

(d) 1,000,000,000 $= 1 \times 10^9$

1,000,000,000,000 $= 1 \times 10^{12}$

(e) $1 \times 10^{14} = 100$ trillion

4. (a) 30 ← 6 × 5 squares

Explanation:

Shape 1: $2 \times 1 = 2$

Shape 2: $3 \times 2 = 6$

Shape 3: $4 \times 3 = 12$

Shape 4: $5 \times 4 = 20$

Shape 5: $6 \times 5 = 30$

(b) Shape n: $(n + 1) \times n = n^2 + n$

(c) $n^2 + n = 90$

$n^2 + n - 90 = 0$

$(n + 10)(n - 9) = 0 \quad \therefore \; n = 9$

5. (a) $\dfrac{x-2}{x+1} - \dfrac{5}{x-1}$ ← Get the common denominator.

$$\frac{(x-1)(x-2) - 5(x+1)}{(x+1)(x-1)}$$

$$\frac{x^2 - 3x + 2 - 5x - 5}{x^2 - 1} \Rightarrow \frac{x^2 - 8x - 3}{x^2 - 1}$$

(b) $\dfrac{x-2}{x+1} - \dfrac{5}{x-1} = \dfrac{1}{3}$

$$\therefore \; \frac{x^2 - 8x - 3}{x^2 - 1} = \frac{1}{3}$$

$$\frac{(x^2 - 8x - 3)(x^2 - 1)}{x^2 - 1} = \frac{1(x^2 - 1)}{3}$$

$$x^2 - 8x - 3 = \frac{1(x^2 - 1)}{3}$$

$$3(x^2 - 8x - 3) = x^2 - 1$$

$$3x^2 - 24x - 9 = x^2 - 1$$

$$2x^2 - 24x - 8 = 0$$

$$x^2 - 12x - 4 = 0$$

$a = 1, \quad b = -12, \quad c = -4$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{12 \pm \sqrt{144 - 4(1)(-4)}}{2}$$

$$x = \frac{12 \pm \sqrt{160}}{2}$$ ← Use a calculator to get the square root of 160.

$$x = \frac{12 \pm 4\sqrt{10}}{2}$$

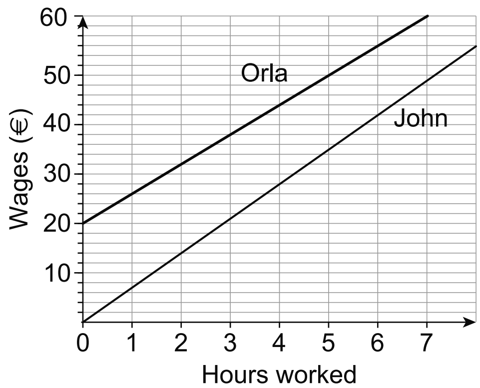
$$x = 6 \pm 2\sqrt{10}$$

6. (a)

John's Wages	Hours worked	1	2	3	4	5	6
	Wages	€7	€14	€21	€28	€35	€42

Orla's Wages	Hours worked	0	1	2	3	4	5
	Wages	€20	€26	€32	€38	€44	€50

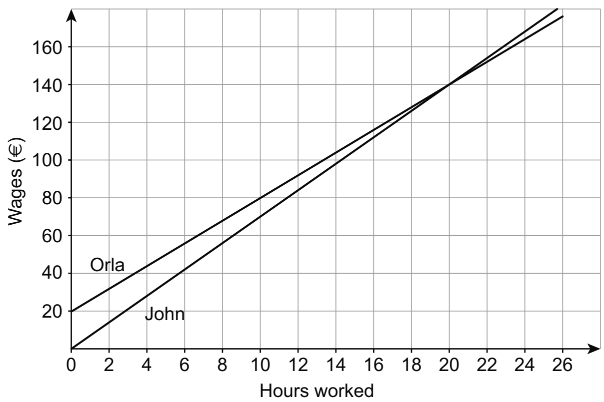
(b)



(c) John: $y = 7x$

Orla: $y = 6x + 20$

(d)



(e) $6 \times 3 = 18$ hours @ €6 per hour = €108 + €20 = €128

(f) From the graph, they earn the same for 20 hours work.

↑ Also: John 7(20) = €140
Orla: 6(20) + 20 = €140

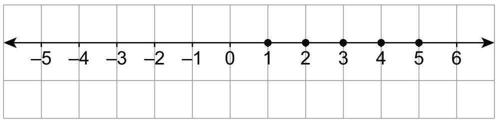
(g) For 25 hours, John earns $25 \times 7 =$ €175

Gross tax on €175 @ r% = (r ÷ 100) × 175 = €1·75r

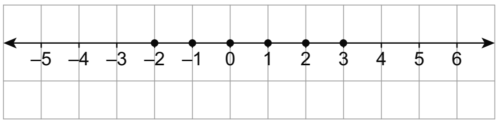
Tax payable = gross tax – tax credit = €1·75r – €30 = €5

€1·75r = €3r = €35 ÷ 1·75 = 20%

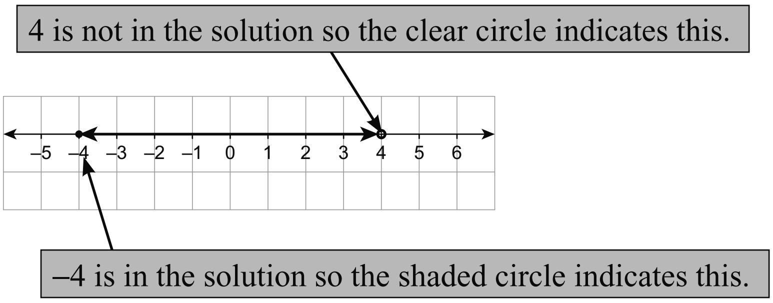
7. (a)



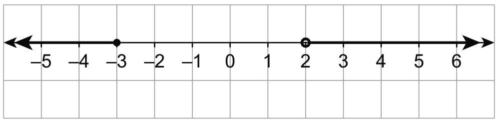
(b)



(c)



(d)



8. (a) $9{\cdot}1 \times 10^{-28}$ $1{\cdot}67 \times 10^{-24}$ $1{\cdot}6726 \times 10^{-24}$ $1{\cdot}6733 \times 10^{-24}$ $2{\cdot}67 \times 10^{-23}$

(b) $2(1{\cdot}67 \times 10^{-24}) + 2(1{\cdot}6726 \times 10^{-24}) + 2(9{\cdot}1 \times 10^{-28})$

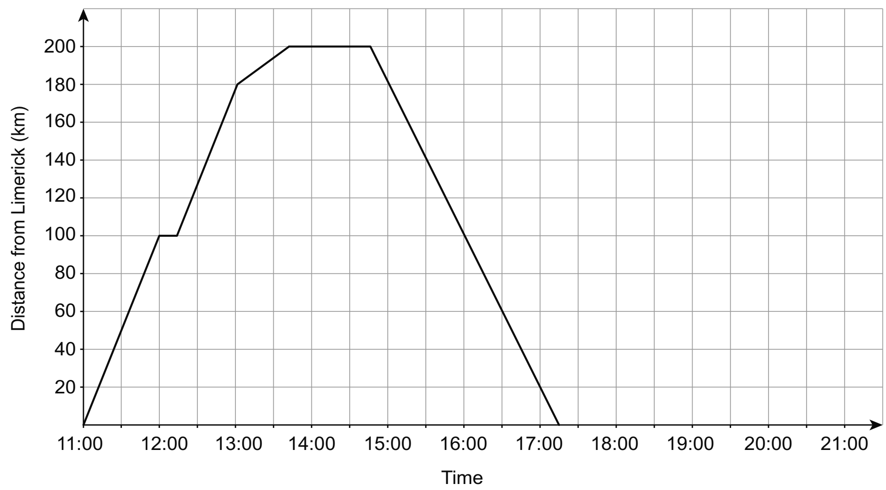
$3{\cdot}34 \times 10^{-24} + 3{\cdot}3452 \times 10^{-24} + 1{\cdot}82 \times 10^{-27}$

$6{\cdot}687 \times 10^{-24}$

(c) $\dfrac{2{\cdot}67 \times 10^{-23}}{1{\cdot}6733 \times 10^{-24}} = 15{\cdot}95$

16 times heavier

9.



(a) **(i)** 12:00

(ii) 15 minutes

(b) Speed = distance ÷ time = 100 km/h

(c) 20 km

(d) See graph

(e) 100 km = 62 miles ← As 1 km = 0·62 miles

$6{\cdot}7 \times 0{\cdot}264$ gallons in 62 miles

This is 1·7688 gallons in 62 miles.

So it is $62 \div 1{\cdot}7688 = 35$ miles/gallon.

10. a is 2 as then both lines will be parallel.

b can have any value provided $a = 2$ for both lines to be parallel.

11. $3 + (x + 2) + (2x) = 3x + (2x) + (3x + 1) + 2$

$$3x + 5 = 8x + 3$$

$$5x = 2$$

$$x = 0{\cdot}4 \text{ cm}$$

12. **(a)** $T = 2\pi\sqrt{\frac{L}{g}}$

$T = 2(3{\cdot}14)\sqrt{\frac{2}{9{\cdot}8}}$

$T = 2{\cdot}8$ seconds

(b) $T = 2\pi\sqrt{\frac{L}{g}}$ ← Square both sides to get rid of the square root.

$T^2 = 4\pi^2\frac{L}{g}$ ← Multiply both sides by g.

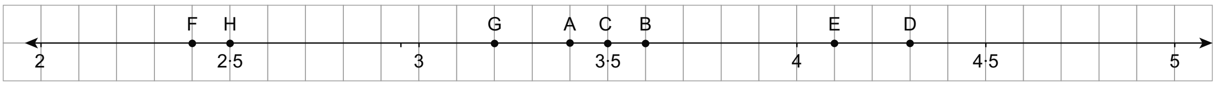
$gT^2 = 4\pi^2 L$ ← Divide both sides by $4\pi^2$.

$L = \frac{gT^2}{4\pi^2}$

13. (a)

	A	B	C	D	E	F	G	H
Number	3·427	$\sqrt{13}$	$\frac{7}{2}$	5sin 60°	410%	$(1{\cdot}54)^2$	$315{\cdot}2 \times 10^{-2}$	$\sqrt[3]{15}$
Decimal number	3·4	3·6	3·5	4·3	4·1	2·4	3·2	2·5

(b)



(c) **(i)** C ÷ H = 3·5 ÷ 2·5

$\frac{3{\cdot}5}{2{\cdot}5} = \frac{35}{25} = \frac{7}{5} = 1{\cdot}4$

(ii) C ÷ H $= \frac{7}{2} \div \sqrt[3]{15} = 1{\cdot}41918$

$= 1{\cdot}4192$

14. (a) $f(1) = 3 - 2(1)^2 = 1$ and $3f(-1) = 3[3 - 2(-1)^2] = 3$

$f(1) + 3f(-1) = 1 + 3 = 4$

(b) $f(-2) = 3 - 2(-2)^2 = -5 : 3f(-2) = 3[3 - 2(-2)^2] = -15$

Is $f(-2) > 3f(-2)$

$-5 > -15$

True as shown

15. (a) Yes

(b) Couples on the graph are (0, 0), (2, 10), (4, 16), (6, 18), (8, 16).

First differences between 0, 10, 16, 18 and 16 are 10, 6, 2, −2.

The second difference between these are −4, −4, −4 which is a constant.

(c) Two points on the graph are (2, 10) and (4, 16).

$$f(t) = at^2 + bt$$

$(2, 10)\ f(2) = a(2)^2 + b(2) = 10 \quad \rightarrow \quad 4a + 2b = 10$

$(4, 16)\ f(4) = a(4)^2 + b(4) = 16 \quad \rightarrow \quad 16a + 4b = 16$

$$4a + 2b = 10$$

$$16a + 4b = 16$$

$$4a + 2b = 10$$

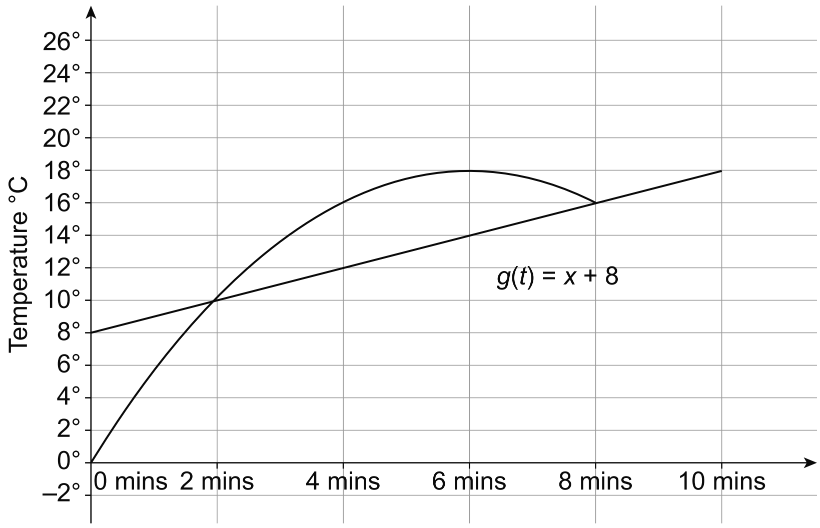
$$4a + b = 4$$

$$b = 6$$

$$a = -\frac{1}{2}$$

$$f(t) = -\frac{1}{2}t^2 + 6t$$

(d)



2 minutes and 8 minutes

Educate.ie Sample 2

Paper 1

1.

Description of number	Number	
Square Number	25	← 5^2
Reciprocal of a Whole Number	$\frac{1}{3}$	← 1 over 3
Prime Number	13	← A number with itself and 1 as the only factors
Irrational Number	$\sqrt{2}$	← A number that cannot be written as a simple fraction
Cubed Number	8	← 2^3
Negative Integer	−7	← A negative whole number
Index Form of a Number	3^4	← A number to a power
Estimate for Number pi	3·14	

2. $F = P(1 + i)^t$ ← See page 30 of *Formulae and Tables*.

$F = 5000(1{\cdot}025)^5$

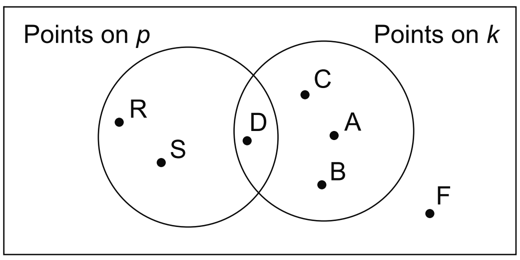
$F =$ €5657·04

Interest = €657·04

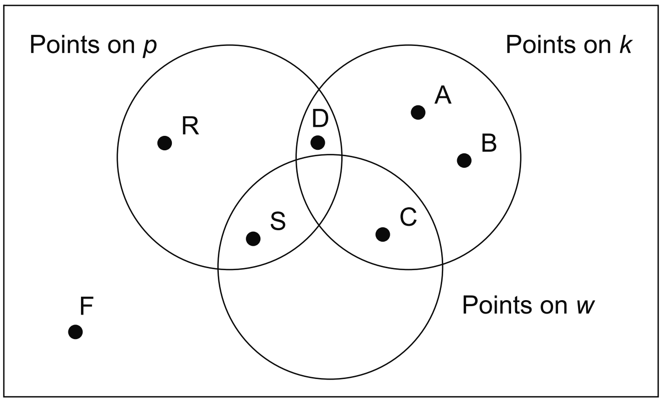
Tax on interest = 25% of €657·04 = €164·26

Value of investment after tax = €5657·04 − €164·26 = €5492·78

3. **(a)**



(b)



4. (a) $5(x + 3) - 3[4 - 3(x - 2)] + 2$

$5x + 15 - 3[4 - 3x + 6] + 2$

$5x + 15 - 12 + 9x - 18 + 2$

$14x - 13$

(b) $\frac{5}{2x+4} - \frac{3}{x}$

Get the common denominator and be careful with the minus between the two fractions.

$\frac{5(x) - 3(2x + 4)}{(2x + 4)(x)}$

$\frac{5x - 6x - 12}{2x^2 + 4x}$

$\frac{-x - 12}{2x^2 + 4x}$

When $x = \frac{1}{2}$

$\frac{-x - 12}{2x^2 + 4x}$, becomes

$\frac{-\left(\frac{1}{2}\right) - 12}{2\left(\frac{1}{2}\right)^2 + 4\left(\frac{1}{2}\right)}$

Use brackets when substituting.

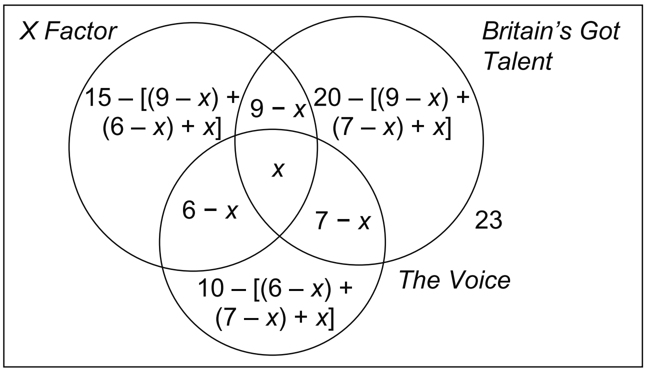
$\frac{-12\frac{1}{2}}{2\left(\frac{1}{4}\right) + 4\left(\frac{1}{2}\right)}$

$\frac{-12\frac{1}{2}}{\frac{1}{2} + 2}$

$\frac{-12{\cdot}5}{2{\cdot}5}$

-5

5. **(a)**



(b) $20 - [(9 - x) + (7 - x) + x] = 20 - [16 - x] = 4 + x$

$15 - [(9 - x) + (6 - x) + x] = 15 - [15 - x] = x$

$10 - [(6 - x) + (7 - x) + x] = 10 - [13 - x] = -3 + x$

$(4 + x) + (x) + (-3 + x) + x + (6 - x) + (9 - x) + (7 - x) + 23 = 50$

$1 + 4x + 22 - 3x + 23 = 50$

$x + 46 = 50$

$x = 4$

6. **(a)** Salary = €65 000 Tax @ 20% = 20% of €32 800 = €6560

Tax @ 41% = 41% of (€65 000 − €32 800) = €13 202

Gross tax = €6560 + €13 202 = €19 762

Net tax = gross tax – tax credit = €19 762 − €2350 = €17 412

(b) Salary = €65 000 PRSI @ 7·5% = 7·5% of €65 000 = €4875

Net salary = gross salary – (net tax + PRSI)

Net salary = €65 000 – (€17 412 + €4875)

Net salary = €65 000 – (€22 287)

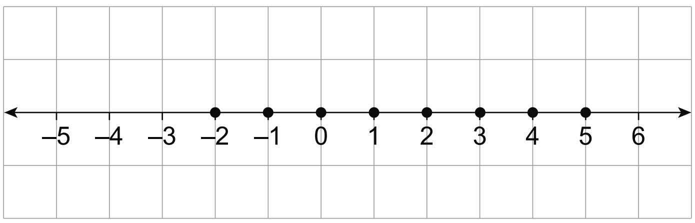
Net salary = €42 713

7. **(a)** $5 \geq 3 - x \geq -2$

$5 - 3 \geq 3 - x - 3 \geq -2 - 3$ ← Subtracting 3 from each part

$2 \geq -x \geq -5$

$-2 \leq x \leq 5$ ← When multiplying by minus, don't forget to change the inequality signs.

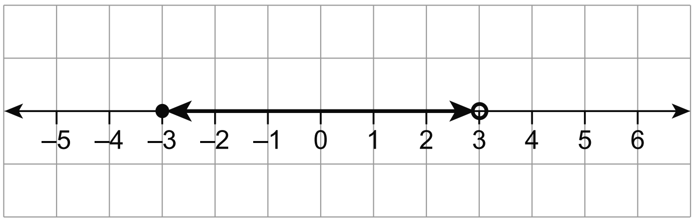


(b) $-3 \le 3 + 2x < 9$

$-3 - 3 \le 3 + 2x - 3 < 9 - 3$ ← Subtracting 3 from each part

$-6 \le 2x < 6$

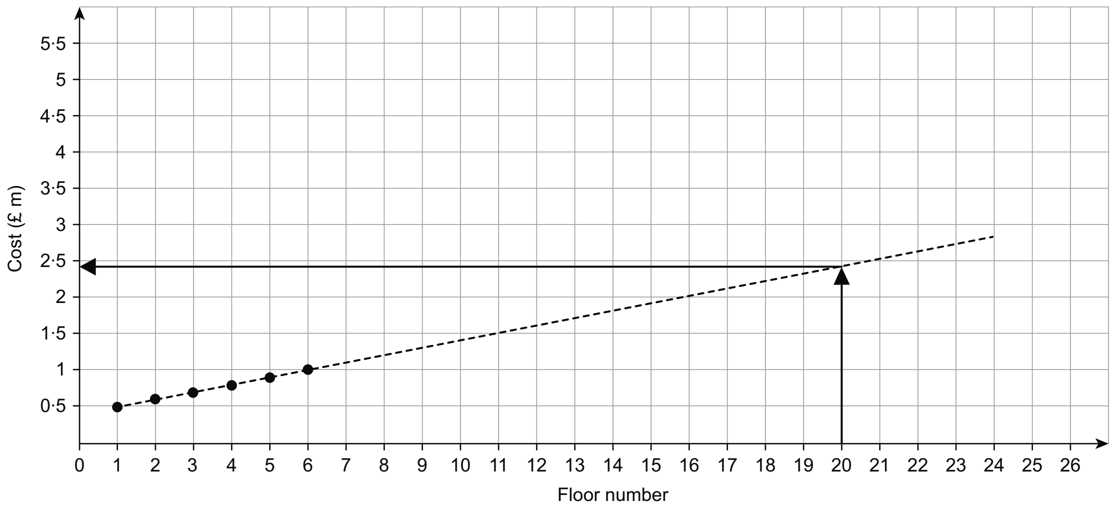
$-3 \le x < 3$ ← Divide each part by 2.



8. (a)

Floor	1^{st}	2^{nd}	3^{rd}	4^{th}	5^{th}	6^{th}
Cost (£ millions)	0·5	0·6	0·7	0·8	0·9	1·0

(b)



(c) Estimation: 2·4 million euro

(d)

1^{st} Floor = 0·5 + 0 = 0·5 + 0(0·1)

2^{nd} Floor = 0·5 + 0·1 = 0·5 + 1(0·1)

3^{rd} Floor = 0·5 + 0·1 + 0·1 = 0·5 + 2(0·1)

4^{th} Floor = 0·5 + 0·1 + 0·1 + 0·1 = 0·5 + 3(0·1)

5^{th} Floor = 0·5 + 0·1 + 0·1 + 0·1 + 0·1 = 0·5 + 4(0·1)

n^{th} Floor = $0{\cdot}5 + (n - 1)0{\cdot}1$

Cost for n^{th} Floor = $0{\cdot}1n + 0{\cdot}4$

(e) **(i)** $T = \frac{n}{2}[2a + (n-1)d]$

$n = 28: \quad a = 0{\cdot}5: \quad d = 0{\cdot}1$

$T = \frac{n}{2}[2a + (n-1)d]$

$T = \frac{28}{2}[2(0{\cdot}5) + (28-1)(0{\cdot}1)]$

$T = 14[1 + 2{\cdot}7]$

$T = 14[3{\cdot}7]$

$T = 51{\cdot}8$ million euro

(ii) $T = \frac{n}{2}[2a + (n-1)d] = 20$

$n = ?: \quad a = 0{\cdot}5: \quad d = 0{\cdot}1$

$\frac{n}{2}[2(0{\cdot}5) + (n-1)(0{\cdot}1)] = 20$

$\frac{n}{2}[1 + 0{\cdot}1n - 0{\cdot}1] = 20$

$n[1 + 0{\cdot}1n - 0{\cdot}1] = 40$

$0{\cdot}9n + 0{\cdot}1n^2 = 40$

$n^2 + 9n - 400 = 0$

$(n + 25)(n - 16) = 0$

$n = 16$

16 Floors

(f) Total cost = £51·8 million

€1 = £0·87

€x = £51·8 million

€x = £51·8 million ÷ 0·87

€x = 59·54 million euro

9. **(a)** $4x^2 - 9x = 0$

$x(4x - 9) = 0$

$x = 0 \quad \text{or} \quad 4x - 9 = 0$

$x = 0 \quad \text{or} \quad x = \frac{9}{4}$

(b) $4x^2 - 9 = 0$

$(2x + 3)(2x - 3) = 0$

$2x + 3 = 0 \quad \text{or} \quad 2x - 3 = 0$

$x = -\frac{3}{2} \quad \text{or} \quad x = \frac{3}{2}$

(c) $4x^2 - 16x - 9 = 0$

$(2x + 1)(2x - 9) = 0$

$2x + 1 = 0 \quad \text{or} \quad 2x - 9 = 0$

$x = -\frac{1}{2} \quad \text{or} \quad x = \frac{9}{2}$

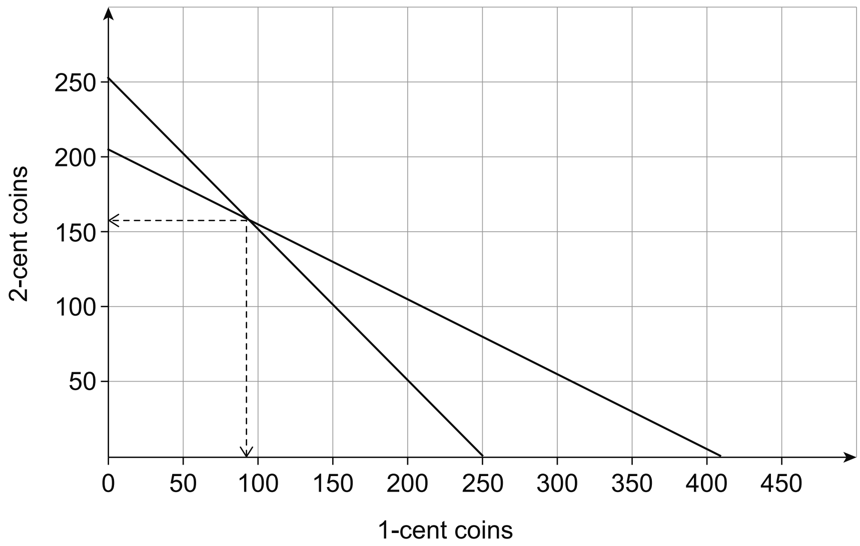
10.

Graph	A	B	C	D	E	F
Story	2	1	5	6	3	4

11. (a) $x + y = 250$

$x + 2y = 410$

(b)



90 one-cent coins and 160 two-cent coins

(c) $x + y = 250$

$\underline{x + 2y = 410}$

$-y = -160 \quad \therefore\ y = 160$

$x + 160 = 250 \quad \therefore\ x = 90$

12. (a) (i) $1{\cdot}534 \times 10^8$

(ii) $2{\cdot}435 \times 10^{-6}$

(b) Given that $(4x^3y^2) \div (xy)^2 = p$

Simplifying we get $\dfrac{4x^3y^2}{x^2y^2} = p$

$\Rightarrow 4x = p$

The question asks to show that $x^2y^3 \div 4(xy)^3$ is the reciprocal of p

Simplifying $\dfrac{x^2y^3}{4x^3y^3}$

$\Rightarrow \dfrac{1}{4x} = \dfrac{1}{p}$ which is the reciprocal of p

13. (a) $a^3 - 2a^2$ ← Common factor a^2

$a^2(a - 2)$

(b) $a^2 - a - b - b^2$

$(a^2 - b^2) + (-a - b)$ ← Grouping

$(a + b)(a - b) - (a + b)$

$(a + b)(a - b - 1)$

14.

	$3x$	$+4$
$2x$	$6x^2$	
$+3$		$+12$

or
Use long division
or
Factorise $6x^2 + 17x + 12$

$3x + 4$

15. (a) $\{2, 4, 6, 8\}$

(b) $\{4, 6, 8, 10\}$

(c) $\{3, 4, 6, 8, 10\}$

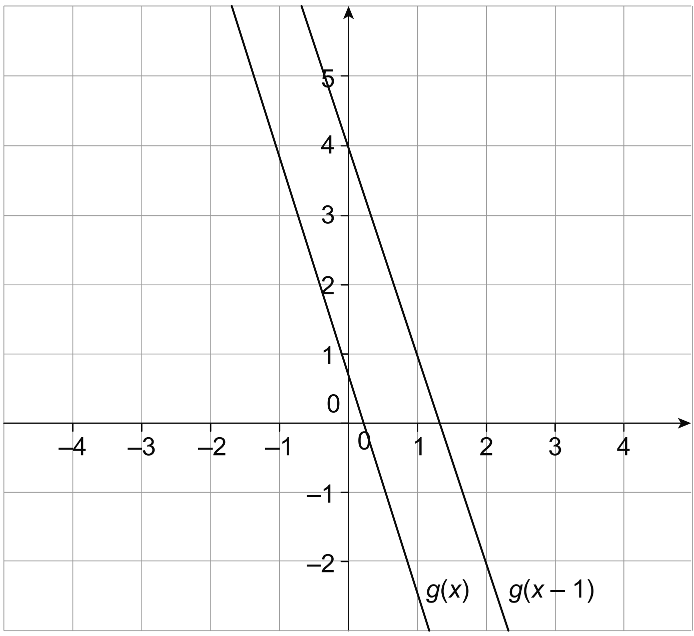
16. (a) $g(x) = 1 - 3x$

$g(x - 1) = 1 - 3(x - 1)$

$= 1 - 3x + 3$

$= 4 - 3x$

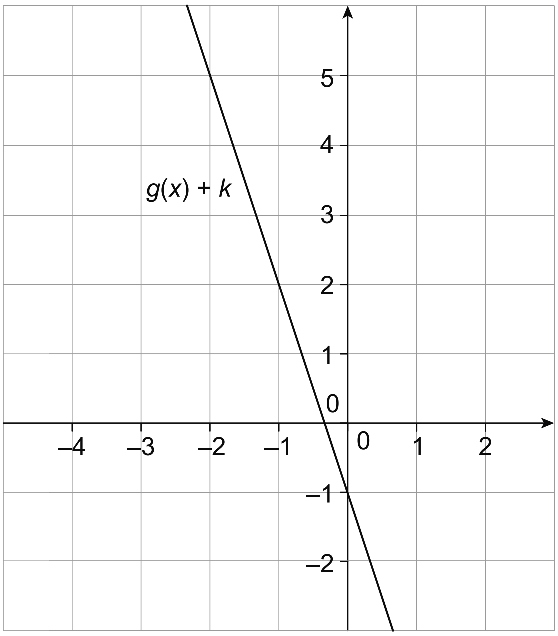
(b)



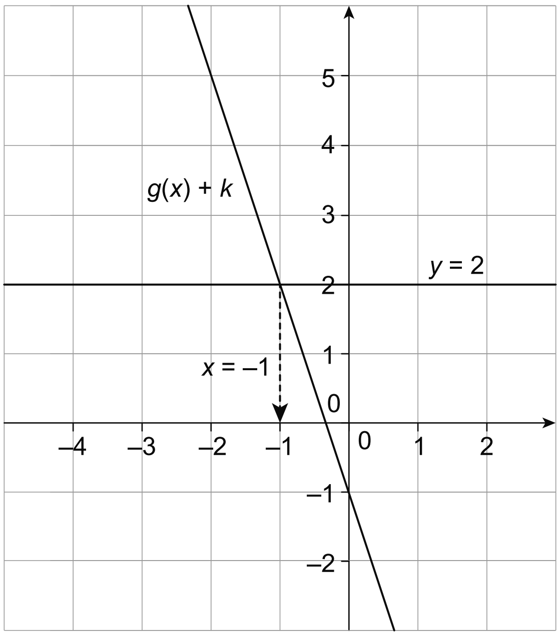
When graphed they form parallel lines.

(c) (i) $g(x) = 1 - 3x + k = 1 - 3x - 2$

$k = -2$



(ii)



Educate.ie Sample 3

Paper 1

1. **(a)** $432 = 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3$

$522 = 2 \times 3 \times 3 \times 29$

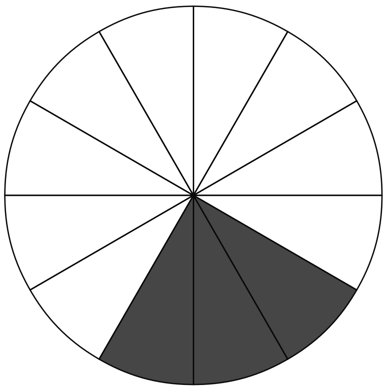
If you have a Casio Calculator (NATURAL– V.P.A.M) to find the prime factors of 432.

Type in 432 then press the equals button.

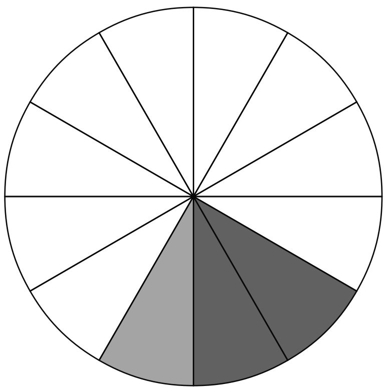
Then press  and the prime factors will show up as $2^4 \times 3^3$.

(b) Highest common factor = 3

2. **(a)**



(b)



$\frac{2}{3} \times \frac{1}{4} = \frac{1}{6}$ means $\frac{2}{3}$ of $\frac{1}{4}$ of the full circle and this is, from the diagram, $\frac{1}{6}$ of the circle.

3. **(a)** Total paid = \$329

€1 = \$1·38

€x = \$329

Use ratio €1 : \$1·38 = €x : \$329

$$\frac{1}{1{\cdot}38} = \frac{x}{329}$$

Therefore €x = (329 × 1) ÷ 1·38

= €238·41

(b) Number of months = May, June, July = 3

$F = P(1 + i)^t$

See page 30 of *Formulae and Tables*.

$F = 238{\cdot}41(1{\cdot}0129)^3$

F = €247·76

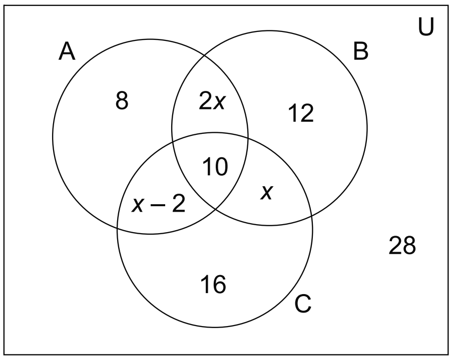
4. **(a)** $8 + 2x + 12 + 10 + x - 2 + x + 16 + 28 = 200$

$4x + 56 + 16 = 200$

$4x + 72 = 200$

$4x = 128$

$x = 32$



(b) $8 + 30 + 16 + 28 = 82$

5. **(a)** 16

(b) 6

(c) 3

(d) 6

(e) 17

(f) 15

6. Copper: 60% of 4·25 g = 2·55 g ⟵ 4·25 × 0·6

Copper: 20% of 4·25 g = 0·85 g ⟵ 4·25 × 0·2

Copper: 20% of 4·25 g = 0·85 g ⟵ 4·25 × 0·2

7.

Statement	Always True	Never True	Example
If $a, b \in Z$ with both a and $b < 0$, then $(a + b) \in N$		✓	If $a = -2$ and $b = -3$: $(-2 - 3) = -5 \notin N$
If $a, b \in Z$ with both a and $b > 0$, then $(a + b) \in N$	✓		If $a = 2$ and $b = 3$: $(2 + 3) = 5 \in N$
If $a, b \in Z$ with $a < b$, then $(a - b) < 0$	✓		If $a = -2$ and $b = -1$: $(-2 + 1) = -1$ True.
If $a, b \in Z$ with both a and $b < 0$, then $a \times b \in N$	✓		If $a = -2$ and $b = -3$: $(-2 \times -3) = 6 \in N$
If $a, b \in Z$ with both a and $b < 0$, then $(a^2 + b^2) < 0$		✓	If $a = -2$ and $b = -3$: $(-2^2 + -3^2) = 13$ is not less than zero.

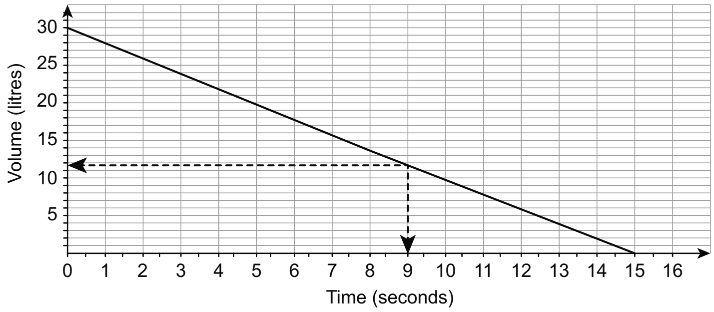
8. **(a)**

Time (sec)	0	2	4	6	8	10	12	14
Volume (l)	30	26	22	18	14	10	6	2

First change 4 4

First change (difference) is a constant therefore linear.

(b)



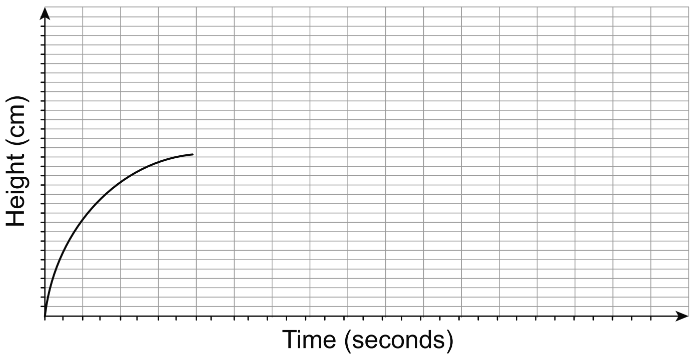
(c) 12 litres ← See broken lines on graph above.

(d) 2 litres/sec ← Rate of change = Slope of Line = $\frac{\text{Rise}}{\text{Run}} = \frac{-2}{1} = -2$

(e) Volume = 30 – 2 × (number of seconds)

V = 30 – 2s

(f)



(g) Volume of cone = $\frac{1}{3}\pi r^2 h$ ← See page 10 of *Formulae and Tables*.

$\frac{1}{3}(3)h^3 = 90\%$ of 30 litres

$h^3 = 27$ litres

$h = \sqrt[3]{27} = 3$ metres

9. (a) $n + (n + 1) + (n + 2)$

$\Rightarrow n + n + 1 + n + 2$

$\Rightarrow 3n + 3$

This expression is divisible by 3. ← $(3n + 3) \div 3 = n + 1$

(b) $(n - 2) + (n - 1) + n$

$\Rightarrow n - 2 + n - 1 + n$

$\Rightarrow 3n - 3$

This expression is divisible by 3. ← $(3n - 3) \div 3 = n - 1$

(c) $n + (n + 1) + (n + 2) + (n + 3)$

$\Rightarrow n + n + 1 + n + 2 + n + 3$

$\Rightarrow 4n + 6$

This expression is not evenly divisible by 4.

10. (a) (i) $a^2 - ac - ab + bc$ ← Grouping

$(a^2 - ac) + (-ab + bc)$

$a(a - c) - b(a - c)$

$(a - b)(a - c)$

(ii) $5x^2 + 5x - 30$ ← Quadratic factors

$5(x^2 + x - 6)$

$5(x + 3)(x - 2)$

(iii) $4x^2 - y^2$ ← Difference of two squares

$(2x + y)(2x - y)$

(b) $x^2 - y^2 = 24$ ← Difference of two squares

$\Rightarrow (x + y)(x - y) = 24$

As $x + y = 3$

$\Rightarrow (3)(x - y) = 24$

$\Rightarrow x - y = 8$

$\therefore\ 2x - 2y = 16$

11. (a) (i) $2x^2 = 5x$

$2x^2 - 5x = 0$

$x(2x - 5) = 0$

$x = 0$ or $2x - 5 = 0$

$x = 0$ or $x = \frac{5}{2}$

(ii) $x^2 + 4 = 8x - 8$

$x^2 - 8x + 12 = 0$

$(x - 2)(x - 6) = 0$

$x - 2 = 0$ or $x - 6 = 0$

$x = 2$ or $x = 6$

(iii) $3 - 4x - 7x^2 = 0$

$7x^2 + 4x - 3 = 0$

$(7x - 3)(x + 1) = 0$

$7x - 3 = 0$ or $x + 1 = 0$

$x = \frac{3}{7}$ or $x = -1$

(b) $2x^2 + 2x - 15 = 0$

$a = 2, b = 2, c = -15$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-2 \pm \sqrt{(2)^2 - 4(2)(-15)}}{2(2)}$$

$$x = \frac{-2 \pm \sqrt{4 + 120}}{4}$$

$$x = \frac{-2 \pm \sqrt{124}}{4}$$

$$x = \frac{-2 \pm 2\sqrt{31}}{4}$$

$$x = \frac{-1 \pm \sqrt{31}}{2}$$

$$x = \frac{-1 \pm 5{\cdot}5677}{2}$$

$$x = \frac{-1 + 5{\cdot}5677}{2} \quad \text{or} \quad x = \frac{-1 - 5{\cdot}5677}{2}$$

$x = 2{\cdot}2838$ or $x = -3{\cdot}2838$

$x = 2{\cdot}28$ or $x = -3{\cdot}28$

12. (a) $\dfrac{1000 \text{ cm}}{x}$ ← 10 m = 1000 cm

(b) $\dfrac{1000}{x + 30}$

(c) $\dfrac{1000}{x} = \dfrac{1000}{x + 30} + 75$

$$\frac{1000}{x} = \frac{1000}{x + 30} + 75$$

$$\frac{1000(x + 30)}{x(x + 30)} = \frac{1000x + 75(x)(x + 30)}{x(x + 30)}$$

$1000x + 30\,000 = 1000x + 75x^2 + 2250x$

$75x^2 + 2250x - 30\,000 = 0$

$x^2 + 30x - 400 = 0$

$(x + 40)(x - 10) = 0$

$x + 40 = 0$ or $x - 10 = 0$

$x = -40$ or $x = 10$

10 cm

13. (a) 1×10^{-9}

(b) $\dfrac{5 \times 10^{-3}}{1 \times 10^{-9}} = \dfrac{5}{10^{-6}} = 5 \times 10^{6} = 5$ million

(c)

Day 1	$4^3 = (2^2)^3 = 2^6$
Day 2	$2 \times 2^6 = 2^7$
Day 3	$2 \times 2^7 = 2^8$
Day 4	$2 \times 2^8 = 2^9$
Day 5	$2 \times 2^9 = 2^{10}$ Critical Value

14. $f(x) = 2^{\cdot x-1} = 54$

$3^{x-1} = 27$

$3^{x-1} = 3^3$

$x - 1 = 3$

$x = 4$

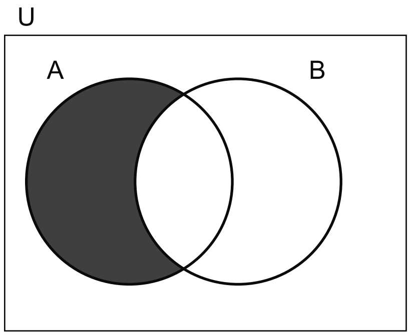
15.

Functions	2^x	2^{x+2}	$5\cdot 2^x$	$2^x - 2$	$3\cdot 2^x$	$2^x + 3$
Graph	1	6	4	3	2	5

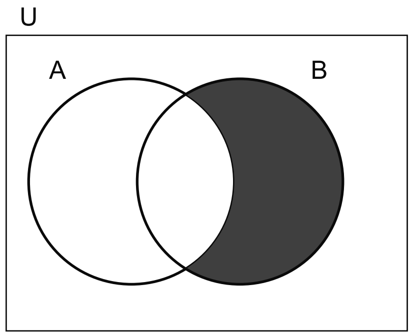
Educate.ie Sample 4

Paper 1

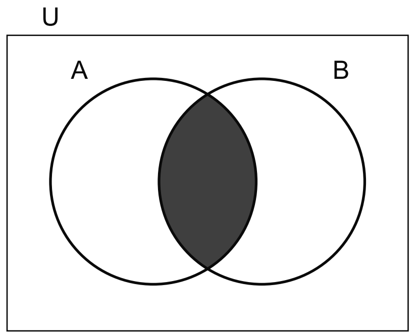
1.



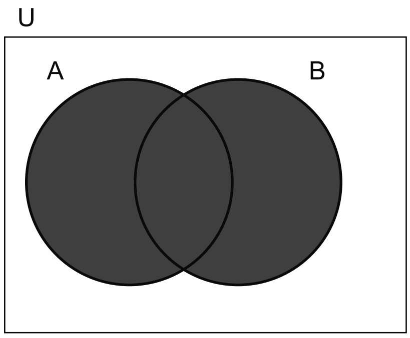
A/B



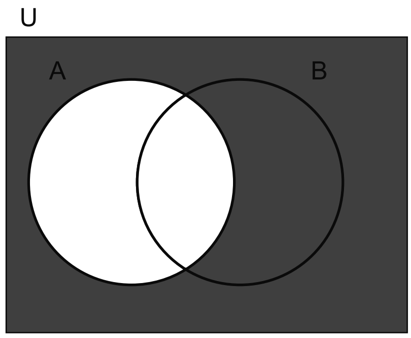
B/A



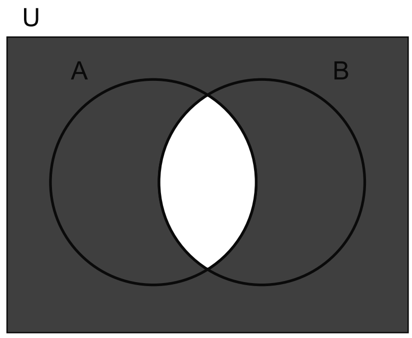
A ∩ B



A ∪ B



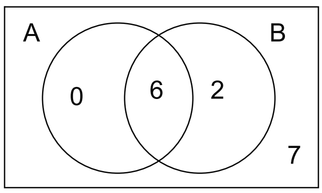
A′ or A^c



(A ∩ B)′ or (A ∩ B)^c

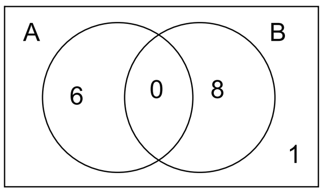
2. **(a)** 6

See diagram for explanation. →



(b) 0

See diagram for explanation. →



(c) 2 ← See diagram in part **(a)**.

3. **(a)** $x^2 - 10x - 24$ ← Quadratic factors

$(x - 12)(x + 2)$

(b) $abx + 2y - by - 2ax$ ← Grouping (rearrange first)

$abx - 2ax + 2y - by$

$ax(b - 2) - y(-2 + b)$

$(b - 2)(ax - y)$

(c) $x^4 - 16$ ← Difference of two squares

$(x^2)^2 - (4)^2$

$(x^2 + 4)(x^2 - 4)$ ← Difference of two squares

$(x^2 + 4)(x + 2)(x - 2)$

4. **(a)** $p + 1 \geq p, p \in \mathbb{Z}$ True ☑ False ☐

Reason: When you add 1 to any integer you will always get a larger integer.

(b) $p^2 + 1 \geq p, p \in \mathbb{Z}$ True ☑ False ☐

Reason: p^2 is always greater than p for $p \in \mathbb{Z}$ because when you square a positive or negative number you will always get a larger positive number.

(c) $p + 1 \geq p, p \in \mathbb{R}$ True ☑ False ☐

Reason: When you add 1 to any real number (including a fraction) you will always get a larger real number.

(d) $2p \geq p, p \in \mathbb{Z}$ True ☐ False ☑

Reason: If p is negative, this statement is not true.

(e) $2p \geq p, p \in \mathbb{N}$ True ☑ False ☐

Reason: p can't be negative so this statement is always true.

5. **(a)** 3% of €15 000 = €450

Balance = €65 000 − €15 000 = €50 000

10% of €50 000 = €5000

Total Levy = €450 + €5000 = €5450

(b) 2% of €10 036 = €200·72

4% of €5980 = €239·20

Balance = €65 000 – (€10 036 + €5980) = €48 984

7% of €48 984 = €3428·88

Total USC = €200·72 + €239·20 + €3428·88 = €3868·80

(c) 20% of €41 800 = €8360

Balance = €65 000 – €41 800 = €23 200

41% of €23 200 = €9512

Total gross tax = €8360 + €9512 = €17 872

Net tax = gross tax – tax credits

Net tax = €17 872 – €4950 = €12 922

(d) Net salary = €65 000 – (pension levy + USC + income tax)

Net salary = €65 000 – (€5450 + €3868·80 + €12 922)

Net salary = €42 759·20

6.

Length (cm)	Width (cm)	Area (cm^2)
0	25	0
2	23	46
4	21	84
6	19	114
8	17	136
10	15	150
12	13	156
14	11	154
16	9	144
18	7	126
20	5	100
22	3	66
24	1	24

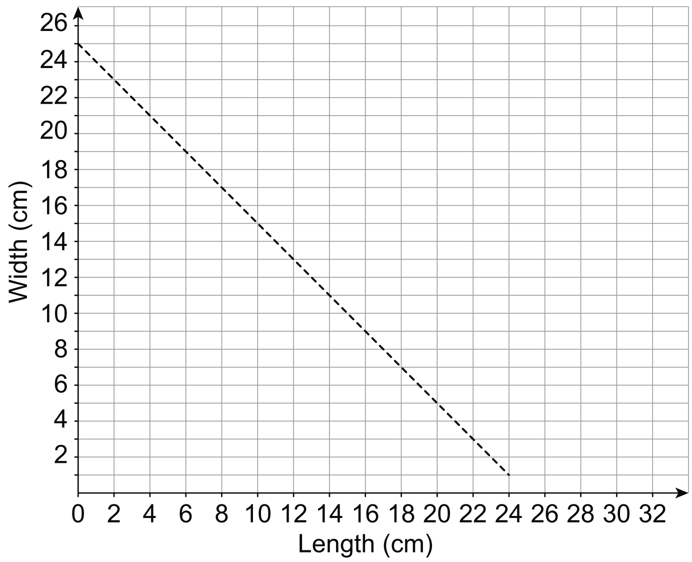
(a) **(i)** Length against width: Prediction: Linear graph

(ii) Length against area: Prediction: Quadratic graph

Explanation for **(i)** The first difference (change) is a constant.

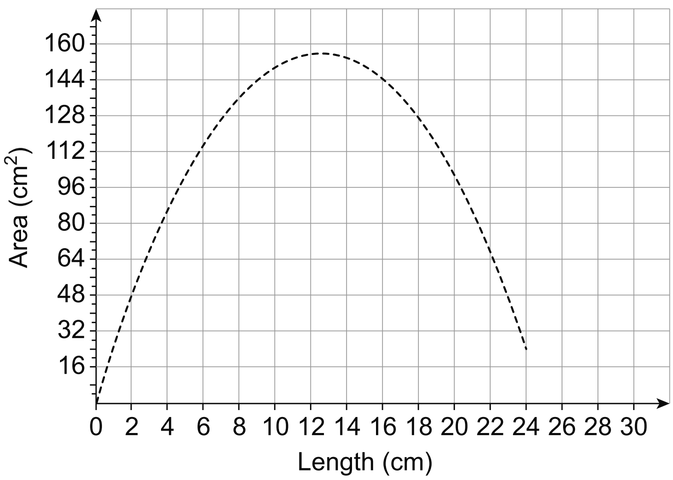
Explanation for **(ii)** The second difference (change) is a constant.

(b)



(c) Width + length = 25 cm

(d)



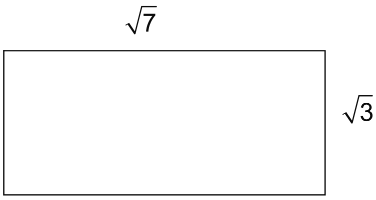
(e) Maximum area = 156 cm^2

See page 23 of *Formulae and Tables*.

7. (a)

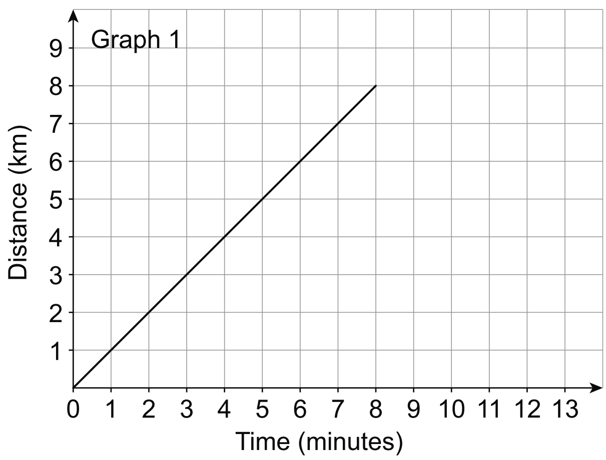
Number/Set	**Natural $\mathbb{N}$**	**Integers $\mathbb{Z}$**	**Rational $\mathbb{Q}$**	**Irrational $\mathbb{R}\backslash\mathbb{Q}$**	**Real $\mathbb{R}$**
7	✓	✓	✓		✓
−12		✓	✓		✓
$\frac{1}{4}$			✓		✓
2π				✓	✓
$\sqrt{3}$				✓	✓
0·001			✓		✓
2×10^3	✓	✓	✓		✓
$\sqrt[3]{64}$	✓	✓	✓		✓
$0{\cdot}33\dot{3}$			✓		✓

(b)



Area = $\sqrt{7} \times \sqrt{3} = \sqrt{21}$

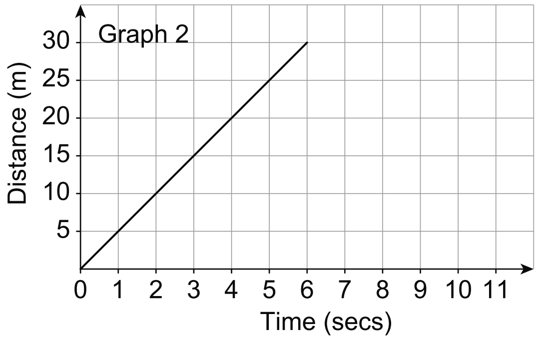
8.



(a) Distance 8 km

(b) Time 8 minutes

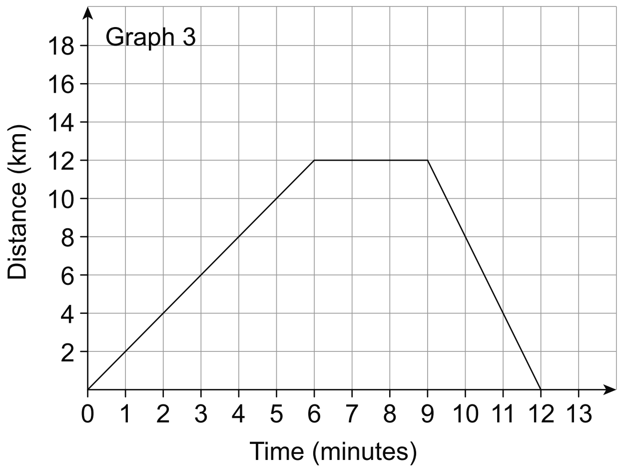
(c) Speed 8 km in 8 minutes = 60 km/h



(a) Distance 30 m

(b) Time 6 seconds

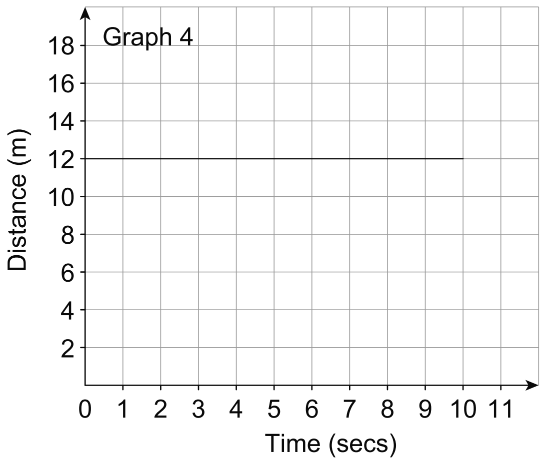
(c) Speed 30 metres in 6 seconds = 18 km/h



(a) Distance 12 + 12 = 24 km

(b) Time 12 minutes

(c) Speed 24 km in 12 minutes = 120 km/h



(a) Distance 0 m

(b) Time 10 seconds

(c) Speed 0 km/h (at rest)

9. (a)

Motor tax		= €302
Insurance		= €908
Petrol	(13 cent)(16 000 km)	= €2080
Oil	(0·16 cent)(16 000 km)	= €25·60
Tyres	(1·8 cent)(16 000 km)	= €288
Servicing	(1·8 cent)(16 000 km)	= €288
Repairs	(6·7 cent)(16 000 km)	= €1072
Total		= €4963·60

(b)

Motor tax		= €390
Insurance		= €940
Petrol	(13 cent)(18 000 km)	= €2340
Oil	(0·16 cent)(18 000 km)	= €28·80
Tyres	(1·82 cent)(18 000 km)	= €327·60
Servicing	(2·2 cent)(18 000 km)	= €396
Repairs	(6·6 cent)(18 000 km)	= €1188
Total		= €5610·40

(c)

Cost: 2013		**Cost: 2014**	
Motor tax	€302	Motor tax	€390
Insurance	€908	Insurance	€940
Total	€1210	Total	€1330

Increase = €1330 − €1210 = €120

% Increase = (120 ÷ 1210) × 100 = 9.9% = 10%

10. (a) $\frac{2x}{5} - \frac{2y}{3} = 3$ Simplify $\Rightarrow \frac{6x - 10y}{15} = \frac{45}{15}$ $\Rightarrow 6x - 10y = 45$

$\frac{x-3}{4} - \frac{y-1}{3} = \frac{2}{3}$ Simplify $\Rightarrow \frac{(3x-9)-(4y-4)}{12} = \frac{8}{12}$ $\Rightarrow 3x - 4y = 13$

$6x - 10y = 45$

$\underline{3x - 4y = 13}$

$6x - 10y = 45$

$\underline{6x - 8y = 26}$

$-2y = 19$ $\Rightarrow y = -9{\cdot}5$

$6x - 10y = 45$

$6x - 10(-9{\cdot}5) = 45$

$95 + 6x = 45$

$x = -\frac{25}{3}$

(b) $\frac{2x}{5} - \frac{2y}{3} = 3$

When $x = -\frac{25}{3}$ and $y = -9{\cdot}5$

$\frac{2\left(\frac{-25}{3}\right)}{5} - \frac{2(-9{\cdot}5)}{3} = 3$

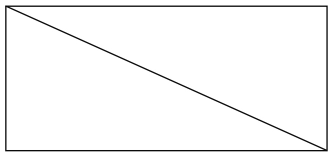
$-\frac{10}{3} + \frac{19}{3} = 3$

$\frac{9}{3} = 3$

$3 = 3$

11. (a) Area = $(x + 1)(x) = 90$

$$x^2 + x = 90$$
$$x^2 + x - 90 = 0$$
$$(x + 10)(x - 9) = 0$$
$$x = 9$$

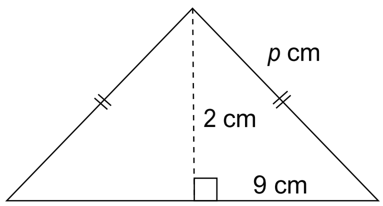


$(\text{Diagonal})^2 = 9^2 + 10^2$

Diagonal = $\sqrt{181}$ cm

(b) Area = $\frac{1}{2}(4x + 2)\left(\frac{x}{2}\right) = 18$

$$2x^2 + x = 36$$
$$2x^2 + x - 36 = 0$$
$$(2x + 9)(x - 4) = 0$$
$$x = 4$$



$p^2 = 9^2 + 2^2$

$p = \sqrt{85}$

Perimeter = $\sqrt{85} + \sqrt{85} + 18$

Perimeter = $18 + \sqrt{340}$ $\quad (2\sqrt{85} = \sqrt{340})$

12. (a) $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ ← See page 20 in *Formulae and Tables*.

$a = 1, \quad b = -3, \quad c = -8$

$$x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(1)(-8)}}{2(1)}$$

$$x = \frac{3 \pm \sqrt{9 + 32}}{2}$$

$$x = \frac{3 \pm \sqrt{41}}{2}$$

$$x = \frac{3 \pm 6{\cdot}4031}{2}$$

$x = \dfrac{3 + 6{\cdot}4031}{2}$ or $x = \dfrac{3 - 6{\cdot}4031}{2}$

$x = 4{\cdot}70155$ or $x = -1{\cdot}70155$

$x = 4{\cdot}70$ or $x = -1{\cdot}70$

(b) $x = 4{\cdot}70155$ or $x = -1{\cdot}70155$

$x = 4{\cdot}70$ or $x = -1{\cdot}70$

$(2t + 3)^2 - 3(2t + 3) - 8 = 0$

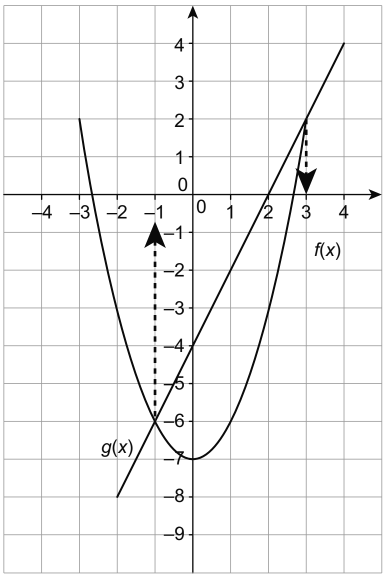
$x = 2t + 3$

$t = \dfrac{x - 3}{2}$

$t = \dfrac{4{\cdot}70155 - 3}{2}$ or $t = \dfrac{-1{\cdot}70155 - 3}{2}$

$t = 0{\cdot}9$ or $t = -2{\cdot}4$

13. (a) (i) Drawn below
(ii) Drawn below



(b) Where the two graphs intersect: $x = -1$ and $x = 3$

(c) $x^2 - 7 = 2x - 4$

$x^2 - 2x - 3 = 0$

$(x + 1)(x - 3)$

$x = -1 \qquad x = 3$

14. (a) A and B are the roots of $x^2 - 8x + 12 = 0$

$(x - 6)(x - 2) = 0$

A(2, 0)   B(6, 0)

(b) A(2, 0) is on the function $k(x)$

$y = -x + b$

$0 = -2 + b$

$b = 2$

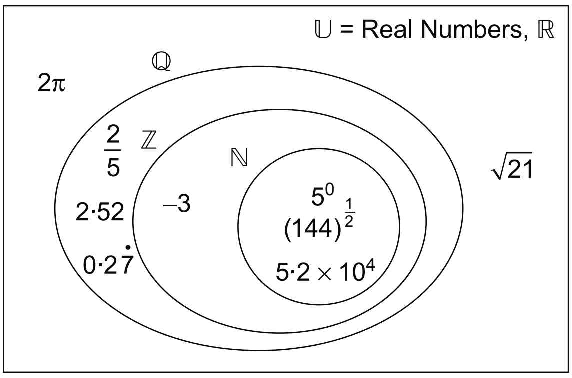
The function $k(x)$ is $-x + 2$

Educate.ie Sample 5

Paper 1

1. **(a)** $0\cdot\dot{6}$ is a rational number because it can be expressed as a fraction $\frac{2}{3}$.

(b)



See page 23 of *Formulae and Tables*.

(c) $-3,\ 0\cdot2\dot{7},\ \frac{2}{5},\ 5^0,\ 2\cdot52,\ \sqrt{21},\ 2\pi,\ (144)^{\frac{1}{2}},\ 5\cdot2 \times 10^4$

2. **(a)** 2 : 5 : 13 means 20 parts in total $\frac{2}{20}, \frac{5}{20}$ and $\frac{13}{20}$

$\frac{1}{20}$ of €13 000 = €650

$\frac{2}{20}$ of €13 000 = €650 × 2 = €1300

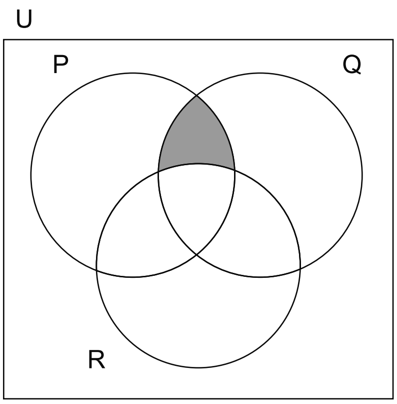
$\frac{5}{20}$ of €13 000 = €650 × 5 = €3250

$\frac{13}{20}$ of €13 000 = €650 × 15 = €8450

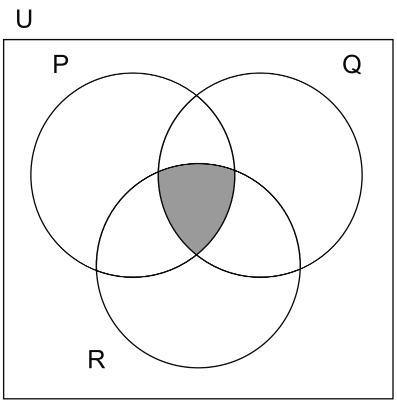
(b) Tax @ 2·5% on €8450 = 8450(0·025) = €211·25

After tax the person had €8450 − €211·25 = €8238·75

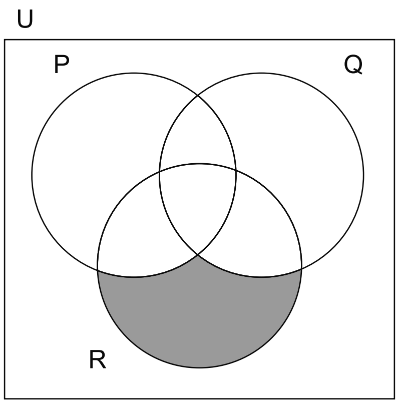
3. **(a)**



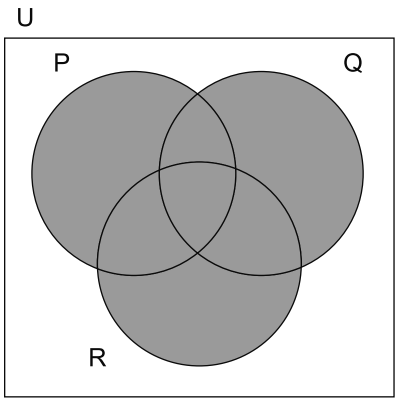
$(P \cap Q) / R$

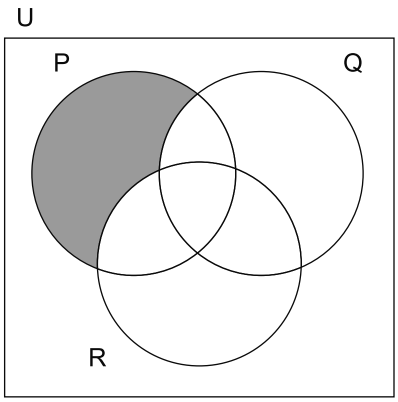


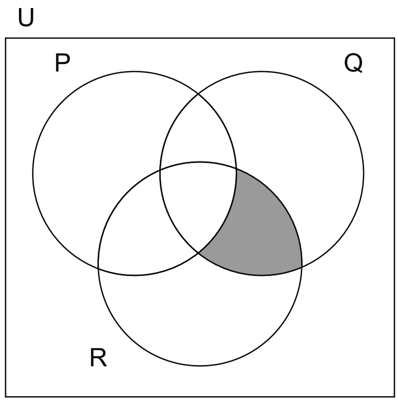
$(P \cap Q \cap R)$



$R / (P \cup Q)$

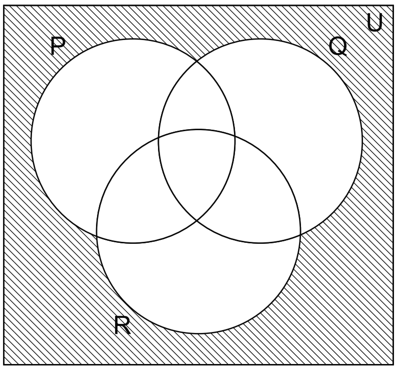






$(P \cup Q \cup R)$ $P / (Q \cup R)$ $(Q \cap R) / P$

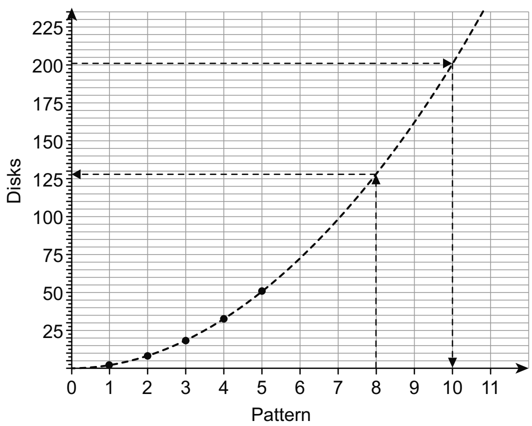
(b)



4. (a)

Pattern	Rows of disks	Columns of disks	Number of disks
1^{st}	1	2	$1 \times 2 = 2$
2^{nd}	2	4	$2 \times 4 = 8$
3^{rd}	3	6	$3 \times 6 = 18$
4^{th}	4	8	$4 \times 8 = 32$
5^{th}	5	10	$5 \times 10 = 50$
.			
.			
.			
n^{th}	n	$2n$	$n \times 2n = 2n^2$

(b)



(c) Explanation on diagram above

(i) 128 disks

(ii) 10^{th} Pattern

(d) **(i)** $2(8)^2 = 128$

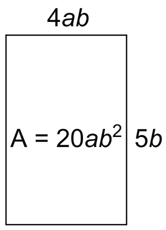
(ii) $2n^2 = 200$

$n^2 = 100$

$n = 10$

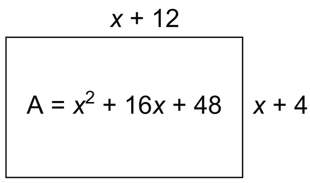
(e) $n \times 3n = 3n^2$

5. **(a)** **(i)**



$$\frac{20ab^2}{4ab} = 5b$$

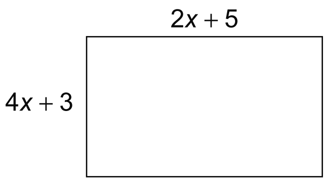
(ii)



$x^2 + 16x + 48$

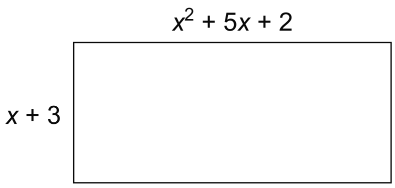
$(x + 12)\ (x + 4)$

(b) **(i)**



$$\begin{aligned} \text{Area} &= (2x + 5)(4x + 3) \\ &= 8x^2 + 6x + 20x + 15 \\ &= 8x^2 + 26x + 15 \end{aligned}$$

(ii)



$$\begin{aligned} \text{Area} &= (x^2 + 5x + 2)(x + 3) \\ &= x^3 + 3x^2 + 5x^2 + 15x + 2x + 6 \\ &= x^3 + 8x^2 + 17x + 6 \end{aligned}$$

6. **(a)** **(i)** $x > -1$

(ii) $x < -3$ and $x \geq 3$

(b) **(i)** $20x + 150 < 10x + 360$

(ii) $20x + 150 < 10x + 360$

$20x - 10x < 360 - 150$

$10x < 210$

$x < 21$

20 days

7. **(a)** Athlete A

(b) Athlete C overtook Athlete B approximately 20 km from the start.
Then Athlete B overtook Athlete C at about the 56 km mark.

(c) Athlete A: Distance = 62 km: Time = 200 minutes:
Speed = Distance ÷ Time = 18·6 km/h

Athlete B: Distance = 62 km: Time = 225 minutes:
Speed = Distance ÷ Time = 16·5 km/h

Athlete C: Distance = 62 km: Time = 235 minutes:
Speed = Distance ÷ Time = 15·8 km/h

(d) 134 minutes

(e) The cycle, because they finished last in the other two legs.

8. **(a)** $\frac{x+4}{x-1} - \frac{x+5}{x+1}$

$\frac{(x+4)(x+1) - (x+5)(x-1)}{(x-1)(x+1)}$

$\frac{x^2 + 5x + 4 - x^2 - 4x + 5}{(x-1)(x+1)}$

$\frac{x+9}{x^2-1}$

(b) $\frac{x+4}{x-1} - \frac{x+5}{x+1} = \frac{x^2}{x^2-1}$

$\frac{x+9}{x^2-1} = \frac{x^2}{x^2-1}$

$x + 9 = x^2$

$x^2 - x - 9 = 0$

$a = 1, b = -1, c = -9$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ ← See page 20 of *Formulae and Tables*.

$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(-9)}}{2(1)}$

Sample 5 Educate.ie P1

$$x = \frac{1 \pm \sqrt{1 + 36}}{2}$$

$$x = \frac{1 \pm \sqrt{37}}{2}$$

$$x = \frac{1 + \sqrt{37}}{2} \quad \text{or} \quad x = \frac{1 - \sqrt{37}}{2}$$

$$x = 3{\cdot}54 \quad \text{or} \quad x = -2{\cdot}54$$

9. (a) Equation 1: $5x + 4y = 9{\cdot}2$

Equation 2: $3x + 6y = 8{\cdot}4$

(b)

$$5x + 4y = 9{\cdot}2$$
$$3x + 6y = 8{\cdot}4$$
$$15x + 12y = 27{\cdot}6$$
$$6x + 12y = 16{\cdot}8$$
$$9x = 10{\cdot}8$$
$$x = 1{\cdot}20$$
$$5(1{\cdot}20) + 4y = 9{\cdot}2$$
$$6 + 4y = 9{\cdot}2$$
$$4y = 3{\cdot}2$$
$$y = 0{\cdot}8$$

Ice creams cost €1·20 and smoothies cost €0·80.

10. (a)

$$\frac{V}{i} = R$$
$$\frac{4}{5{\cdot}8} = R$$
$$R = 0{\cdot}689655$$

(b)

$$\frac{V}{i} = R$$
$$V = Ri$$
$$\frac{V}{R} = i$$

11. See how much interest €100 would earn.

$F = P(1 + i)^t$ ← See page 30 of *Formulae and Tables*.

$F = 100(1{\cdot}1)^5$

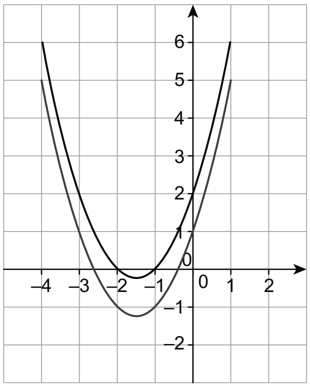
$F = €161{\cdot}05$

Interest = €161·05 − €100 = €61·05

€100 .... €61·05

€x .... €305·26

$$x = \frac{305{\cdot}26 \times 100}{61{\cdot}05} = €500$$

12. 

(a) $a = -4, \quad b = 1, \quad p = 2$

(b) $x = -2, \quad x = -1$

(c) In grey on graph above

(d) $x = -4$ and $x = 1$

$x + 4 = 0$ and $x - 1 = 0$

$\therefore \ (x + 4)(x - 1) = 0$

$\therefore \ x^2 + 3x - 4 = 0$

$\therefore \ p = -4$

(e) $x^2 + 3x = 0$

$x(x + 3) = 0$

$x = 0 \quad \textit{or} \quad x = -3$

13. (a) $f(0) = 41$

$f(3) = 47$

$f(6) = 71$

41, 47, 71 are all prime numbers.

(b) $f(40) = 1601$

$f(41) = 1681$

1601 is also a prime number but 1681 is not as it is a square number.

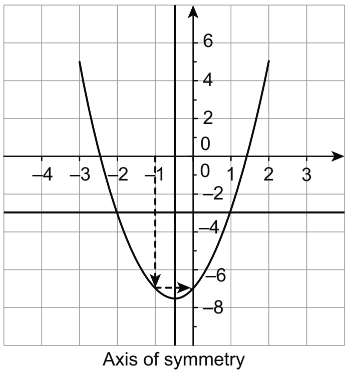
(c) $f(3) + f(6) = 47 + 71 = 118$

$f(3 + 6) = f(9) = 113$

$\therefore \ f(3) + f(6) \neq f(3 + 6)$

Sample 5
Educate.ie P1

14. Shown on diagram below.



(a) $x = -2$ and $x = 1$

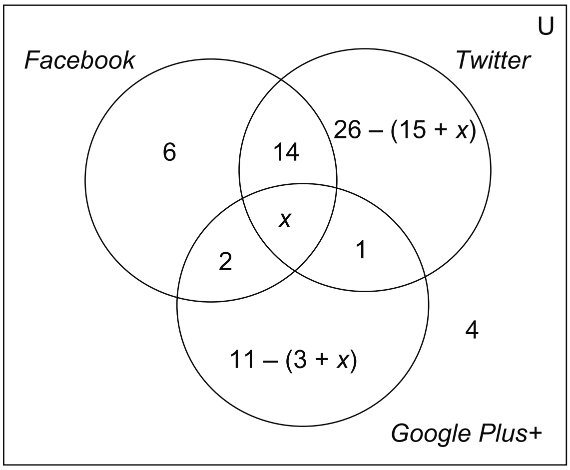
(b) -7

(c) Axis of symmetry: $x = -0{\cdot}5$

Educate.ie Sample 6

Paper 1

1. (a)



(b) $6 + 14 + x + 2 + 1 + 26 - 15 - x + 11 - 3 - x + 4 = 40$

$20 + x + 3 + 11 - x + 8 - x + 4 = 40$

$20 + 14 + 8 - x + 4 = 40$

$46 - x = 40$

$x = 6$

2. (a) $50x^2 - 2$ ← 2 is a common factor.

$2(25x^2 - 1)$ ← Difference of two squares also

$2(5x + 1)(5x - 1)$

(b) $p^2 - 9x^2 - 6x - 2p$ ← Grouping

$(p^2 - 9x^2) + (-6x - 2p)$

$(p + 3x)(p - 3x) - 2(3x + p)$

$(p + 3x)(p - 3x - 2)$

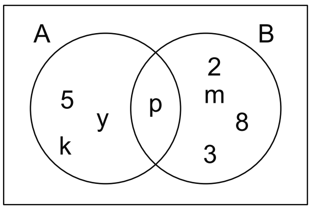
$(p + 3x)(p - 3x - 2)$

(c) $3x^3 + 4x^2 - 4x$ ← x is a common factor.

$x(3x^2 + 4x - 4)$ ← Quadratic factors

$x(3x - 2)(x + 2)$

3. (a)



(i) 4

(ii) 5

(iii) 1

(iv) 8

(b)

Statement	**True or False**	**Reason**
#(A/B) = #(B/A)	False	#{5, k, y} $\neq$ #{m, 8, 2, 3}
$(A \cap B) \subset (A \cup B)$	True	{p} is a subset of {5, k, y, p, m, 8, 2, 3}
$\#(A \cap B) = \#B - 3$	False	$1 \neq 5 - 3$
$\#[(A/B) \cup (B/A)] = 7$	True	$[(A/B) \cup (B/A)]$ = {5, k, y, m, 8, 2, 3} which has 7 elements.

4. Choice (4, −1)

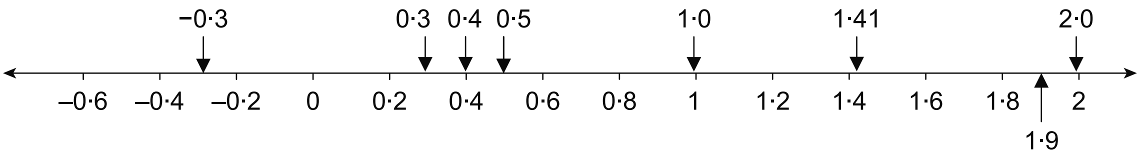
Reason: It satisfies both equations: $2x - 3y = 11$: $2(4) - 3(-1) = 11 \quad \Rightarrow 8 + 3 = 11$

$3x + 2y = 10$: $3(4) + 2(-1) = 10 \quad \Rightarrow 12 - 2 = 10$

5. (a)

	A	**B**	**C**	**D**	**E**	**F**	**G**	**H**
Number	$\left(\frac{1}{4}\right)^{\frac{1}{2}}$	$\sqrt{2}$	$-\frac{1}{3}$	2 tan 45°	25%	$(1{\cdot}25)^0$	4×10^{-1}	$\sqrt[3]{7}$
Decimal Number	0·5	1·4	−0·3	2·0	0·3	1·0	0·4	1·9

(b)



(c) $2 - \frac{1}{\sqrt{2}} = \frac{2\sqrt{2} - 1}{\sqrt{2}}$

$\frac{2\sqrt{2} - 1}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{4 - \sqrt{2}}{2} = \frac{2^2 - \sqrt{2}}{2}$

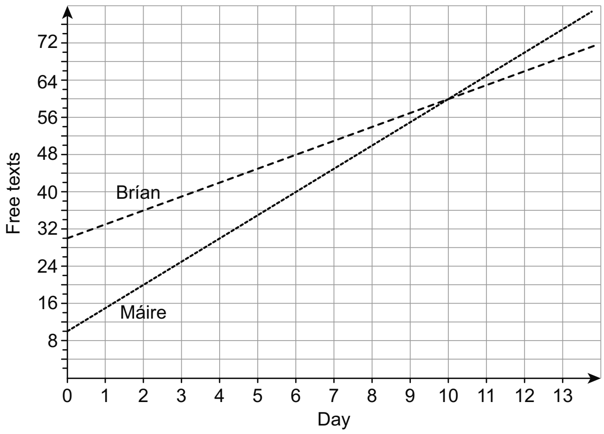
(d) **(i)** $\frac{1}{0{\cdot}91} + 6{\cdot}23 \times 2{\cdot}7 = 1 + 6 \times 3 = 19$

(ii) Using calculator: $\frac{1}{0{\cdot}91} + 6{\cdot}23 \times 2{\cdot}7 = 17{\cdot}92$

6. (a)

Day	Brían	Máire
0	30	10
1	33	15
2	36	20
3	39	25
4	42	30
5	45	35
6	48	40
7	51	45

(b)



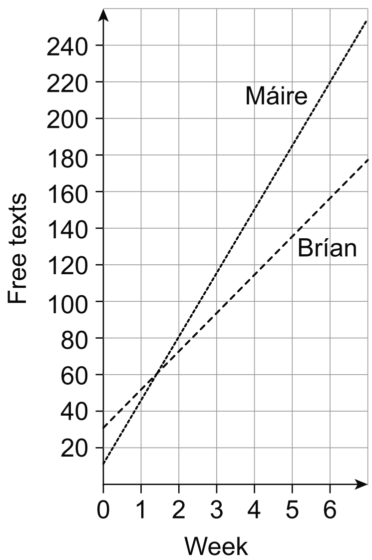
(c) Brían: Total free texts = 30 + 3 times the number of days

Máire: Total free texts = 10 + 5 times the number of days

(d) Brían: $T = 30 + 3d$: d = number of days, T = total number of free texts

Máire: $T = 10 + 5d$: d = number of days, T = total number of free texts

(e)



Sample 6 Educate.ie P1

(f) **(i)** Day 10

(ii) 66 texts

(iii) 135 + 185 = 320

(g) 50

7. **(a)** $3 = \frac{2}{x} + \frac{5}{x^2}$

$\frac{3x^2}{x^2} = \frac{2x + 5}{x^2}$

$3x^2 = 2x + 5$

$3x^2 - 2x - 5 = 0$

$(3x - 5)(x + 1) = 0$

$x = \frac{5}{3}$ or $x = -1$

(b) $\frac{2}{x - 1} - \frac{3}{x - 2} = -2$

$\frac{2(x - 2) - 3(x - 1)}{(x - 1)(x - 2)} = \frac{-2(x - 1)(x - 2)}{(x - 1)(x - 2)}$

$2x - 4 - 3x + 3 = -2(x^2 - 3x + 2)$

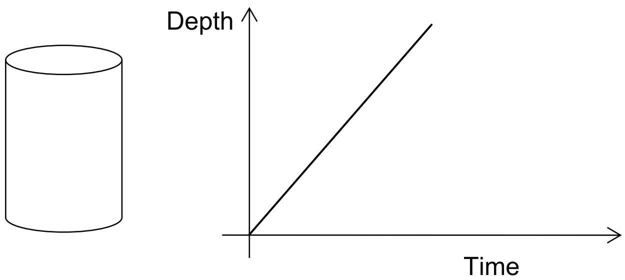
$-x - 1 = -2x^2 + 6x - 4$

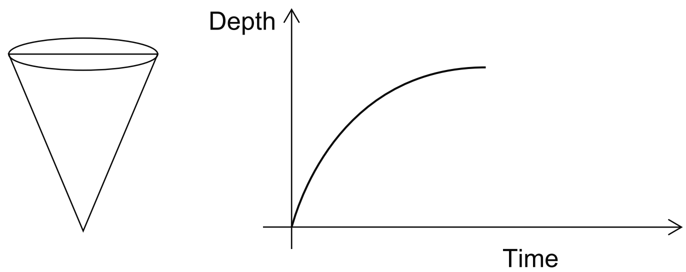
$2x^2 - 7x + 3 = 0$

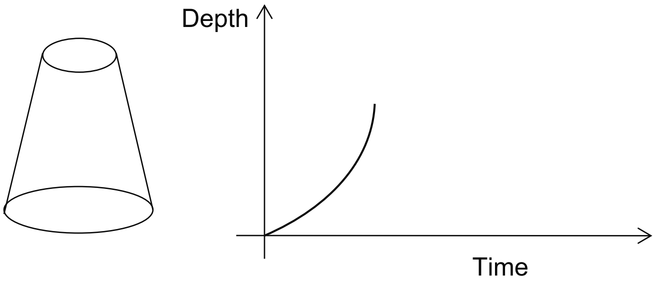
$(2x - 1)(x - 3) = 0$

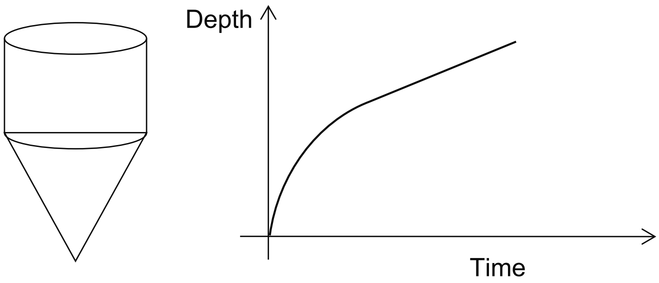
$x = \frac{1}{2}$ or $x = 3$

8.

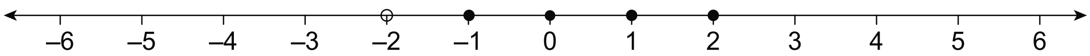




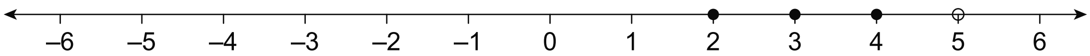




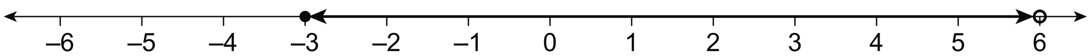
9. (a) (i)



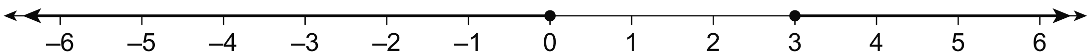
(ii)



(iii)



(iv)



(b) $-9 < x - 4 \leq -2$

$+4 \qquad +4 \qquad +4$

$-5 < x \leq 2$

Solution set: $\{-4, -3, -2, -1, 0, 1, 2\}$

10. (a) 1 yottagram $= 10^{24}$ grams $= \dfrac{10^{24}}{1000}$ kg $= \dfrac{10^{24}}{10^3} = 10^{21}$ kg

(b) 1 tonne $= 10^3$ kilograms

20 tonne $= 20 \times 10^3$ kilograms $= 100 \times 20 \times 10^3$ decagrams $= 2 \times 10^6$ decagrams

(c) 1 cow = 0·5 tonne

24 cow = 12 tonnes

(d) $1{\cdot}5 \times 10^{21}$ kg $+ 4{\cdot}2 \times 10^{17}$ kg $= 1{\cdot}50042 \times 10^{21}$ kg

$1{\cdot}50042 \times 10^{18}$ tonnes

11. (a) $z = \sqrt{\dfrac{xy - z^2}{y}}$

$z^2 = \dfrac{xy - z^2}{y}$

$z^2 y = xy - z^2$

$z^2 y - xy = -z^2$

$y(z^2 - x) = -z^2$

$y = \dfrac{-z^2}{z^2 - x}$

$y = \dfrac{z^2}{x - z^2}$

(b) $y = \dfrac{z^2}{x - z^2}$

$y = \dfrac{\left(\frac{1}{4}\right)^2}{\left(\frac{3}{2}\right) - \left(\frac{1}{4}\right)^2}$

$y = \left(\frac{1}{16}\right) \div \left(\frac{3}{2} - \frac{1}{16}\right)$

$y = \left(\frac{1}{16}\right) \div \left(\frac{24}{16} - \frac{1}{16}\right)$

$y = \left(\frac{1}{16}\right) \div \left(\frac{23}{16}\right)$

$y = \left(\frac{1}{16}\right) \times \left(\frac{16}{23}\right)$

$y = \dfrac{1}{23}$

12. Joint salary before tax = €109 650

20% of €65 600 = €13 120

41% of (€109 650 − €65 600) = 41% of €44 050 = €18 060·50

Total gross tax = €13 120 + €18 060·50 = €31 180·50

Net tax = gross tax – tax credits

Net tax = €31 180·50 – €9220 = €21 960·50

Net salary = gross salary – net tax

Net salary = €109 650 – €21 960·50 = €87 689·50

13. (a) $x = \dfrac{2}{3} \Rightarrow 3x - 2 = 0$ $\qquad x = -\dfrac{1}{2} \Rightarrow 2x + 1 = 0$

$\Rightarrow (3x - 2)(2x + 1) = 0 \qquad \Rightarrow 6x^2 - x - 2 = 0$

$\therefore a = 6, b = -1, c = -2$

(b) $x = \sqrt{3} \Rightarrow x - \sqrt{3} = 0$ $\quad\quad$ $x = -\sqrt{3} \Rightarrow x + \sqrt{3} = 0$

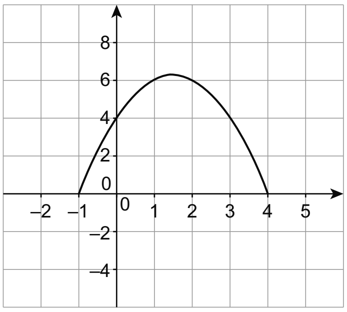
$\Rightarrow [x - \sqrt{3}][x + \sqrt{3}] = 0$

$x^2 + x\sqrt{3} - x\sqrt{3} - 3 = 0$

$x^2 - 3 = 0$

$\therefore\ a = 1,\ b = 0,\ c = -3$

14.



(a) 6 metres

(b) 6·25 metres

15.

Functions	$x^2 + 2$	$2x + 2$	$-x^2$	$2x - 2$	$-(x - 2)^2$	x^2
Graph	2	3	5	4	6	1

Sample 6
Educate.ie P1

Educate.ie Sample 7

Paper 1

See page 23 of *Formulae and Tables*.

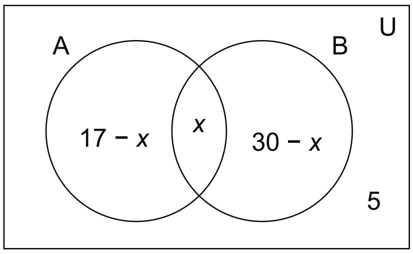
1.

Description of number	**Numbers**
Natural Numbers	$7, 5^0, 144$
Integers	$-9, 7, 5^0, 144$
Prime Numbers	7
Irrational Numbers	$\sqrt{3}, 2\pi$
Squared Number	144
Negative Integer	-9
Reciprocal of a, where $a \in \mathbb{N}$	$\frac{1}{5}, 6^{-1}, 5^0$
Recurring Decimal	$2{\cdot}\dot{3}$

2. **(a)** 5

(b) 2

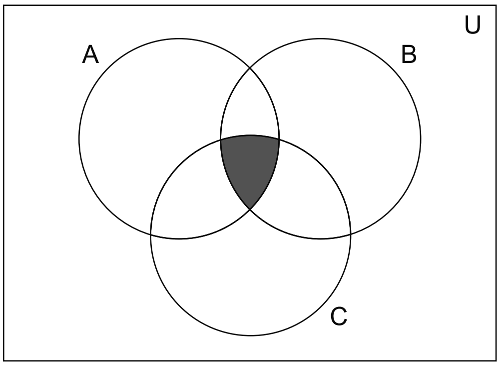
(c) 15



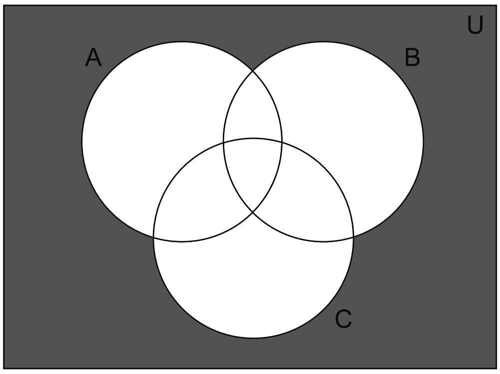
$$17 - x + x + 30 - x = 45$$

$$x = 2$$

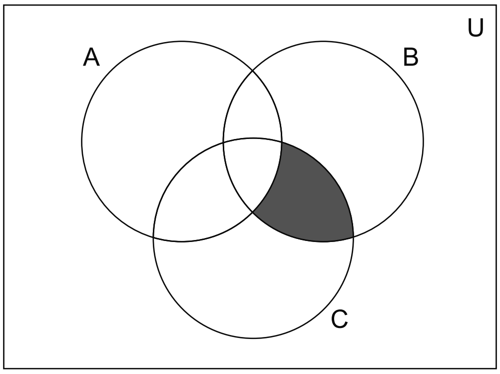
3. **(a)** $(A \cap B \cap C)$



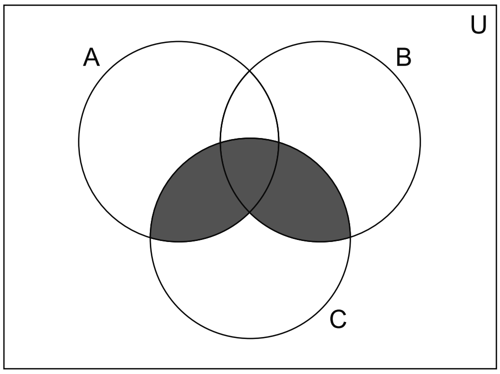
(b) $(A \cup B \cup C)^c$



(c) $(B \cap C) \setminus A$



(d) $(A \cup B) \cap C$



4. (a) $\dfrac{4xy^2}{y^n} = (2x)(2y^6)$

$4xy^2y^{-n} = 4xy^6$

$4xy^{2-n} = 4xy^6$

$y^{2-n} = y^6$

$2 - n = 6 \quad \Rightarrow n = -4$

(b)

	x^2	$7x$	6
x	x^3	$7x^2$	$6x$
1	x^2	$7x$	6

$x^3 + x^2 + 7x^2 + 7x + 6x + 6 = x^3 + 8x^2 + 13x + 6$

Sample 7
Educate.ie P1

(c)

$$\begin{array}{r} 5x^2 - 6x + 1 \\ 2x + 3 \,\overline{)\,10x^3 + 3x^2 - 16x + 3} \\ \underline{10x^3 + 15x^2} \\ -12x^2 - 16x + 3 \\ \underline{-12x^2 - 18x} \\ 2x + 3 \\ 2x + 3 \end{array}$$

or

	$5x^2$	$-6x$	1
$2x$	$10x^3$	$-12x^2$	$6x$
3	$15x^2$	$7x$	3

$3x^2$

5. **(a)** $F = P(1 + i)^t$ ← See page 30 of *Formulae and Tables*.

$F = 1{\cdot}2(1{\cdot}09)^6$

$F = 2{\cdot}0125$

201 cm

(b) $F = P(1 + i)^t$

$2{\cdot}44 = 1{\cdot}5(1 + i)^{10}$

$(1 + i)^{10} = \frac{2{\cdot}44}{1{\cdot}5}$

$(1 + i)^{10} = 1{\cdot}62666$

$1 + i = \sqrt[10]{1{\cdot}62666}$

$1 + i = 1{\cdot}0489$

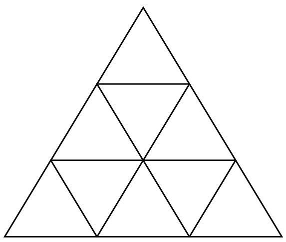
$i = 0{\cdot}0489$

$i = \frac{r}{100} = 0{\cdot}0489$

$r = 4{\cdot}89$

$r = 5\%$

6. **(a)**



Total cards = 3 + 6 + 9 = 18

(b) Working from the top down

1st Row	3(1) = 3
2nd Row	3(2) = 6
3rd Row	3(3) = 9
10th Row	3(10) = 30

3 + 6 + 9 + 12 + 15 + 18 + 21 + 24 + 27 + 30 = 165 cards

(c) $3(1) + 3(2) + 3(3) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 3(65)$

$3[1 + 2 + 3 + 4 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 65]$

$3[55 + 155 + 255 + 355 + 455 + 555 + 315]$ ←

$3[2145]$

6435

> Use calculator and patterns.
> Look up Gauss on Google.
> $\frac{n(n+1)}{2}$
> In this case it is $\frac{65(65+1)}{2} = 2145$
> $3(2145) = 6435$

7. $(x - 3)(x + 4) = 0$

$x^2 + x - 12 = 0$

$\Rightarrow p = 1, w = -12$

8.

Statement	Always true	Never true	Sometimes true	Example
$x^2 - 1 < x, x \in \mathbb{Z}$, and $-2 \leq x \leq 2$			✓	If $x = -2$: $4 - 1 < -2$, False If $x = 1$: $1 - 1 < 1$, True
$x + 2 < x, x \in \mathbb{N}$, and $0 < x \leq 5$		✓		If $x = 0$: $0 + 2 < 0$, False If $x = 4$: $4 + 4 < 4$, False
$x - 2 < x, x \in \mathbb{N}$, and $0 \leq x \leq 5$	✓			If $x = 0$: $0 - 2 < 0$, True If $x = 5$: $5 - 2 < 0$, True
$x^2 < x, x \in \mathbb{Z}$ and $-5 \leq x < 0$		✓		If $x = -5$: $25 < -5$, False If $x = -1$: $1 < -1$, False

9. (a) $\frac{x}{y} + \frac{y}{x}$

$\frac{x^2 + y^2}{yx}$

(b) $(2x - 3)^2 - (x - 6)^2$

$(4x^2 + 9 - 12x) - (x^2 + 36 - 12x)$

$4x^2 + 9 - 12x - x^2 - 36 + 12x$

$3x^2 - 27$

$[= 3(x^2 - 9) = 3(x + 3)(x - 3)]$

(c) $\frac{6x^2 - 23x + 20}{2x - 5}$

$\frac{(3x - 4)(2x - 5)}{2x - 5}$

$3x - 4$

10. (a) Area of A $= (6x + 3)(3x + 1)$ Area of B $= 5x(4x + 2)$

Area of A $= 18x^2 + 15x + 3$ Area of B $= 20x^2 + 10x$

$$18x^2 + 15x + 3 = 20x^2 + 10x$$

$$2x^2 - 5x - 3 = 0$$

$$(2x + 1)(x - 3) = 0$$

$$x = -\frac{1}{2} \quad \text{or} \quad x = 3$$

$$x = 3$$

Sample 7 Educate.ie P1

(b) Perimeter of A = 2(9x + 4) Perimeter of B = 2(9x + 2)

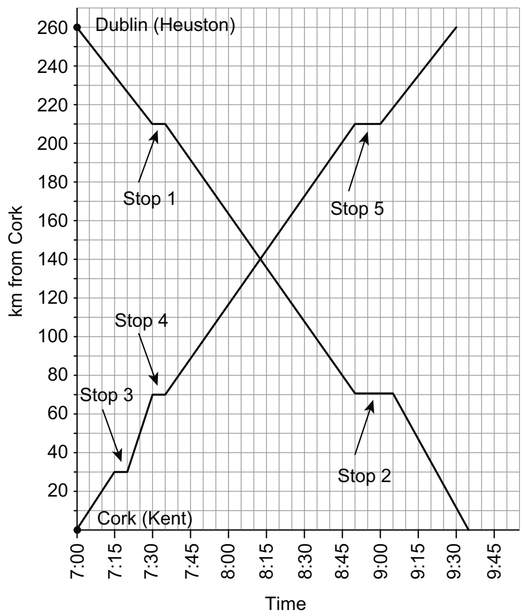
Perimeter of A = 18x + 8 Perimeter of B = 18x + 4

Rectangle A has the longest perimeter.

Perimeter of A = 18(3) + 8 Perimeter of B = 18(3) + 4

Perimeter of A = 62 Perimeter of B = 58

11.



(a) 260 – 210 = 50 km

(b) Dublin/Cork express: 2 stops Cork/Dublin express: 3 stops

(c) Stop 1: 5 minutes Stop 2: 15 minutes Stop 3: 5 minutes Stop 4: 5 minutes

Stop 5: 10 minutes

(d) Dublin/Cork express: 9:35 am Cork/Dublin express: 9:30 am

(e) Speed = Distance ÷ Time

Dublin/Cork express: 70 ÷ 30 min = 140 km/h

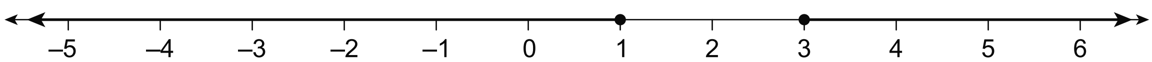
Cork/Dublin express: 50 ÷ 30 min = 100 km/h

(f) Where: 120 km from Dublin or 140 km from Cork: Time: 8:12 am

12. (a) $-5 \leq -5x \leq -15$

$-1 \leq -x \leq -3$

$1 \geq x \geq 3$



(b) $6(x + 5) > 2(7 - x)$

$6x + 30 > 14 - 2x$

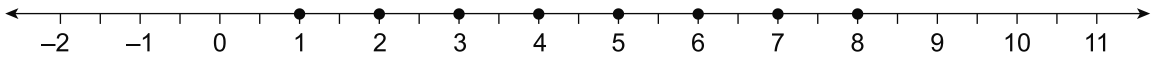
$8x > -16$

$x > -2$

(c) $-5 \leq 3x - 8 \leq 16$

$3 \leq 3x \leq 24$

$1 \leq x \leq 8$



13. (a) $\frac{x}{5} - 60 = 41$

$\frac{x}{5} = 41 + 60$

$\frac{x}{5} = 101$

$x = €505$

(b) 2% of €193 + 4% of €115 + 7% of €197

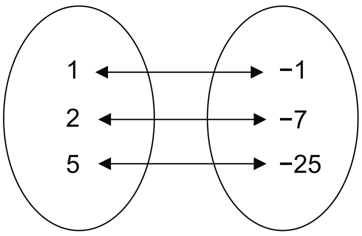
€3·86 + €4·60 + €13·79

€22·25

(c) 4% of €505= €20·20

(d) €505 – (€20·20 + €22·25 + €41) = €421·55

14.



$f(x) = 5 - 2ax$

$f(1) = 5 - 2a(1) = -1$

$5 - 2a = -1$

$-2a = -6$

$a = 3$

$f(x) = 5 - 2(3)x = 5 - 6x$

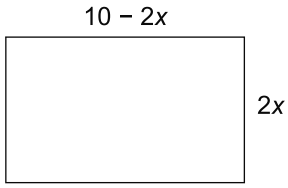
$f(2) = 5 - 6(2) = -7$

$f(x) = 5 - 6(x) = -25$

$6x = 30$

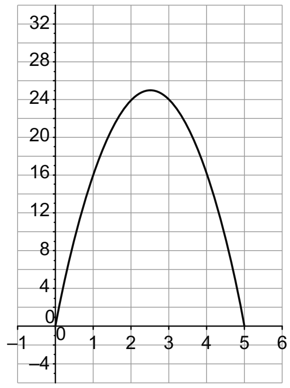
$x = 5$

15. (a)



Area = $2x(10 - 2x) = 20x - 4x^2$

(b)



(c) **(i)** 25 m^2

(ii) Maximum area is a square of sides 5 m.

2014 SEC Paper 1 (Phase 3)

1. (a) $1{\cdot}4, \sqrt{2}, \frac{3}{2}\left(\sqrt{2} = 1{\cdot}414..., \frac{3}{2} = 1{\cdot}5\right)$

Change $\sqrt{2}$ and $\frac{3}{2}$ into decimals with your calculator if you need to.

(b) Answer: π

Reason: It cannot be written as a fraction.

Rational numbers can be expressed as the ratio of two integers i.e. as a fraction.

(c) (i)

n	$\frac{4n^2+1}{13}$
17	$\frac{4\times(17)^2+1}{13} = \frac{1157}{13} = 89$
19	$\frac{4\times(19)^2+1}{13} = \frac{1445}{13}$ or $111\frac{2}{13}$
21	$\frac{4\times(21)^2+1}{13} = \frac{1765}{13}$ or $135\frac{10}{13}$

Replace n with 17, 19, 21 in turn in the formula.

(ii) Answer: 89

The other answers are non-whole numbers.

Reason: It is a positive whole number.

2. (a) (i)

p	$6p + 1$	$6p + 5$
0	1	5
1	7	11
2	13	17
3	19	23
4	25	29
5	31	35

Replace p in the formula with the values under p in the first column

(ii) There are a number of different reasons – **any** two will suffice.

Note: 1 is non-prime as it doesn't have two factors.

Reasons related to "**all** prime numbers":

The formulas do not generate 2, which is prime.

The formulas do not generate 3, which is prime.

Reasons related to "**only** prime numbers":

The formulas generate 1, which is not prime.

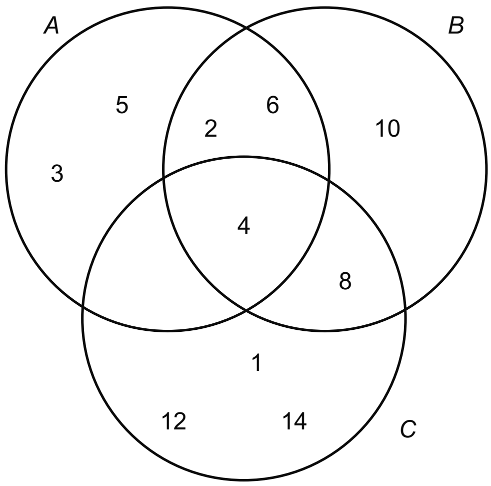
The formulas generate 25, which is not prime.

The formulas generate 35, which is not prime.

(b) $41^2 - 41 + 41 = 41^2$, which has 41 as a factor.

$41^2 = 1681$ which obviously has 41 as a factor along with 1 and 1681.

3. (a) (i)



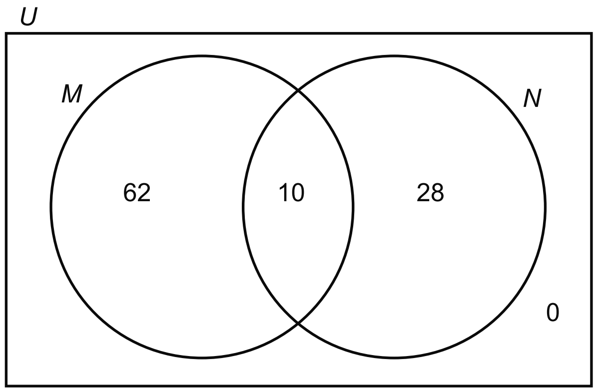
(ii) $A \cap B = \{2, 4, 6\}$ ← $A \cap B$: means what is in both A and B.

$B \backslash (A \cap C) = \{2, 6, 8, 10\}$ ← $B \backslash (A \cap C)$: B less (A and C).

$(B \backslash A) \cup (B \backslash C) = \{2, 6, 8, 10\}$ ← $(B \backslash A) \cup (B \backslash C)$: ($B$ less A) united with (B less C)

(iii) $(A \cap C) \backslash B$ or equivalent ← A null set contains no elements.

(b) (i)



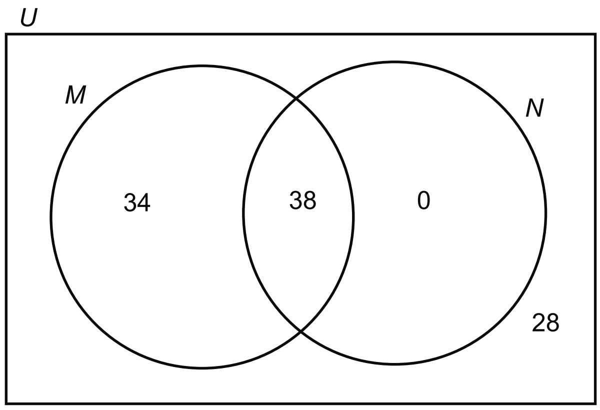
$72 + 38 = 110$ ← OR maximise $M \cup N$

$110 - 100 = 10$

Minimum = 10

To make $\#(M \cap N)$ as small as possible, make $\#(M \cup N)' = 0$.

(ii)



Maximum = 38 ← OR minimise $M \cup N$

To make $M \cap N$ as big as possible, make the smaller set a subset of the larger set.

4. **(a)** $9a^2 - 6ab + 12ac - 8bc = 3a(3a - 2b) + 4c(3a - 2b)$
$= (3a - 2b)(3a + 4c)$

> Factors by grouping

(b) $9x^2 - 16y^2 = (3x - 4y)(3x + 4y)$

> The difference of two squares

(c) $$\frac{2x^2 + 4x}{2x^2 + x - 6} = \frac{2x(x+2)}{(x+2)(2x-3)} = \frac{2x}{2x-3}$$

> Factorise numerator and denominator
> Note: $\frac{x+2}{x+2} = 1$

5. One method:

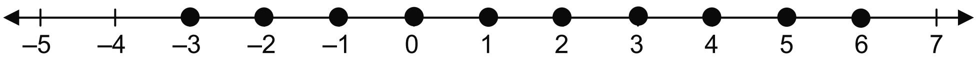
> "Dots" on the number-line as the question involves integers

$$-17 \leq 1 - 3x < 13$$
$$-1: \quad -18 \leq -3x < 12$$
$$\div(-3): \quad 6 \geq x > -4$$
$$\text{i.e.} \quad -4 < x \leq 6$$

Or:

$-17 \leq 1 - 3x$ and $1 - 3x < 13$
$3x \leq 18$ and $-3x < 12$
$x \leq 6$ and $x > -4$

i.e. $-4 < x \leq 6$

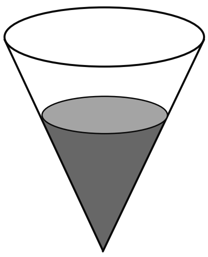


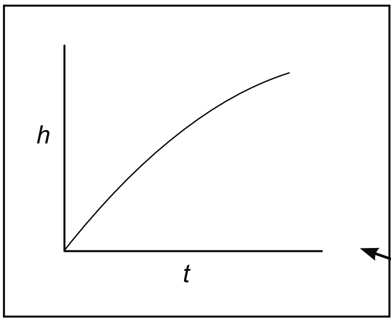
6. **(a)**

Container	1	2	3
Graph	C	A	B

> Reason your way through this type of question e.g. container 3 has two sections so the graph will also (B)
> The rate of change of container 2 is constant and so must be graph (A), etc.

(b)





> The container fills quickly at first and then slows down.

7. **(i)** USC @ 2%: 0·02 × 10 036 = €200·72
USC @ 4%: 16 016 − 10 036 = €5980, and 0·04 × 5980 = €239·20
USC @ 7%: 36 960 − 16 016 = €20 944, and 0·07 × 20 944 = €1466·08
Total USC = €1906

(ii) Tax @ 20%: 0·20 × 32 800 = €6560·00
Tax @ 41%: 36 960 − 32 800 = €4160, and 0·41 × 4160 = €1705·60
Gross Tax: €8265·60
Tax Credits: 8265·60 − 4965·60 = €3300

(iii) Total Deductions: 1906 + 4965·60 = €6871·60

> $\frac{\text{Deductions}}{\text{Gross}} \times 100\%$

Total Deductions as % of Gross Income:

$\frac{6871 \cdot 60}{36{,}960} \times 100 = 18 \cdot 59 = 19\%$, correct to the nearest percent

8. (i) First difference: 3·1 2·3 1·5 0·7 −0·1 −0·9

Second difference: −0·8 −0·8 −0·8 −0·8 −0·8

Answer: Quadratic ← Examine the differences.

Reason: The first differences are not all the same, but the second differences are.

(ii) 5·2 metres ← Use the differences to approximate the heights.

Second difference:			−0·8		−0·8		
First difference:		−0·1		−0·9		−1·7	
Height (m):	7·9		7·8		6·9		5·2
Time (s):	2		2·5		3		3·5

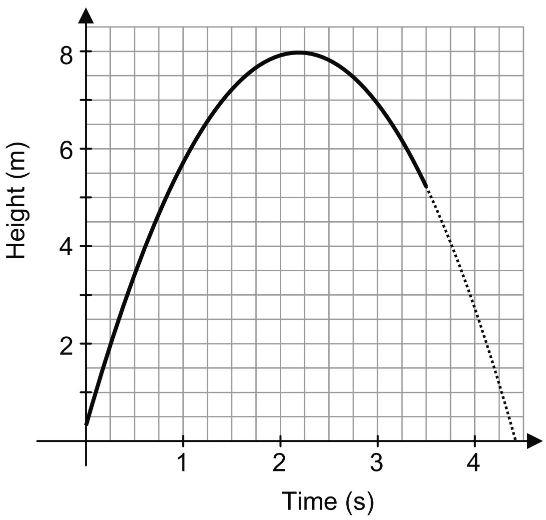
(iii) *Continuing the method for* **(ii)**:

Second difference:			−0·8		−0·8		−0·8		−0·8		
First difference:		−0·1		−0·9		−1·7		−2·5		−3·3	
Height (m):	7·9		7·8		6·9		5·2		2·7		−0·6
Time (s):	2		2·5		3		3·5		4		4·5

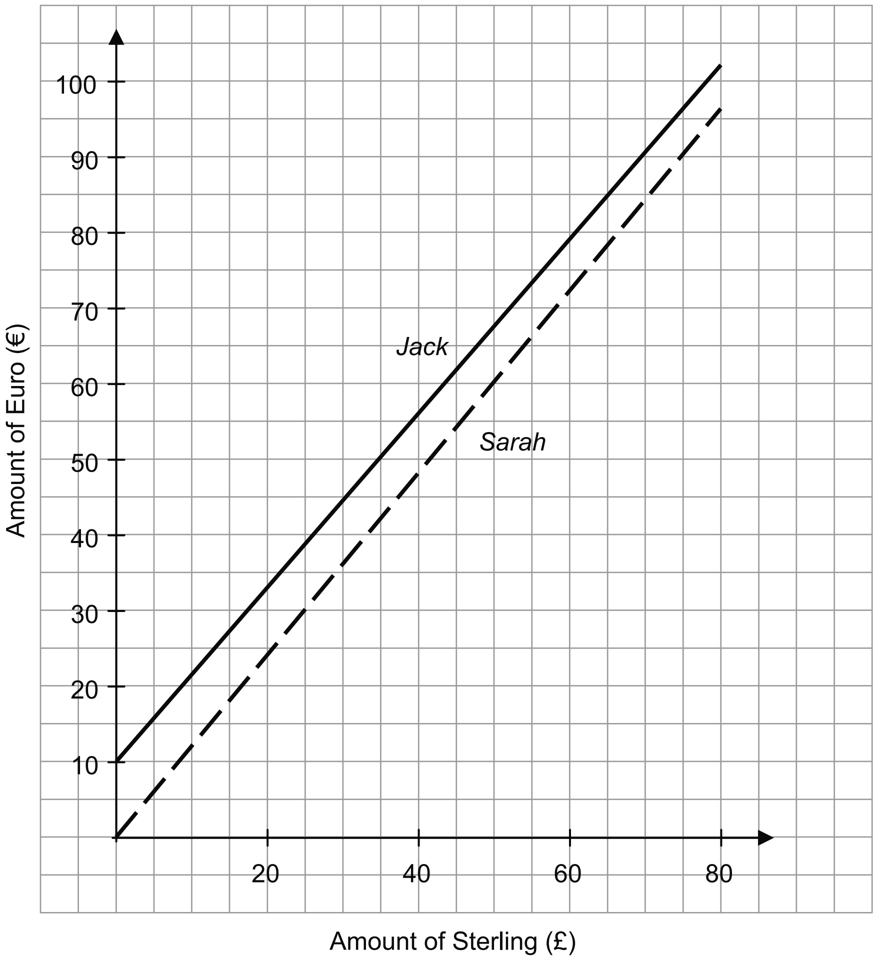
Answer: The ball spends roughly 4·4 seconds in the air. Its height is 0 just before 4·5 seconds.

Or, graphically:

From the graph, the ball spends roughly 4·4 seconds in the air.



9. (i)



(ii) Slope $= \dfrac{56 - 33}{40 - 20} = \dfrac{23}{20}$, *or* 1·15 ← Slope formula. See page 18 of *Formulae and Tables*.

Explanation: Each extra £1 costs Jack an extra €1·15.

Or:

Explanation: Each £1 costs Jack €1·15, after an initial fee of €10.

(iii) $e = 1{\cdot}15s + 10$, where s is the amount in sterling, and e is the amount in euro.

(iv) Slope $= \dfrac{48 - 24}{40 - 20} = \dfrac{6}{5}$, *or* 1·2, y-intercept = 0 ← Slope formula. See page 18 of *Formulae and Tables*.

$e = 1{\cdot}2s$, where s is the amount in sterling, and e is the amount in euro.

(v) Using formulas: ← Solve simultaneously

$e = 1{\cdot}15s + 10$ and $e = 1{\cdot}2s$, so $1{\cdot}15s + 10 = 1{\cdot}2s$,

i.e. $s = 200$ and $e = 240$.

Amount of sterling: £200

From table:

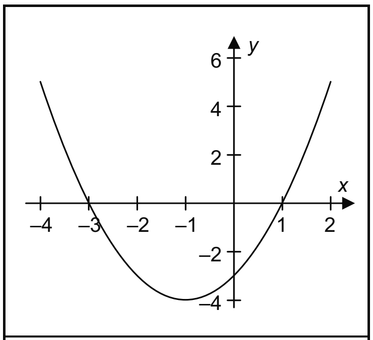
Each time the amount of sterling goes up by 20, the difference between the costs decreases by €1.

This difference is €9 for £20.

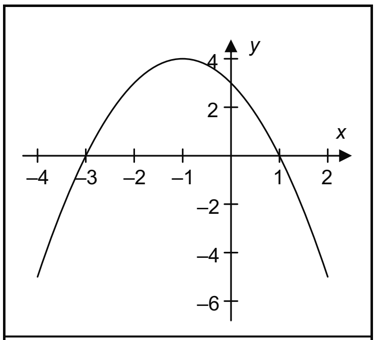
So after 9 increases, i.e. increase of 9 × 20 = £180, the costs are the same, i.e. for £200.

2014 SEC P1

10. (a)



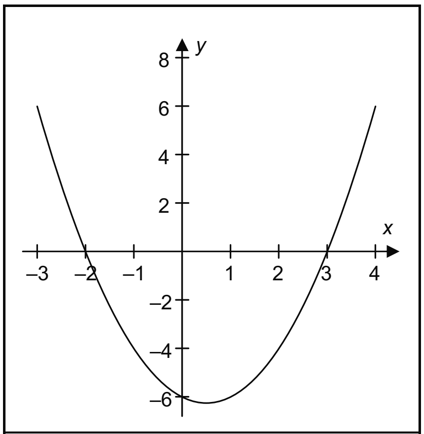
Answer: $f(x)$



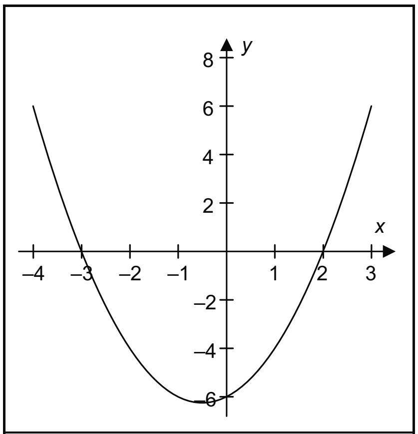
Answer: $g(x)$

> $+x^2$ will have ∪ shape.
> $-x^2$ will have ∩ shape.

(b)



$h(x)$



$k(x)$

> "Roots": points of intersection of the graph and x-axis

> If $x = 3$ is a root, then $(x–3)$ is a factor.

Roots of $h(x)$: $x = -2$ and $x = 3$

Equation: $h(x) = (x + 2)(x - 3)$, or $h(x) = x^2 - x - 6$

Check y-intercept is correct, i.e. co-efficient of x^2 is correct: $h(0) = -6$, which corresponds to the graph.

Roots of $k(x)$: $x = -3$ and $x = 2$

Equation: $k(x) = (x + 3)(x - 2)$, or $k(x) = x^2 + x - 6$

Check y-intercept is correct, i.e. co-efficient of x^2 is correct: $k(0) = -6$, which corresponds to the graph.

11. (i) Increase x by 1: $x + 1$

Decrease x by 2: $x - 2$

(ii) $(x + 1)(x - 2) = 1$ or equivalent.

> "Product" means multiply.

(iii) $(x + 1)(x - 2) = 1$

> "3 decimal places" is a hint to use the quadratic formula. See page 20 of *Formulae and Tables*.

$\Rightarrow x^2 - x - 3 = 0$

$$\Rightarrow x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(-3)}}{2(1)}$$

$\Rightarrow x = 2{\cdot}3028$ and $x = -1{\cdot}3028$

$\Rightarrow x = 2{\cdot}303$ and $x = -1{\cdot}303$, correct to three decimal places.

12. (a) $(6x - 3)(2x - 1) = 12x^2 - 12x + 3$ ← Multiply carefully!

(b) ← Long division

$$\begin{array}{r} 3x^2 + x - 2 \\ x - 1 \overline{)\, 3x^3 - 2x^2 - 3x + 2} \\ \underline{3x^3 - 3x^2} \quad\quad\quad\quad \\ x^2 - 3x + 2 \\ \underline{x^2 - x} \quad\quad \\ -2x + 2 \\ \underline{-2x + 2} \\ 0 \end{array}$$

Answer: $3x^2 + x - 2$.

Or:

	$3x^2$	x	-2
x	$3x^3$	x^2	$-2x$
-1	$-3x^2$	$-x$	2

Answer = $3x^2 + x - 2$

(c) (i) ← Choose the letter to "get rid of" carefully. (y here)

(1) × 3: $6x - 9y = 54$

(2): $\underline{5x + 9y = -10}$

$11x = 44$

$\div 11$: $x = 4$

Substitute in $x = 4$ in:

$2(4) - 3y = 18$

$8 - 3y = 18$

$-3y = 18 - 8$

$-3y = 10$

$\times(-1)$: $3y = -10$

$\div 3$: $y = -10 \div 3 = -\frac{10}{3}$ *or equivalent*

Answer: $x = 4$ and $y = -\frac{10}{3}$.

(ii) Note: You only need to check the equation that **wasn't** used to find the second variable. In this case, we only need to use (2). ← Verify using substitution.

$5(4) + 9\left(-\frac{10}{3}\right) = 20 - 30 = -10.$

13. (i) $x = \sqrt{3^2 + 3^2}$ ← Use Pythagoras's Theorem or $\sin A = \frac{\text{Opposite}}{\text{Hypotenuse}}$

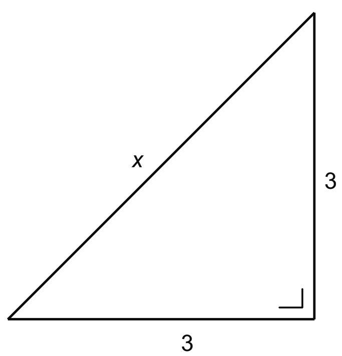
$= \sqrt{18}$ or $3\sqrt{2}$

Or:

$\sin 45° = \frac{3}{x}$

$\frac{1}{\sqrt{2}} = \frac{3}{x}$

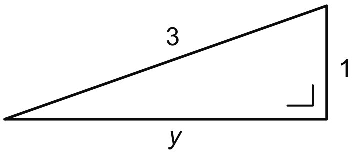
$x = 3\sqrt{2}$



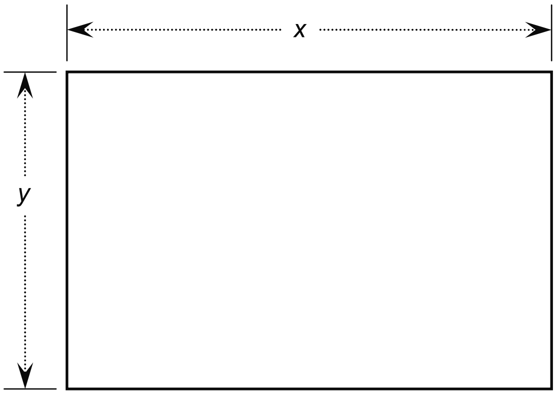
2014 SEC P1

(ii) $y = \sqrt{3^2 - 1^2} = \sqrt{8}$ or $2\sqrt{2}$

Apply Pythagoras's Theorem.



(iii)



Note: $\sqrt{8} = \sqrt{4}\sqrt{2} = 2\sqrt{2}$

$\sqrt{18} = \sqrt{9}\sqrt{2} = 3\sqrt{2}$

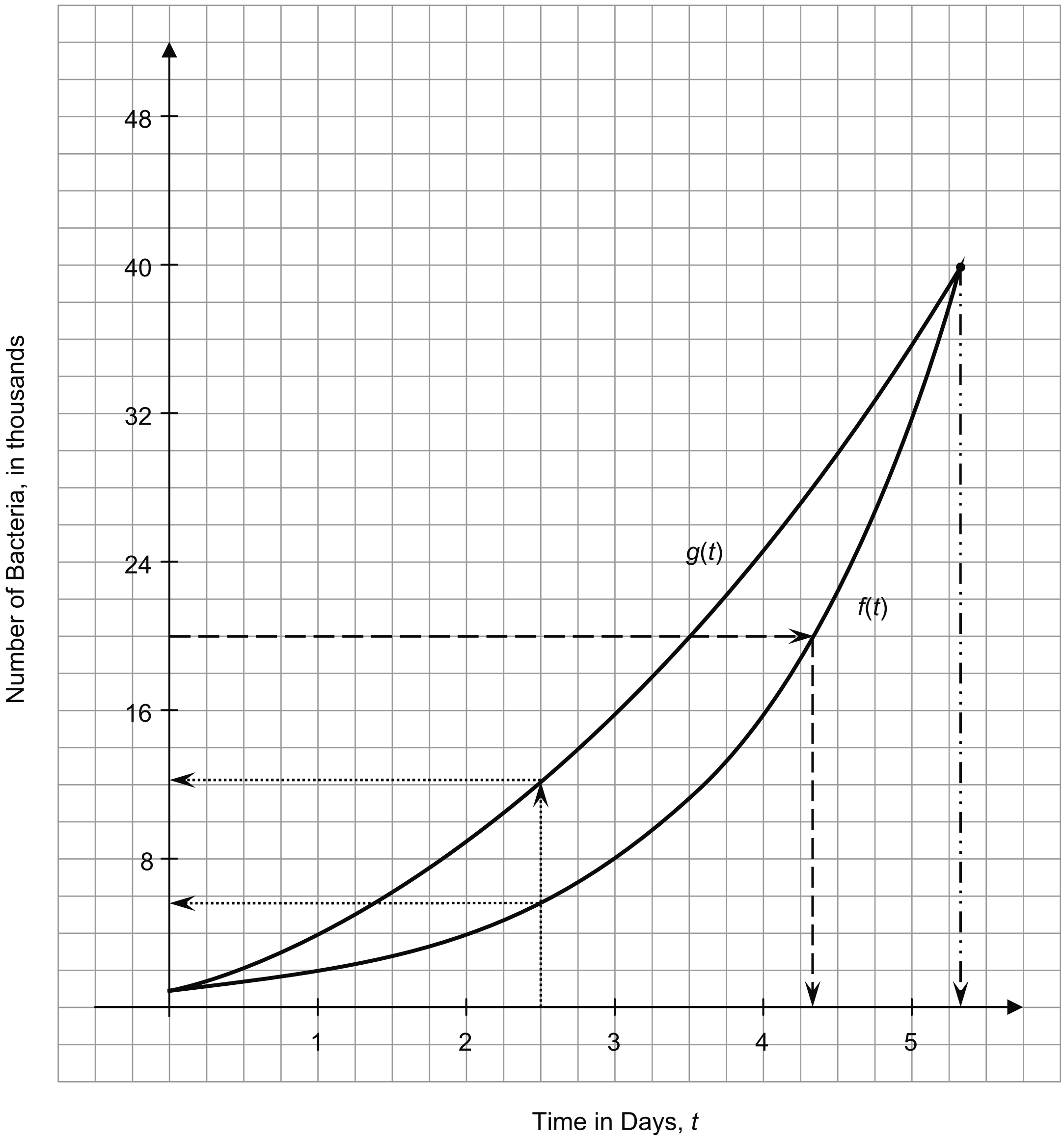
Perimeter $= 2x + 2y$

$= 2\sqrt{18} + 2\sqrt{8}$

$= 2(3\sqrt{2}) + 2(2\sqrt{2})$

$= 10\sqrt{2}$

14. (i)



$f(0) = 1$ $g(0) = 1$ ← Note: f is an exponential function. g is a quadratic function.

$f(1) = 2$ $g(1) = 4$

$f(2) = 4$ $g(2) = 9$ ← Show workings in the grid after the question. Use your calculator to verify that the points are correct.

$f(3) = 8$ $g(3) = 16$

$f(4) = 16$ $g(4) = 25$

$f(5) = 32$ $g(5) = 36$

$g(t) = t^2 + 2t + 1$

t	t^2	$+2t$	$+1$	y
0	0	0	+1	1
1	1	+2	+1	4
2	4	+4	+1	9
3	9	+6	+1	16
4	16	+8	+1	25
5	25	+10	+1	36

2014 SEC P1

$g(t) = t^2 + 2t + 1$

$g(0) = (0)^2 + 2(0) + 1 = 0 + 0 + 1 = 1$

$g(1) = (1)^2 + 2(1) + 1 = 1 + 2 + 1 = 4$

$g(2) = (2)^2 + 2(2) + 1 = 4 + 4 + 1 = 9$

$g(3) = (3)^2 + 2(3) + 1 = 9 + 6 + 1 = 16$

$g(4) = (4)^2 + 2(4) + 1 = 16 + 8 + 1 = 25$

$g(5) = (5)^2 + 2(5) + 1 = 25 + 10 + 1 = 36$

(ii) Marie after 2·5 days: 12 000 bacteria, approximately

Paul after 2·5 days: 6000 bacteria, approximately

Difference: 12 000 − 6000 = 6000 bacteria

> Read each separately from the graph and subtract.

(iii) $t \geq 4{\cdot}3$ days

> "Range" implies that your answer will involve an inequality. Read your answer from the graph.

(iv) $t = 5{\cdot}3$ days

(v) Answer: Paul, i.e. $f(t)$

> Substitute $t = 14$ into both formulae and compare.

Reason: $f(14) = 16\,384 = 1{\cdot}6 \times 10^4$, so Paul predicts $1{\cdot}6 \times 10^4 \times 1000 = 1{\cdot}6 \times 10^7$.

$g(14) = 225 = 2{\cdot}3 \times 10^2$, so Marie predicts $2{\cdot}3 \times 10^2 \times 1000 = 2{\cdot}3 \times 10^5$.

2013 SEC Paper 1 (Phase 3)

1. (a) (i)

Number/Set	$\mathbb{N}$	$\mathbb{Z}$	$\mathbb{Q}$	$(\mathbb{R}/\mathbb{Q})$	$\mathbb{R}$
$\sqrt{5}$	No	No	No	Yes	Yes
8	Yes	Yes	Yes	No	Yes
-4	No	Yes	Yes	No	Yes
$3\frac{1}{2}$	No	No	Yes	No	Yes
$\frac{3\pi}{4}$	No	No	No	Yes	Yes

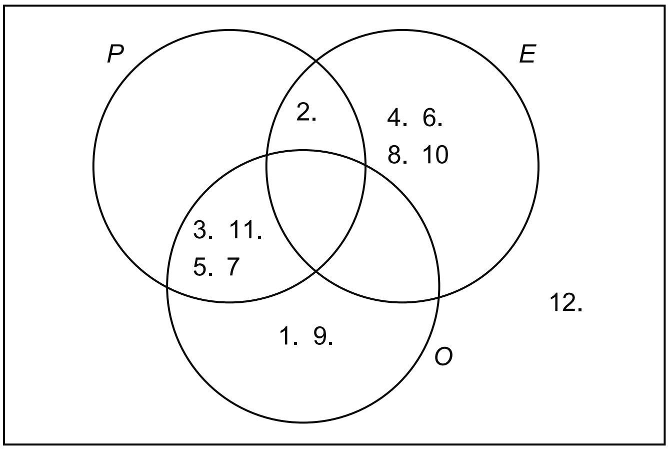
Rational numbers ($\mathbb{Q}$) can be expressed as the ratio of two integers (i.e. as a fraction).

$\frac{3\pi}{4}$ is not an element of $\mathbb{Q}$ (rationals) since 3π *is* not an integer. Also, $3\frac{1}{2} = \frac{7}{2}$.
You can also use your calculator to check whether a number is rational or not.

(ii) $\sqrt{5}$ cannot be written as a fraction.

(b) $7\sqrt{2} - 3\sqrt{2} + \sqrt{2} = 5\sqrt{2}$

2. (a) *U*



Prime numbers have only themselves and 1 as factors.

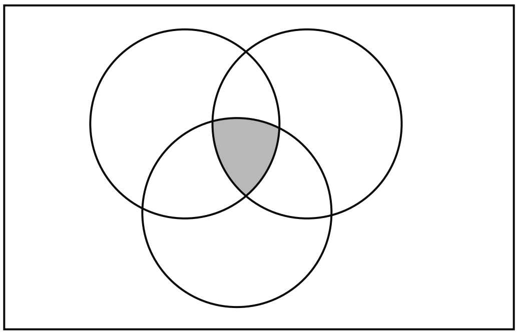
(b) P\(E∪O), P∩E∩O, E∩O, (E∩O)\P

A null set doesn't contain any elements.

(c) $\frac{1}{5}$

$\text{Probability} = \frac{1 \text{ (the number 2)}}{5 \text{ (primes less than 12)}}$

3. (a) (i) *U*

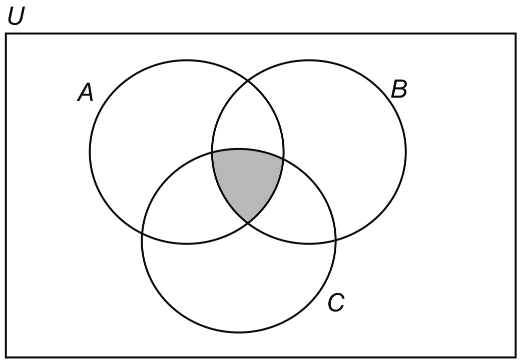


A∩B∩C

$A \cap B \cap C$ means what is common to all 3 sets

2013 SEC P1

(ii)



(A∩B) \ C

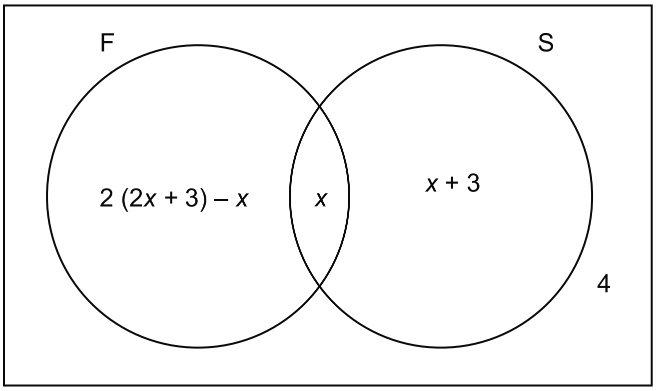
$(A \cap B) \setminus C$ means is common to (A and B) less C

(b) **(iv)** A\B = B\A or (iii) (A\B)\C = A\(B\C)

Diagram or explanation

Examine each statement separately, preferably by drawing diagrams.

(c)



Let x represent the number of people who had been to both countries.

$2(2x + 3) - x + x + x + 3 = 34 \Rightarrow x = 5$

4. **(a)** 8·65 × 0·7 = 6·055 which is 6·06 correct to two decimal places or any other check.

(b) 8·65 × 0·94 = €8·13

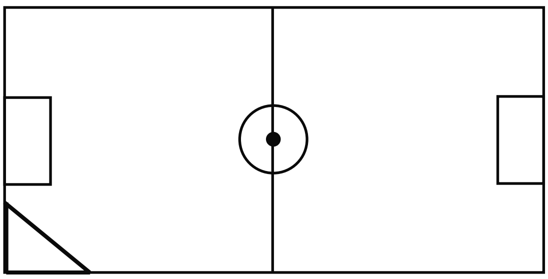
Or find 6% of €8·65 and subtract

(c) No.

8·13 × 1·06 = €8·62

This is not as high as the original starting point.

5. **(a)**



Start at the corner flag. Use the tape measure to measure a certain distance out along the side-line. e.g. 5 m.

Then measure a certain distance out along the goal-line. e.g. 4 m.

Then measure the distance between these two end points

Using Pythagoras's Theorem, see if the calculated distance is the same as the measured distance.

(b) Use the trundle wheel to measure the radius, i.e. the distance from the centre spot to anywhere on the circumference.

Use circumference = $2\pi r$ to calculate the circumference.

Then use the trundle wheel to measure the circumference on the circle and see if the two match.

6. **Car A**: (Time to reach D) $T = \frac{D}{S} = \frac{70}{50} = 1{\cdot}4$ h

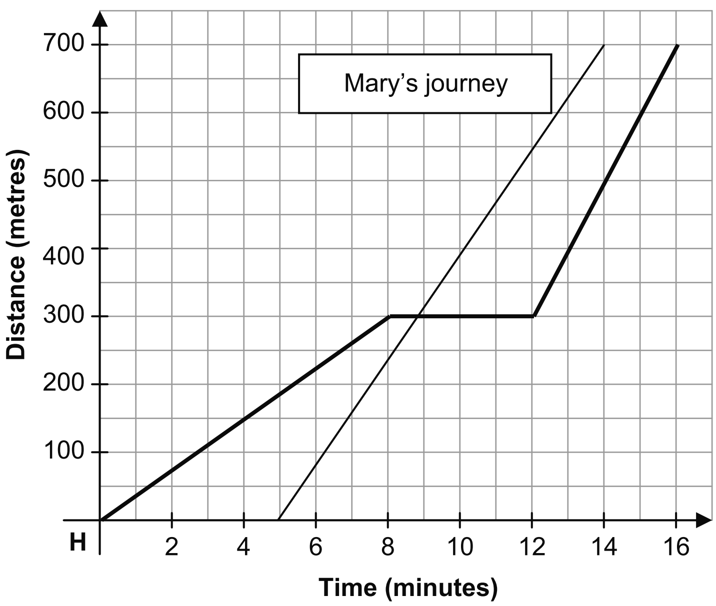
Car B: Distance travelled $45 \times 1{\cdot}4 = 63$ km ← Car B: Distance = Speed × Time

7. **(a)**

Story	Tick one story (✓)
Angela walks at a constant pace and stops at 5.08 for four minutes. She then walks at a slower pace and arrives at practice at 5.16.	
Angela walks at a constant pace and stops at 5·12 for four minutes. She then walks at a faster pace and arrives at practice at 5.16.	
Angela walks at a constant pace and stops at 5.08 for five minutes. She then walks at a faster pace and arrives at practice at 5.16.	
Angela walks at a constant pace and stops at 5.08 for four minutes. She then walks at a faster pace and arrives at practice at 5.16.	✓
Angela walks at a constant pace and stops at 5.08 for four minutes. She then walks at the same pace and arrives at practice at 5.16.	

Try to understand WHY each of the other stories is incorrect.

(b)



"Constant pace": Mary's journey will have to be represented by a straight line segment.

8. **(a)** $\dfrac{4(5-x)+5(x-4)}{20} = \dfrac{x}{20}$ ← Common denominator is needed to add these two fractions.

(b) $3x^2 + 11x - 4 = 0$ $\qquad$ $3x^2 + 11x - 4 = 0$ ← Rearrange to form a quadratic equation before you solve for x.

$(3x-1)(x+4) = 0$ $\qquad$ $x = \dfrac{-11 \pm \sqrt{11^2 - 4(3)(-4)}}{2(3)}$

$x = \dfrac{1}{3}, x = -4$ $\qquad$ $x = \dfrac{-11 \pm 13}{6}$

$x = \dfrac{1}{3}, x = -4$

(c) Method A ← Long division: Try to learn the Method B shown in the solutions as it is more convenient.

$$\begin{array}{r} 2x^2 - 5x + 2 \\ x + 3 \,\big)\, 2x^3 + x^2 - 13x + 6 \\ \underline{2x^3 + 6x^2} \qquad\qquad \\ -5x^2 - 13x \qquad \\ \underline{-5x^2 - 15x} \qquad \\ 2x + 6 \\ \underline{2x + 6} \end{array}$$

Method B

	ax^2	bx	c
x	ax^3	bx^2	cx
+3	$3ax^2$	$3bx$	$3c$

$ax^3 = 2x^3 \Rightarrow a = 2$

$x^2(3a + b) = -5 \Rightarrow 3a + b = -5$

$\Rightarrow 6 = b = -5 \Rightarrow b = -11$

$3c = 6 \Rightarrow c = 2$

(d) $5x + 4y = 30 - 7{\cdot}90 = 22{\cdot}10$ ← Represent this situation with simultaneous equations.

$2x + 6y = 30 - 8{\cdot}40 = 21{\cdot}60$

$\Rightarrow x = €2{\cdot}10$

$y = €2{\cdot}90$

9. **(a)** $\left(\dfrac{1}{0{\cdot}2 + 0{\cdot}1}\right) = 3\dfrac{1}{3}$ or $\dfrac{10}{3}$ or $3{\cdot}3$ ← Substitute the values for S and P into the formula.

(b) The denominator increases so the value of the fraction decreases. ← You could check this by substituting values of P (>0.1) in the fraction in the solution to part (a) and taking a look at the effect.

(c) $M = \dfrac{1}{S+P}$ ← Multiply both sides by $(S + P)$ Subtract MS from both sides Divide both sides by M

$MS + MP = 1$

$MP = 1 - MS$

$P = \dfrac{1 - MS}{M}$ or $P = \dfrac{1}{M} - S$

10. (a) $2(7) = 14$ ← Substitute the values in.

$2(7) + 1 = 15$

Even numbers differ by 2

(b) (i) 2 is the lowest even number so adding 2 onto an even number will give the next even number.

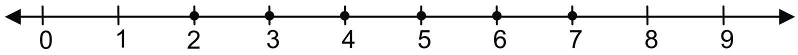
(ii) $\frac{x+2}{2} - \frac{x}{3} = 8$ ← NB "is subtracted from"

$x = 42$

Multiply each part of the inequality by 2.

Add 6 to each part of the inequality.

11. (a) $2 \leq x < 8$



(b) (i) $32x \leq 3000 - 800$ or similar

(ii) $32x \leq 2200$

$x \leq €68{\cdot}75$

Divide both sides by 32 (positive so it won't affect the inequality sign).

1 m = 1000 mm

12. (a) Width 27 mm Height 15 mm

(b) Make height of logo = 1000 mm

$\frac{15}{27} = \frac{1000}{x}$

$x = 1800$ mm (= width) (or 1·8 m) (or 180 cm)

Scale Factor $= \frac{1000}{15} = 66{\cdot}6$

$27 \times 66{\cdot}66 = 1799{\cdot}82$ mm

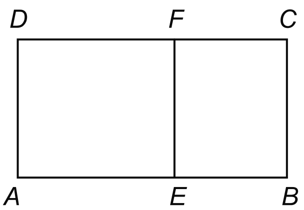
OR

Make height of logo = 1 m

$\frac{15}{27} = \frac{1}{x}$

$x = 1{\cdot}8$ m

13. (i) $x - 1$



(ii) $\frac{x}{1} = \frac{1}{x-1}$ ← "Two decimal places" is a hint to use the quadratic formula. (See page 20 of *Formulae and Tables*)

$x(x-1) = 1 \Rightarrow x^2 - x - 1 = 0$

$x = \frac{1 \pm \sqrt{5}}{2}$

$x = 1{\cdot}618\ldots = 1{\cdot}62$ cm (discard neg. value)

14.

Term 1	Term 2	Term 3	Term 4	Term 5
$2a - b + 2c$	$8a - 2b + 2c$	$18a - 3b + 2c$	$32a - 4b + 2c$	$50a - 5b + 2c$

Subtract to get the differences.

$2a - b + 2c$

$8a - 2b + 2c$ Diff $= 6a - b$

$18a - 3b + 2c$ Diff $= 10a - b$ Diff $= 4a$

$32a - 4b + 2c$ Diff $= 14a - b$ Diff $= 4a$

$50a - 5b + 2c$ Diff $= 18a - b$ Diff $= 4a$

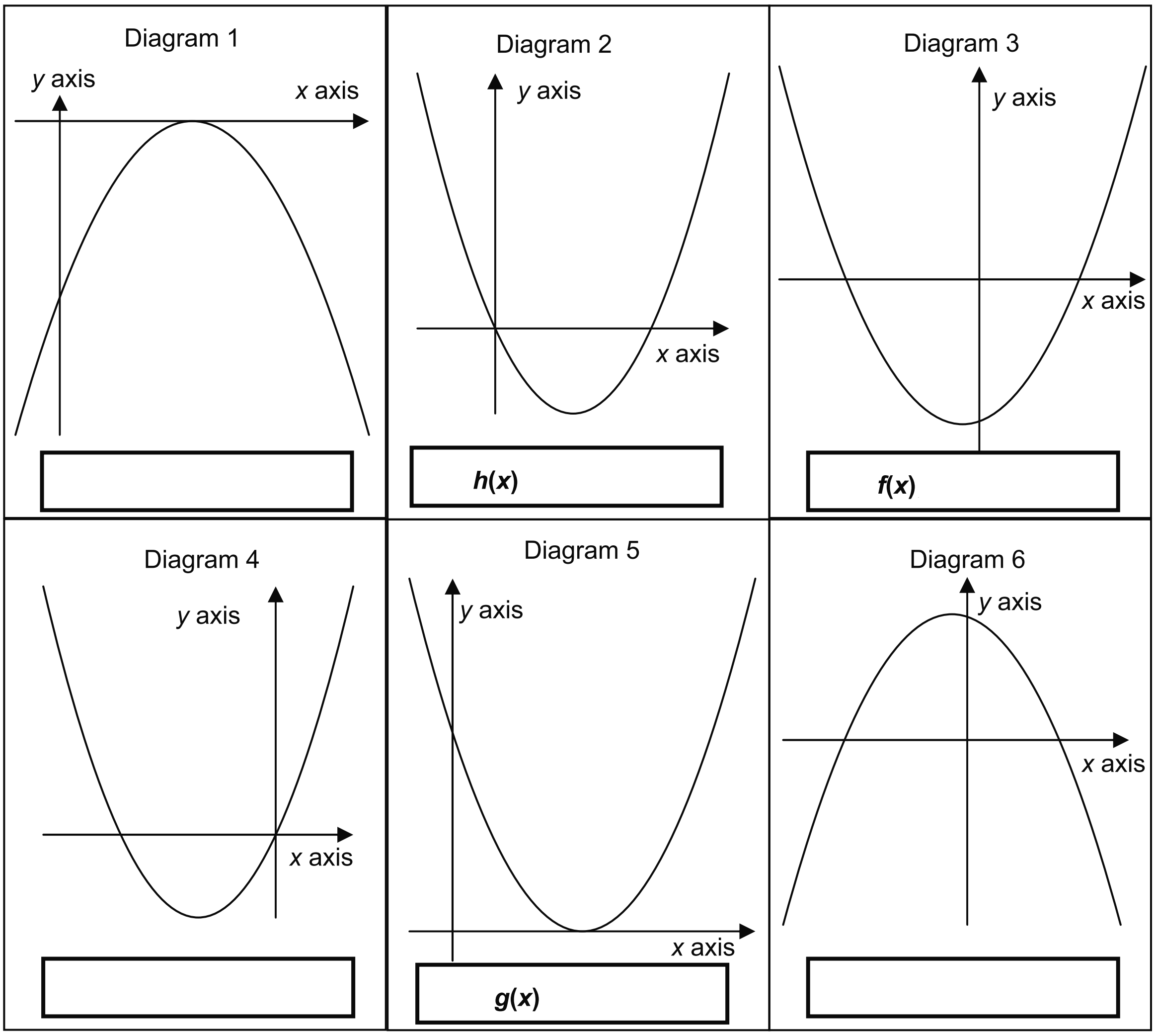
2nd difference is constant therefore the relationship is quadratic.

15. (a)

Solve $f(x) = 0$	Solve $g(x) = 0$	Solve $h(x) = 0$
$(2x - 3)(x + 2) = 0$	$(x - 3)(x - 3) = 0$	$x(x - 2) = 0$
$x = \frac{3}{2}, x = -2$	$x = 3$	$x = 0, x = 2$

The solutions to these equations are called the "roots" of the functions.

(b)



Roots represent the point(s) of intersection of the function with the x-axis.

Use the answers from (a) to decide on the appropriate sketch.

2012 SEC Paper 1 (Phase 2)

1. (a) Reason 1:

$-7{\cdot}3$ is not a positive number.

OR $-7{\cdot}3$ is a minus number.

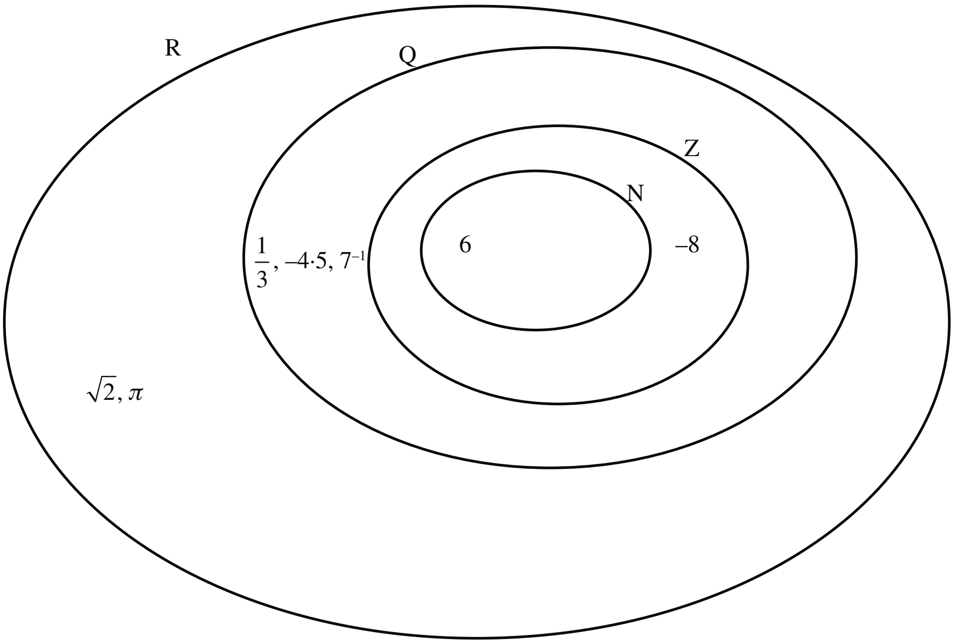
Reason 2:

$-7{\cdot}3$ is not a whole number.

OR $-7{\cdot}3$ is a decimal.

Natural numbers: Positive whole numbers excluding 0

(b)

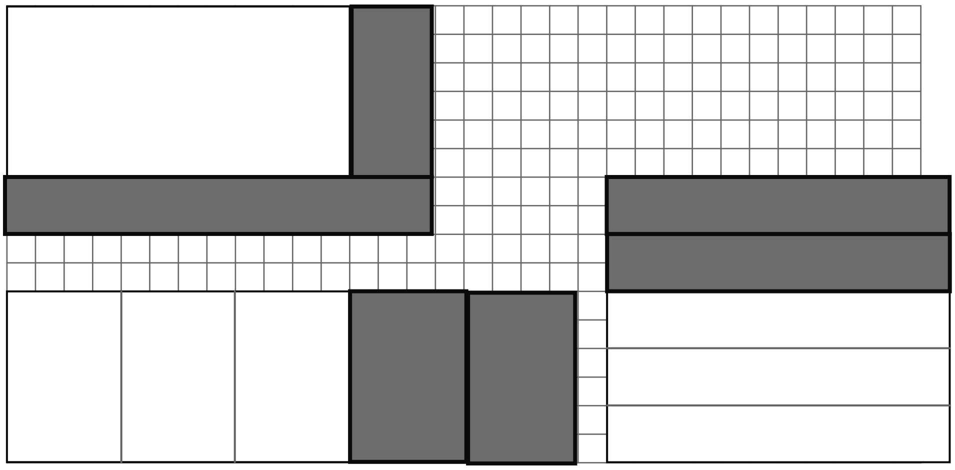


Notes: $-4{\cdot}5 = \frac{-9}{2}$ $\therefore$ Rational

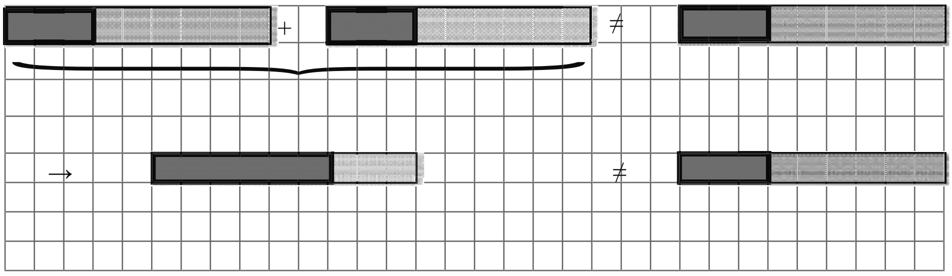
$7^{-1} = \frac{1}{7}$ $\therefore$ Rational

π and $\sqrt{2}$ can't be written as fractions $\therefore$ Irrational

2. **(a)**



(b)



3. **(a)** **(i)** €1200 × 1·405 = $1685·4

$1 = €1·4045 (Multiply both sides by 1200.)

(ii) $1685 × 0·97 = $1634·84

Or find 3% and subtract

(b) $\frac{2047}{97} \times 100 = 2110{\cdot}3$ $\quad$ 360 × 0·03 = 91·8

2110·3 × R = 3060 $\quad$ OR $\quad$ 3060 − 91·8 = 2968·2

R = 1·45 $\quad$ $\frac{2968{\cdot}2}{2047} = 1{\cdot}45$

Try to understand both methods in the solutions.

(c) 1 Euro = £0·87315 $\quad$ €1 = anything greater than 0·87315

⇒ £1 = 1·1453 Euro

£1 = anything less than €1·1453

4. **(a)** 2250 + 2600 + 150 = 5000

To get each proportion: $\frac{\text{Minutes played}}{\text{Total minutes}}$

$\frac{150\,000}{5000} \times 2250 = €67\,500$

$\frac{150\,000}{5000} \times 600 = €78\,000$

$\frac{150\,000}{5000} \times 150 = €4500$

(b) **(i)**

Michael	Paul	John
1	1·5	2·5

Let Michael be represented by 1 when building up the ratio.

50% = 140 000

$\frac{140\,000}{2{\cdot}5} \times 5 = €280\,000$ $\quad$ ⇒ 100% = €280 000

(ii) $\frac{280\,000}{5} = 56\,000$ (one part)

Michael 56 000 × 1 = €56 000

Paul 56 000 × 1·5 = €84 000

5. (a) 10 036 × 0·02 = 200·72 (2% USC charge)

5980 × 0·04 = 239·2 (4% USC charge)

[45 000 − (10 036 + 5980)] × 0·07 = 2028·88 (7% USC charge)

2028·88 + 239·2 + 200·72 = €2468·8 Total USC Charge

> Finding 2%: Multiply by 0·02
> Finding 4%: Multiply by 0·04 etc.

(b) 1650 + 1650 = €3300

(c) 32 800 × 0·2 = 6560 Tax at lower rate

45 000 − 32 800 = 12 200

12 200 × 0·41 = 5002 Tax at upper rate

6560 + 5002 = 11 562 Total gross Tax

11 562 − 3300 = €8262 Total Net Tax

> Standard Rate Cut Off Point: First €32 800 is taxed at the lower rate (20%)

(d) 45 000 − (2468·8 + 8262) = €34 269·2

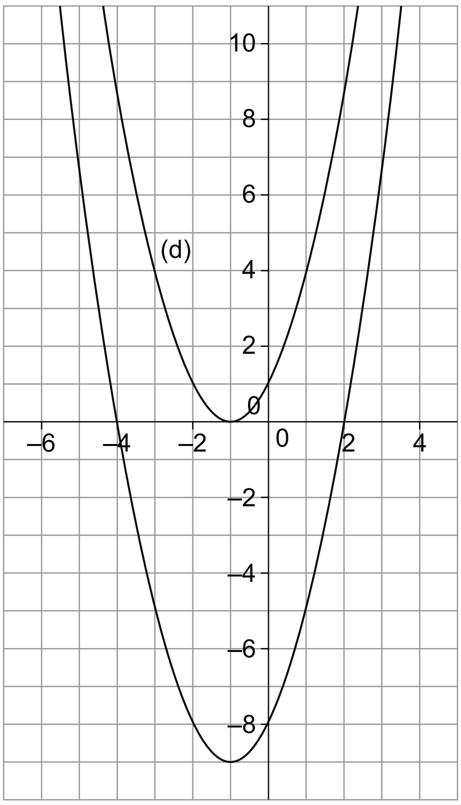
> Net Pay = Take Home Pay

2468·8 + 8262 = 10 730·8

45 000 − 10 730·8 = €34 269·2

6. (a) Roots 2 and −4

> Roots: Points of intersection of the graph and the x-axis. These are found by solving the equation.



(b) $f(x) = x^2 + 2x + k$

$2 = 3^2 + 2(3) + k$

$2 = 9 + 6 + k$

$k = -13$

> Substitute (3, 2) into the function and it will lead to an equation in k.

2012 SEC P1

(c) $(x + t)^2 = x^2 + 2x + k$

$x^2 + 2tx + t^2 = x^2 + 2x + k$

$\Rightarrow 2t = 2$

$t = 1$

(d) Shown in part (a). ← Two roots are the same: x-axis is a tangent to the graph of the function.

(e) $(x + 5)(x - 3) = 0$

$x^2 + 2x - 15 = 0$ ← If $x = 5$ is a root then $(x - 5)$ is a factor etc.

$\Rightarrow k = -15$

Constant is product of roots

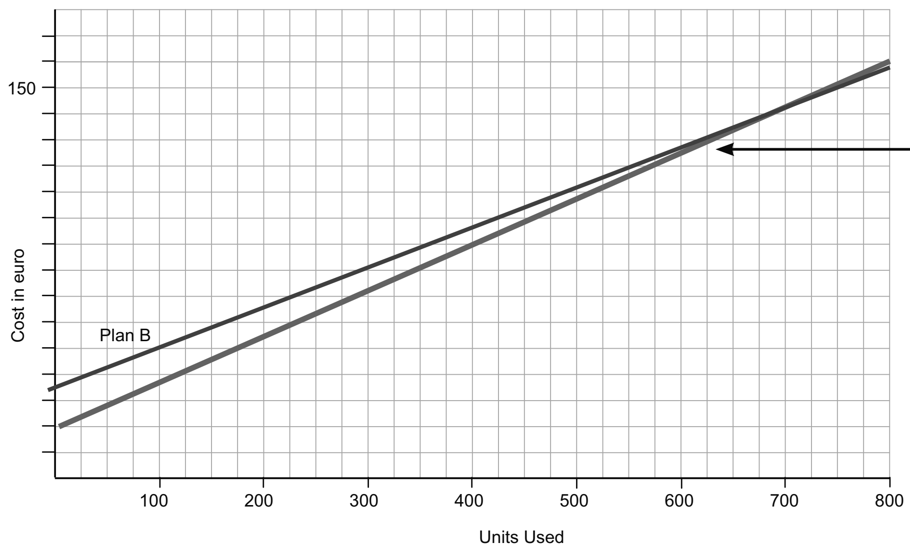
$-5 \times 3 = -15$

$\Rightarrow k = -15$

7. (a) $56 - 38 = 18$, $74 - 56 = 18$, $92 - 74 = 18$, $110 - 92 = 18$, $128 - 110 = 18$, $146 - 128 = 18$, $164 - 146 = 18$ ← The first difference is constant for all linear functions.

Common first difference of 18

(b)



A continuous line is needed.

(c) €20 ← Point of intersection with the y-axis.

(d) Method:

When the units used go down by 100 then the cost goes down by 18.

$\Rightarrow 38 - 18 = 20$

$m = \dfrac{56 - 38}{200 - 100} = 0{\cdot}18 \left(\text{or } \dfrac{9}{50}\right)$ ← Slope formula: See page 18 of *Formulae and Tables*.

$y - 38 = 0{\cdot}18\,(x - 100)$ ← Equation formula: $y - y_1 = m(x - x_1)$ See page 18 of *Formulae and Tables*.

$0{\cdot}18x - y + 20 = 0$

sub $x = 0$

$\Rightarrow y = 20$

Standing charge: €20

(e) Cost $= 20 + 0{\cdot}18x$

(f) $650 \times 0{\cdot}18 + 20 = 137$

$155{\cdot}5 - 137 = 18{\cdot}5$

$\dfrac{18{\cdot}5}{137} \times 100 = 13{\cdot}5\%$ VAT

$650 \times 18 + 2000 = 13\,700$

$15\,550 - 13\,700 = 1850$

$\dfrac{1850}{13\,700} \times 100 = 13{\cdot}5\%$ VAT

Rate $= \dfrac{\text{Amount of VAT}}{\text{Total}} \times 100\%$

(g)

Units Used	Plan B Cost in euro
100	€51·50
200	€67·00
300	€82·50
400	€98·00
500	€113·50
600	€129·00
700	€144·50
800	€160·00

(h) **Scenario 1: Concentrates on 650 units**

[$36 + 0{\cdot}155 \times 650 = €136{\cdot}75$]

The cost of Plan A and Plan B are very similar therefore it doesn't really matter which plan Lisa chooses.

Scenario 2: Concentrates on low and/or high usage

If Lisa tends to use a low number of units on average, then plan A is better but if she uses a high number of units on average then Plan B is better.

OR

Lisa should choose plan B as it is 25c cheaper. See graph in part (c).

(i) Continuous line again.

(j) 640 units ← Point of Intersection

8. **(a)** $W = \dfrac{1}{2}CV^2$

$W = \dfrac{1}{2}(2500)(32)^2$ ← Substitute in the values for *C* and *V*.

$W = 1\,280\,000$

(b) $W = \frac{1}{2}CV^2$

$2W = CV^2$

$\frac{2W}{C} = V^2$

$\sqrt{\frac{2W}{C}} = V$

Multiply by 2.
Divide both sides by C.
Take the square root of both sides.

9. (a) 2w + 2d = 12

1w + 5d = 10

2w + 2d = 12

−2w − 10d = −20

−8d = −8

8d = 8

⇒ D = 1 point

Substitute d = 1

2w = 12 − 2

2w = 10

W = 5 points

Use Simultaneous Equations

Trial and error with verification of both solutions is awarded full marks.

(b) With the new system, the ratio of win:draw is higher which rewards a victory more and might encourage a team to go for a win.

10. $\frac{1}{2}(2x)(x + 3) = 10$

$x^2 + 3x - 10 = 0$

$(x + 5)(x - 2) = 0$

⇒ $x = -5$ (not possible) and $x = 2$ cm

Area of a Triangle Formula:
$A = \frac{1}{2}\, b \times h$
See page 9 of *Formulae and Tables*.
Note: x must be positive (length).

11. (i) $5x^2(x - 2)$ ← H.C.F

(ii) $(2x - 9y)(2x + 9y)$ ← Difference of two squares

(iii) $a(a - b) + 3(a - b)$

$(a + 3)(a - b)$ ← Factor by grouping terms

12. (a) (i) $(x - 6)(x + 1) = 0$

⇒ $x = 6$ or $x = -1$

$a = 1, b = -5, c = -6$ ← Quadratic equation

$x = \frac{5 \pm \sqrt{(-5)^2 - 4(1)(-6)}}{2(1)}$

$x = \frac{5 \pm \sqrt{(25 + 24)}}{2}$

$x = \frac{5 \pm 7}{2}$

$x = 6, x = -1$

(ii) $(4x - 1)(2x - 3) = 0$

⇒ $x = \frac{1}{4}$ or $x = \frac{3}{2}$

$x = 14 \pm \frac{\sqrt{(-14)^2 - 4(8)(3)}}{2(8)}$ ← Quadratic equation

$x = \frac{14 \pm \sqrt{196 - 96}}{2(8)}$

$x = \frac{1}{4}, x = \frac{3}{2}$

(iii) $2(2x + 5) - 3(4x - 1) = 3(-1)$ ← Multiply across by 6

$4x + 10 - 12x + 3 = -3$

$8x = 16$

$x = 2$

(b) $x = \frac{7 \pm \sqrt{49 - 4(2)(-6)}}{4}$ ← Roots are found by solving the equation. 'Two decimal places' is a hint to use the quadratic formula (page 20, *Formulae and Tables*)

$x = \frac{7 \pm \sqrt{49 + 48}}{4}$

$x = \frac{7 \pm \sqrt{97}}{4}$

$x = \frac{7 \pm 9{\cdot}848859}{4}$

$x = 4{\cdot}21$ or $x = -0{\cdot}71$

13.

Statement	**Always true**	**Never true**	**Sometimes true**
If $a > b$ and $b > c$, then $a > c$	✓		
If $-a < 4$ and $b < -4$, then $a < b$		✓	
If $a > b$, then $-a > -b$		✓	
If $a > b$ and $b < c$, then $a < c$			✓
If $3a + 1 > 2$, then $a > 0$	✓		
If $2b - 4 < 3b - 8$, then $b > 4$	✓		
If a and b are both positive and $a < b$, then $\frac{1}{a} < \frac{1}{b}$		✓	

Examine each statement carefully. Substitution of real numbers could help to understand what is being asked each time.

14. (a) $g(3) = 2^0 = 1$ ← Replace x with 3

(b) **(i)** $h(t) = t^2 - 3t$

$h(2t + 1) = (2t + 1)^2 - 3(2t + 1)$ ← Replace x with t and $(2t + 1)$

$= 4t^2 + 4t + 1 - 6t - 3$

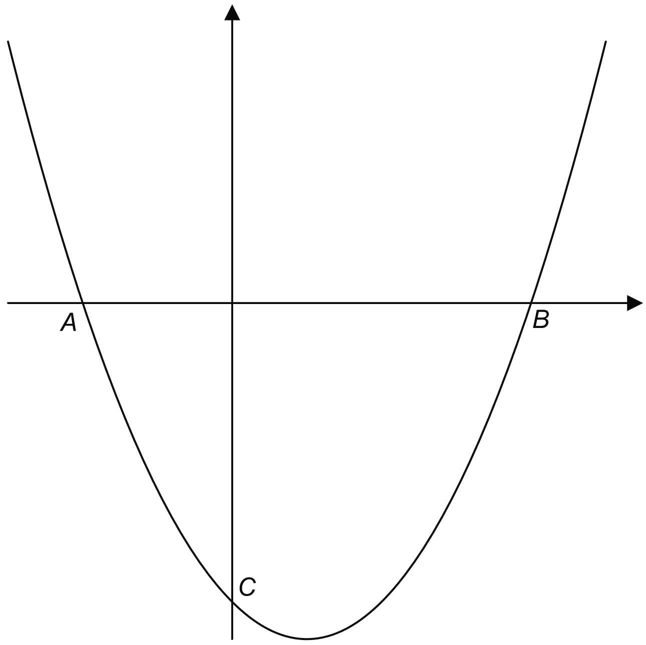
(ii) $4t^2 - 2t - 2 = t^2 - 3t$ ← Equate

$3t^2 + 2t - 2 = 0$

$(3t - 2)(t + 1) = 0$

$t = \frac{2}{3}, t = -1$

(c)



(i) C | A, B

Sub $x = 0$ | $f(x) = 0$

$y = -8$ | $(x - 4)(x + 2) = 0$

$C(0, -8)$ | $\Rightarrow x = 4$ or $x = -2$

> A and B are roots which are found by solving the equation.
> C is the y-intercept which is the constant term ($x = 0$).

(ii) $-2 \leq x \leq 4$

> Where the function is on or below the x-axis

Educate.ie Sample 1

Paper 2

1. **(a)** $\frac{1}{3}\pi r^2 h = \frac{1}{3}\pi(3)(3)(4) = 12\pi \text{ cm}^3$

Volume of a cone: See page 10 of *Formulae and Tables*.

(b) **(i)** $12 \times 5 \times H = 90$

$60H = 90$

$H = 90 \div 60$

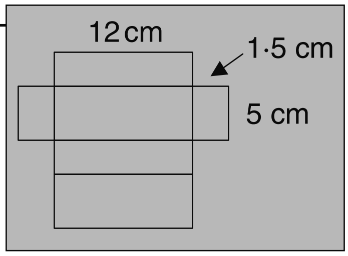
$H = 1{\cdot}5 \text{ cm}$

(ii) $2(L \times W) + 2(L \times H) + 2(W \times H)$

$2(12 \times 5) + 2(12 \times 1{\cdot}5) + 2(5 \times 1{\cdot}5)$

$= 120 + 36 + 15$

$= 171 \text{ cm}^2$



(iii) $1 \text{ Block } 90 \text{ cm}^3 = 90 \times 8{\cdot}4 = 756 \text{ g}$

$113{\cdot}4 \div 0{\cdot}756 = 150 \text{ blocks}$

(c) **(i)** $2\pi rh + 4\pi r^2$

$= 2(3{\cdot}14)(42)(170) + 4(3{\cdot}14)(42)(42)$

$= 44{,}839{\cdot}2 + 22{,}155{\cdot}84$

$= 66{,}995{\cdot}04$

$= 67{,}000 \text{ cm}^2$

Surface Area (SA) of capsule = SA of cylinder + SA of two hemispheres

(ii) $\pi r^2 h + \frac{4}{3}\pi r^3$

$= (3{\cdot}14)(0{\cdot}42)(0{\cdot}42)(1{\cdot}70) + (1{\cdot}333)(3{\cdot}14)(0{\cdot}42)(0{\cdot}42)(0{\cdot}42)$

$= 0{\cdot}9416232 + 0{\cdot}310104214$

$= 1{\cdot}251727415$

$= 1{\cdot}25 \text{ m}^3$

Volume of capsule = Volume of cylinder + Volume of two hemispheres

2. **(a)**

$x - 2y = -2$

$x + 2y = 6$

$2x = 4$

$x = 2$

$2 - 2y = -2$

$2 + 2 = 2y$

$4 = 2y$

$2 = y$

Point (2, 2)

Solving simultaneously

(b) $x - 2y + 2 = 0$ let $x = 0$

$0 - 2y + 2 = 0$

$2 = 2y$

$1 = y$

$C = (0, 1)$

(c) $(0, 3) \in x + 2y - 6 = 0$

$0 + 2(3) - 6 = 0$

$0 = 0$

(d) Slope of $l = \frac{1}{2}$

Slope of $m = -2$ Point $(4, -2)$

$y + 2 = -2(x - 4)$

$y + 2 = -2x + 8$

$2x + y = 8 - 2$

$2x + y = 6$

Equation of a line formula: $y - y_1 = m(x - x_1)$. See page 18 of *Formulae and Tables*.

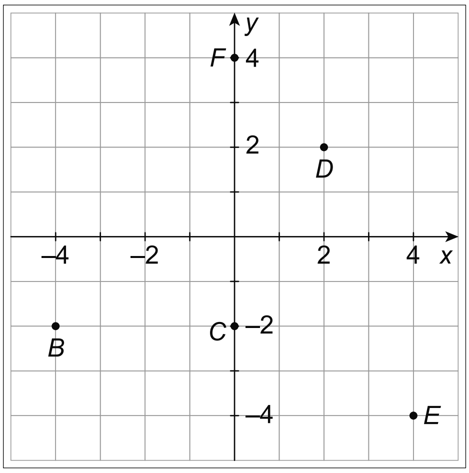
3. A = Axial symmetry in the y axis

B = Central symmetry

C = Translation

4. (a) $A = (-5, 3)$

(b)



(c) $\left(\frac{-4 + 0}{2}, \frac{-2 - 2}{2}\right) = \left(\frac{-4}{2}, \frac{-4}{2}\right) = (-2, -2)$

Midpoint formula: See page 18 of *Formulae and Tables*.

(d) $\frac{4 + 2}{0 + 4} = \frac{6}{4} = \frac{3}{2}$

Slope formula: See page 18 of *Formulae and Tables*.

(e) $y - 4 = \frac{3}{2}(x - 0)$

$2y - 8 = 3x$

$3x - 2y + 8 = 0$

Equation of a line formula: $y - y_1 = m(x - x_1)$. See page 18 of *Formulae and Tables*.

5. (a) Approx. 35 times

(b) Approx. 105 times

Probability $= \frac{\text{Favourable outcomes}}{\text{Total outcomes}}$

(c) Approx. 105 times

6. (a)

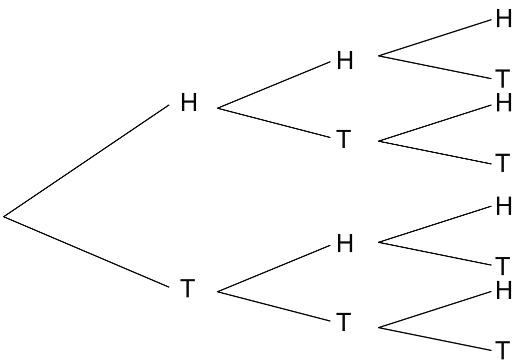
Class A		Class B
	6	3 3 5 7 9
9 8 7 6	7	0 2 6 7 8
4 3 2 2 0	8	1 1 3
7 7 4 2 1 1	9	2
	10	0
		Key 9|2 = 92

(b) 83 ← Median is the middle value if the results are arranged in ascending/descending order.

(c) The lowest score was 76%.

(d) One student scored 100%.

7. (a)



← Tree diagram

Sample space
(HHH), (HHT), (HTH), (HTT)
(THH), (THT), (TTH), (TTT)

(b) $\frac{1}{8}$

(c) $\frac{3}{8}$ ← Probability $= \frac{\text{Favourable outcomes}}{\text{Total outcomes}}$

(d) $\frac{1}{8}$

(e) $\frac{7}{8}$

8. (a) $Y = 9x + 10$ line 3, as the slope is 9.

(b) Lines 1 and 4 both have the same slope 3. ← The lines are in the form of $y = mx + c$. m is the slope of the line.

(c) Lines 5 and 6 since $-\frac{1}{2} \perp 2$. ← The slopes are $-\frac{1}{2}$ and 2. As these multiply to give -1, the lines are perpendicular.

9. **(a)** $A(2, 3) \xrightarrow{+3\,-4} B(5, -1)$

$P(6, 7) \longrightarrow Q(9, 3)$

$Q = (9, 3)$

(b) $\sqrt{(5-2)^2 + (-1-3)^2} = \sqrt{(9-6)^2 + (3-7)^2}$

$\sqrt{(3)^2 + (-4)^2} = \sqrt{(-3)^2 + (-4)^2}$

$5 = 5$

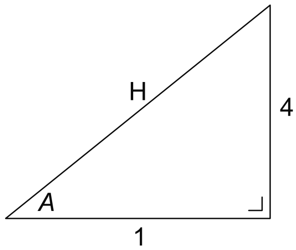
Distance formula:
$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
See page 18 of *Formulae and Tables*.

10. **(a)** $\text{Tan } A = \dfrac{\text{Opposite}}{\text{Adjacent}} = \dfrac{4}{1}$

$\text{Hypotenuse} = \sqrt{4^2 + 1^2}$

$= \sqrt{17}$

$\text{Cos } A = \dfrac{\text{Adjacent}}{\text{Hypotenuse}} = \dfrac{1}{\sqrt{17}}$



(b) 60 mins = 384 km

1 mins = 384 ÷ 60 = 6·4 km

25 mins = 6·4 × 25

= 160 km

OR

$\text{Speed} = \dfrac{\text{Distance}}{\text{Time}}$

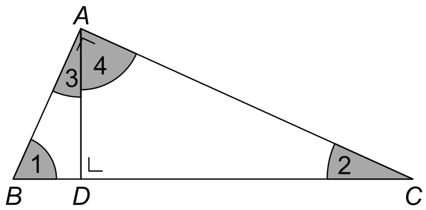
$384 = \dfrac{\text{Distance}}{0.4166666}$

Distance = (384)(0·41666666) = 160 km

(c) $\sin 36 = \dfrac{|AB|}{160}$

$|AB| = 160 \sin 36 = 94$ km

11. **Diagram**



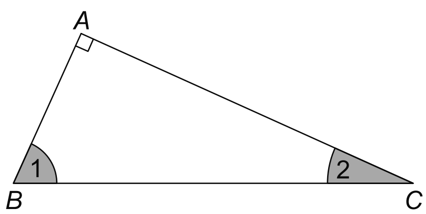
Given: A triangle *ABC* in which *A* is 90°

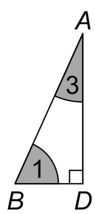
To prove: $|BC|^2 = |AB|^2 + |AC|^2$

Construction: Draw $AD \perp BC$ and mark in the angles 1, 2, 3 and 4.

Proof: Consider the triangles *ABC* and *ABD*

Standard proof





∠1 is common to both triangles.

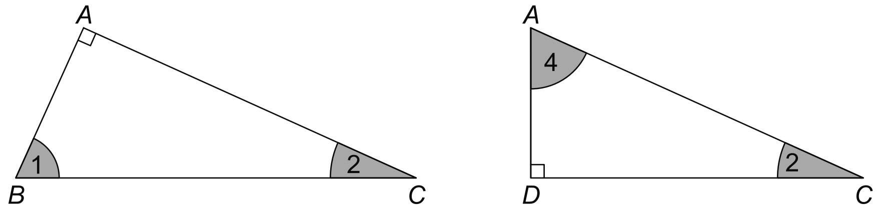
$|\angle BAC| = |\angle ADB| = 90°$

ΔABC and ΔABD are similar.

$\dfrac{|BC|}{|AB|} = \dfrac{|AB|}{|BD|}$

$|AB|^2 = |BC| \cdot |BD|$ (equation 1)

Now consider the triangles *ABC* and *ADC*.



$\angle 2$ is common to both.

$|\angle BAC| = |\angle ADC| = 90°$

so ΔABC and ΔADC are similar.

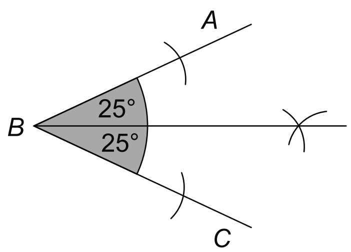
$$\frac{|AC|}{|DC|} = \frac{|BC|}{|AC|}$$

$|AC|^2 = |BC| \cdot |DC|$ (equation 2)

Adding equation 1 and equation 2 we get

$$|AB|^2 + |AC|^2 = |BC| \cdot |BD| + |BC| \cdot |DC|$$
$$= |BC| \cdot (|BD| + |DC|)$$
$$= |BC| \cdot |BC|$$
$$|AB|^2 + |AC|^2 = |BC|^2$$
$$|BC|^2 = |AB|^2 + |AC|^2$$

12.



Steps:
1. Place the compass on the point *B* and draw two arcs on the arms of the angle.
2. Place the compass point where the arcs cut the arms and draw two arcs to cut each other.
3. Join *B* to the point where the arcs cut.
4. This line is the bisector of the angle *ABC*.

13. (a) 60–80 interval

(b) 40–60 km/h

(c)

10	30	50	70	90
8	24	40	18	10

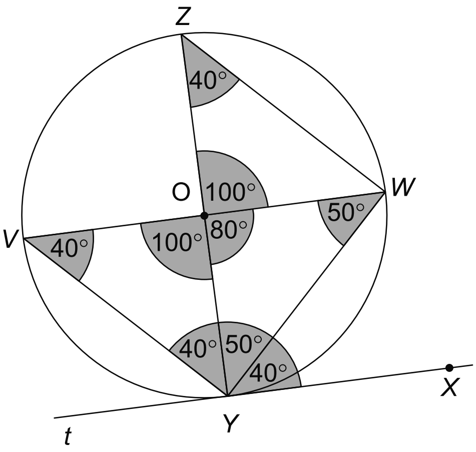
$$= \frac{(10 \times 8) + (30 \times 24) + (50 \times 40) + (70 \times 18) + (90 \times 10)}{100}$$

$$= \frac{80 + 720 + 2000 + 1260 + 900}{100}$$

$$= \frac{4960}{100} = 49{\cdot}6 \text{ km/hr}$$

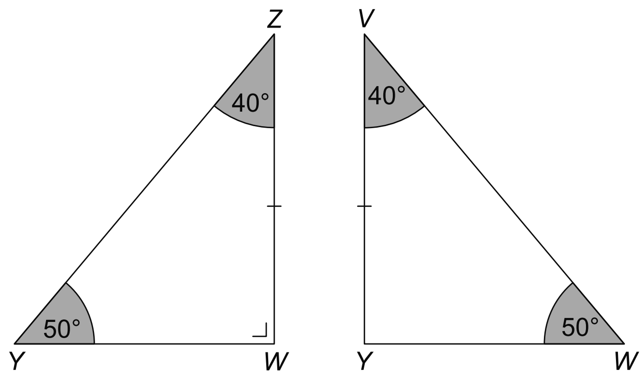
(d) 18 + 10 = 28

14.



(a) 40°

(b)



Both triangles are congruent since

$\|\angle YZW\| = \|\angle YVW\|$	40°
$\|ZY\| = \|VW\|$	diameter
$\|\angle ZYW\| = \|\angle VWY\|$	50°

15. $(10)^2 = (6)^2 + (h)^2$ ← Pythagoras's Theorem

$100 = 36 + h^2$

$64 = h^2$

$8 \text{ cm} = h$

16. 4, 4, 6, 7, 9 ← If the mode is 4 then we must have at least two 4s. As the median is 6 and there are five numbers, 6 is the middle number. As the range is 5 and I am assuming that 4 is the lowest, the maximum number is 9. As the mean of the five numbers is 6 then the total must be 30. So the final number must be 7.

17. (a)

	(1, 0)	**(2, 0)**	**(3, 0)**
(0, 1)	$\sqrt{2}$	$\sqrt{5}$	$\sqrt{10}$
(0, 2)	$\sqrt{5}$	$\sqrt{8}$	$\sqrt{13}$
(0, 3)	$\sqrt{10}$	$\sqrt{13}$	$\sqrt{18}$

← Use Pythagoras's Theorem. Example (0, 2) and (3, 0)

$\sqrt{(3)^2 + (2)^2} = \sqrt{13}$

(b) Looking for greater than $\sqrt{9}$,

$\Rightarrow \frac{5}{9}$

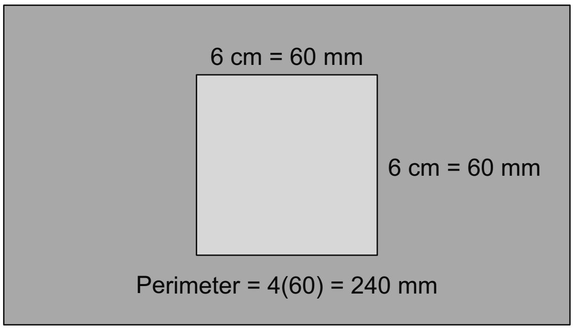
(c) All have a negative slope ← Use diagram to find slopes of the line segments. They all have negative slopes.

$\Rightarrow 1$

Educate.ie Sample 2

Paper 2

1. (a)



Area = 36 cm^2 = 6 cm × 6 cm
Each side is 6 cm or 60 mm.

(b) (i) Area of $\frac{1}{4}$ of circle = $\frac{1}{4}\pi r^2$

$\frac{1}{4}\pi(14)^2 = 49\pi$ cm^2

Area of a disc = πr^2
See page 8 of *Formulae and Tables*.

(ii) Length of $\frac{1}{4}$ of a circle = $\frac{1}{4}2\pi r$

$\frac{3{\cdot}14 \times 14}{2} = 21{\cdot}98$ cm

Length of a circle = $2\pi r$
See page 8 of *Formulae and Tables*.

Perimeter of shaded region = 21·98 + 14 + 14 = 49·98 cm

14 cm = radius

(c) (i) Volume of hemisphere = $\frac{2}{3}\pi r^3$

$= \frac{2}{3}\pi\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right) = \frac{\pi}{12}$ cm^3

Volume of a sphere = $\frac{4}{3}\pi r^3$
See page 10 of *Formulae and Tables*.

(ii) Volume of disk: $\pi r^2 x = \frac{\pi}{12}$

$3^2 x = \frac{1}{2}$

$x = \frac{1}{108} = 0{\cdot}00925$ cm = 0·09 mm

Volume of a cylinder = $\pi r^2 h$
See page 10 of *Formulae and Tables*.

2. 3 × 8 = 24 choices

This is the Fundamental Principle of Counting.
3 options and then 8 options give a total of 3 × 8 outcomes.

3. (a) 18 outcomes

This is the Fundamental Principle of Counting.
3 options and then 6 options give a total of 3 × 6 outcomes.

(b)

	A	B	C	D	E	F
1	**1, A**	**1, B**	**1, C**	**1, D**	**1, E**	**1, F**
2	**2, A**	**2, B**	**2, C**	**2, D**	**2, E**	**2, F**
3	**3, A**	**3, B**	**3, C**	**3, D**	**3, E**	**3, F**

(c) 3 outcomes

(d) $\frac{3}{18}$ or $\frac{1}{6}$ ← Probability = $\frac{\text{Favourable outcomes}}{\text{Total outcomes}}$

(e) $\frac{6}{18}$ or $\frac{1}{63}$

4. **(a)** Generally this type of data is ignored.

Examples: Row 12, Row 4, Row 21, Row 22

(b) Boys: Mean = 1713 ÷ 12 = 142·75 cm

Girls: Mean = 1697 ÷ 12 = 141·42 cm

According to this data the average height of the boys in this class seems to be greater than that of the girls.

(c)

Girls							**Boys**		
						12	8	8	9
		8	5	5	2	13	0	2	6
5	5	2	2	0	0	14	0	2	3
				0	0	15	1		
						16	2		
						17			
						18			
						19	2		
						20			

Key 13|2 means 132 cm

(d) Any two such as: The girls have a smaller range of heights. The boys have a bigger range of heights. There is one boy who is very tall compared with the rest of the boys.

5. Area of shaded triangle = $\frac{1}{2}(8) \times 8 = 32$ cm^2 ← Area of triangle = $\frac{1}{2}$ base × perp. height

Area of shaded rectangle = $4 \times 8 = 32$ cm^2 ← Area of rectangle = base × height

6.

Equation	A	B	C	D
Graph	4	1	3	2

7. **Proof:**

$|\angle AOC| = |\angle DOB|$ … … … Vertically opposite

$|\angle AO| = |\angle OB|$ … … … Radii

$|\angle CO| = |\angle OD|$ … … … Radii

$\therefore\ \Delta AOC$ and ΔDBO are congruent … … … $SAS = SAS$

8. **(i)** $\frac{3}{6}$ or $\frac{1}{2}$ ← Probability $= \frac{\text{Favourable outcomes}}{\text{Total outcomes}} = \frac{\text{Three 1s}}{6} = \frac{3}{6} = \frac{1}{2}$

(ii) $\frac{2}{6}$ or $\frac{1}{3}$ ← Probability $= \frac{\text{Favourable outcomes}}{\text{Total outcomes}} = \frac{\text{Two 2s}}{6} = \frac{2}{6} = \frac{1}{3}$

(iii) $\frac{1}{6}$ ← Probability $= \frac{\text{Favourable outcomes}}{\text{Total outcomes}} = \frac{\text{One 3}}{6} = \frac{1}{6}$

9. **(i)** Chimney A: $\tan 64° = \frac{\text{Approximate height}}{100}$

Approximate height = 100 × tan 64° = 205 m

Chimney B: $\tan 25° = \frac{\text{Approximate height}}{75}$

Approximate height = 75 × tan 25° = 35 m

Ratio of the heights $= \frac{205}{35} = 41 : 7$

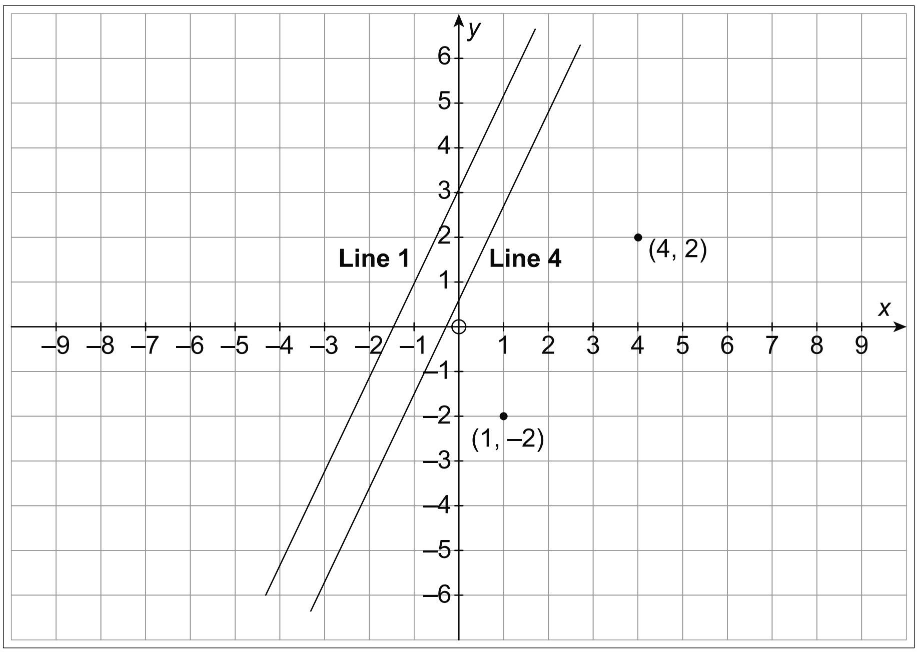
This is approximately 6 : 1

(ii) Any two ways such as:

- Taking her own height into account.
- Taking the diameters of the bases of the chimneys into account.
- Any other reasonable way.

10. **(a)** 1 and 4: 2 and 6: 3 and 5:

(b)



No

(c) On the axes above, plot the points represented by the couples (1, –2) and (4, 2).

(d) $\frac{y_2 - y_1}{x_2 - x_1} = \frac{2+2}{4-1} = \frac{4}{3}$ Point (4, 2)

$y - y_1 = m(x - x_1)$

$y - 2 = \frac{4}{3}(x - 4)$

$3y - 6 = 4x - 16$

$4x - 3y - 10 = 0$

(e) No ← Because the slope of this line is $\frac{4}{3}$ and none of the lines 1 to 6 have a slope of $\frac{4}{3}$.

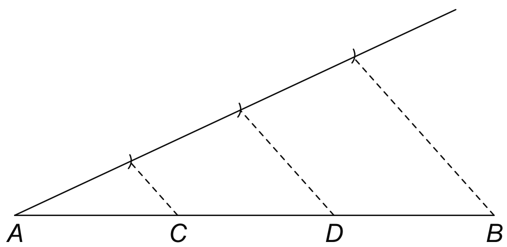
11. (a) cos $\angle ABC = 12 \div 15 = 0{\cdot}8$

sin $\angle BCA = 12 \div 15 = 0{\cdot}8$

(b) $|\angle ABC| = 37°$ ← Calculator:  36·869°

$|\angle BCA| = 53°$ ← Calculator:  53·13°

12. (a)



← Steps:
1. Draw another line from *A* at an angle to the first line.
2. With a compass, draw three arcs of equal length to cut this line.
3. Join the outside arc to *B*.
4. Through the other arc, draw a line parallel to the line going through the first arc. *AB* will now be divided into three equal segments.
5. Label the points *C* and *D* on the line segment *AB*.

(b) $|AC|$ = Depends on individual diagram.

$|CD|$ = Depends on individual diagram.

$|DB|$ = Depends on individual diagram.

Should all be equal

13. *A* = (–5, 3) *B* = (5, 3) *C* = (5, –3) *D* = (–5, –3)

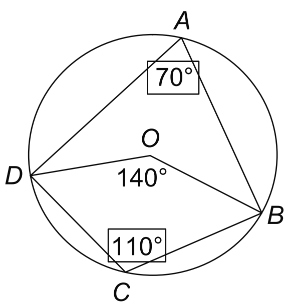
B is the image of *A* under Axial symmetry in the *y* axis.

C is the image of *A* under Central symmetry in the origin.

D is the image of *A* under Axial symmetry in the *x* axis.

14. If the sides of two triangles are proportional then the two triangles are similar.

15.



← This question is based on the theorem: "The angle at the centre of a circle is twice the angle at the circle standing on the same arc."

Educate.ie Sample 3

Paper 2

1. **(a)** **(i)** $\pi r^2 = (3\cdot14)(11)(11) = 379\cdot94 = 380 \text{ cm}^2$ ← Area of a disc: $= \pi r^2$ See page 8 of *Formulae and Tables*.

(ii) $\frac{120}{360} \times (3\cdot14)(11)(11) = 127 \text{ cm}^2$

(b) **(i)** $2\pi rh = 2(\pi)(7)(20) = 280\pi \text{ cm}^2$ ← Length of a circle: $= 2\pi r$ See page 8 of *Formulae and Tables*.

(ii) $2\pi rh + \pi r^2 + \frac{1}{2}(4\pi r^2)$

$= 2(\pi)(7)(20) + \pi(7)(7) + 2\pi(7)(7)$

$= 280\pi + 49\pi + 98\pi$

$= 427\pi \text{ cm}^2$

(c) **(i)** $\frac{1}{3}\pi(x)(x)(3x) = x^3\pi \text{ units}^3$ ← Volume of a cone: $= \frac{1}{3}\pi r^2 h$ See page 10 of *Formulae and Tables*.

(ii) Volume of 2nd cone $= \frac{1}{3}(\pi)(2x)(2x)(1\cdot5x)$

$= 2x^3\pi \text{ units}^3$

Ratio $= 2x^3\pi : x^3\pi$

$= 2:1$

2. **(a)** Categorical **(b)** Numerical **(c)** Categorical **(d)** Numerical **(e)** Categorical

3. **(a)** $H^2 = 2^2 + 7^2$ ← Use Pythagoras's Theorem

$H^2 = 4 + 49$

$H = \sqrt{53}$

(b) $\sin A = \frac{\text{Opposite}}{\text{Hypotenuse}}$

$= \frac{7}{\sqrt{53}}$

(c) $\cos A = \frac{\text{Adjacent}}{\text{Hypotenuse}} = \frac{2}{\sqrt{53}}$

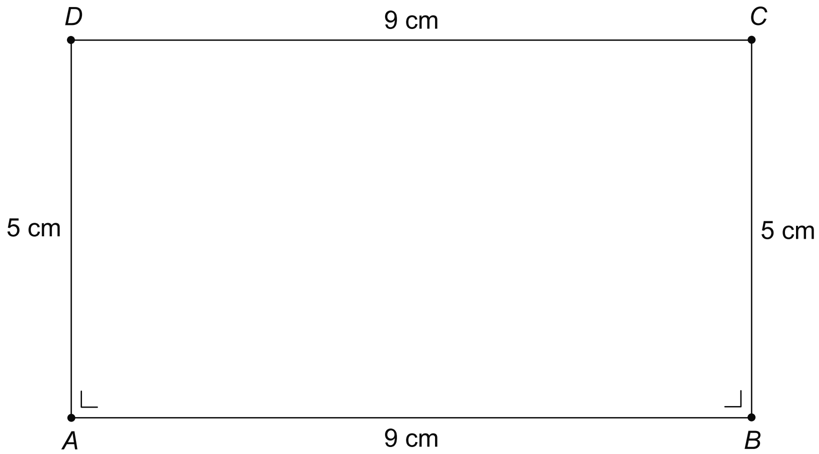
(d) $\cos A = \frac{2}{\sqrt{53}}$

$\cos A = 0{\cdot}274721127$

$A = \cos^{-1}(0{\cdot}274721127)$

$A = 74°$ ← Calculator:  74·06˚

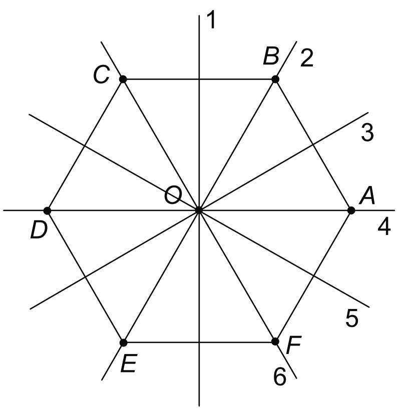
4.



Steps:
1. Draw a line 9 cm in length with a ruler. Label as *AB*.
2. With the centre of the protractor on *A*, mark out an angle of 90º.
3. With a compass, mark off 5 cm along the 90º line. Label as *D* where the arc cuts the line.
4. With the centre of the protractor on *B*, mark out an angle of 90º.
5. With a compass, mark off 5 cm along this 90º line. Label as *C* where the arc cuts the line.
6. Join *D* to *C*.

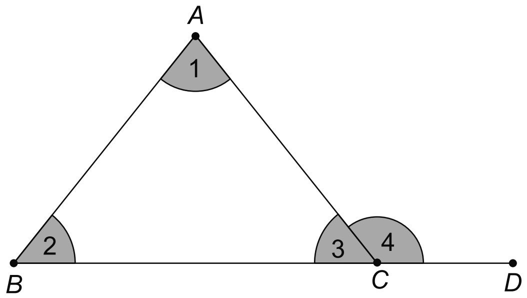
5. (a) 6

(b)



(c) *A* translation *A* to *C* or *A* translation *F* to *D*.

6. Diagram



Given: A triangle *ABC* with the side [*BC*] extended to the point *D*. The angles marked 1, 2, 3 and 4 are also given.

To prove: $|\angle 4| = |\angle 1| + |\angle 2|$

Construction: None

Proof: $|\angle 3| + |\angle 4| = 180°$ — Straight angle

$|\angle 1| + |\angle 2| + |\angle 3| = 180°$ — Angles in a triangle ← Standard proof

$|\angle 3| + |\angle 4| = |\angle 1| + |\angle 2| + |\angle 3|$ — Both equal 180°

$|\angle 4| = |\angle 1| + |\angle 2|$ — Subtract $|\angle 3|$

7. (a) $|\angle BCA| = 90°$ since the angle at the centre of the circle is a straight angle of 180° and is twice the size of $|\angle BCA|$.

(b) Since $|\angle BCA| = 90°$ then $|\angle CAB| = 45°$ which, by theorem, is equal to $|\angle CDB|$.
Answer $|\angle CDB| = 45°$

(c) By Pythagoras's Theorem

$|AB|^2 = |AC|^2 + |BC|^2$

$(12)^2 = 2(BC)^2$

$144 = 2(BC)^2$

$72 = BC^2$

$\sqrt{72} = |BC|$

(d) Area $= \sqrt{72} \times \sqrt{72} = 72 \text{ cm}^2$

8. (a) $\dfrac{6 + 11 + 15 + 16 + 17}{5} = \dfrac{65}{5} = 13$

(b) $\dfrac{6 + 11 + 15 + 16 + 17 + x}{6} = 14$

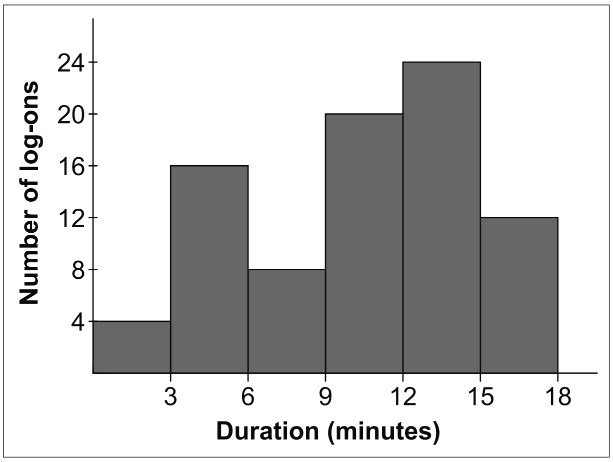
$65 + x = 6(14)$

$65 + x = 84$

$x = 84 - 65$

$x = 19$

9. (a)



A histogram is chosen as it is suitable to display intervals in time.

(b) $4 + 16 + 8 + 20 + 24 + 12 = 84$

(c) Median value = 42 which is in the interval 9–12

> The median line will divide the area of the histogram into two equal parts.

10.

	Male	**Female**	**Total**
Wearing glasses	16	18	34
Not wearing glasses	9	7	16
Total	25	25	50

(a) $\frac{25}{50} = \frac{1}{2}$

> **Probability** $= \frac{\text{Favourable outcomes}}{\text{Total outcomes}} = \frac{\text{25 males}}{50} = \frac{1}{2}$

(b) $\frac{16}{50} = \frac{8}{25}$

> **Probability** $= \frac{\text{Favourable outcomes}}{\text{Total outcomes}} = \frac{\text{16 not wearing glasses}}{50} = \frac{8}{25}$

(c) $\frac{16}{50} = \frac{8}{25}$

> **Probability** $= \frac{\text{Favourable outcomes}}{\text{Total outcomes}} = \frac{\text{16 males wear glasses}}{50} = \frac{8}{25}$

11. (a) $\tan 56{\cdot}31 = \frac{|XY|}{50}$

$|XY| = 50 \tan 56{\cdot}31$

> Calculator: Key as you see: Answer = 74·9718

$|XY| = 75$ m

(b) $(100)^2 = (75)^2 + (KX)^2$

> Use Pythagoras's Theorem

$(KX)^2 = 10{,}000 - 5{,}625$

$|KX| = \sqrt{4375}$

$|KX| = 66$ m

$|KT| = 66 - 50 = 16$ m

12. (a) $|\angle BCD| = 180 - 53 = 127°$

(b) $12h = 90$

$h = 90 \div 12 = 7{\cdot}5$ cm

13. **(a)** Line 4: the slope is $\frac{1}{4}$ or 0·25

(b) Line 2 and line 5. Both slopes are –3

(c) Line 1 and line 4

Since

$-4 \times \frac{1}{4} = -1$

(d) Cuts x-axis at $y = 0$

$0 = -3x + 12$

$3x = 12$

$x = 4 \quad (4, 0)$

Cuts y-axis at $x = 0$

$y = -3(0) + 12$

$y = 12 \quad (0, 12)$

14. **(a)** (1, 1) (1, 2) (1, 3) (1, 4) (1, 5) (1, 6)

(2, 1) (2, 2) (2, 3) (2, 4) (2, 5) (2, 6)

(3, 1) (3, 2) (3, 3) (3, 4) (3, 5) (3, 6)

(4, 1) (4, 2) (4, 3) (4, 4) (4, 5) (4, 6)

(5, 1) (5, 2) (5, 3) (5, 4) (5, 5) (5, 6)

(6, 1) (6, 2) (6, 3) (6, 4) (6, 5) (6, 6)

(b) Scores that add up to 9 are

(6, 3) (4, 5) (5, 4) (3, 6)

$= \frac{4}{36} = \frac{1}{9}$

(c) Scores that add up to 10 are

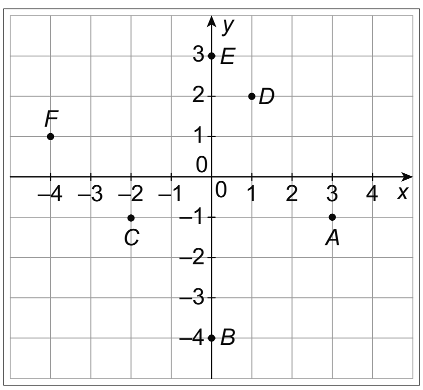
(4, 6) (6, 4) (5, 5) $= \frac{3}{36} = \frac{1}{12}$

(d) 4 or less = (1, 1) (1, 2) (1, 3) (2, 1) (2, 2) (3, 1) $= \frac{6}{36} = \frac{1}{6}$

(e) $\frac{4}{36} + \frac{3}{36} = \frac{7}{36}$

15. **(a)** (3, –1)

(b)



(c) $\left(\frac{-2-4}{2}, \frac{-1+1}{2}\right) = \left(\frac{-6}{2}, \frac{0}{2}\right) = (-3, 0)$

Midpoint formula: See page 18 of *Formulae and Tables*.

(d) $\frac{-1+4}{-2-0} = \frac{3}{-2} = \frac{-3}{2}$ or $-1\frac{1}{2}$

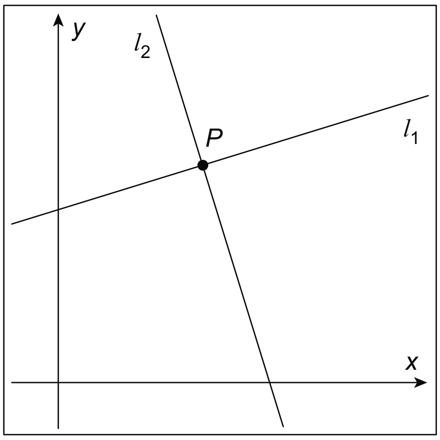
Slope formula: See page 18 of *Formulae and Tables*.

(e) $\frac{2+1}{1-3} = \frac{3}{-2} = -1\frac{1}{2}$

Yes [*AD*] is parallel to [*BC*].

They have the same slope.

16.



Slope of l_1 is $\frac{1}{3} = \frac{Rise}{Run}$ and it is in the positive direction.

Slope of l_1 is –3 so it is perpendicular to the line l_1.

17. (a) $P(\text{May}) = \frac{31}{365} = 0{\cdot}085$

(b) $P(\text{June}) = \frac{30}{365} = 0{\cdot}082$

(c) $P(\text{May or June}) = \frac{30+31}{365} = \frac{61}{365} = 0{\cdot}17$

Educate.ie Sample 4

Paper 2

1. (a) $\pi r^2 h = (3{\cdot}14)(4{\cdot}5)(4{\cdot}5)(9) = 572{\cdot}3 \text{ cm}^3$

> Volume of a cylinder: $= \pi r^2 h$
> See page 10 of *Formulae and Tables*.

(b) (i) $4x = 96$

$x = 96 \div 4 = 24$ m

Area $= x^2 = (24)(24) = 576 \text{ m}^2$

> Curved Surface Area of cylinder: $= 2\pi rh$
> See page 10 of *Formulae and Tables*.

(ii) $2\pi rh = 2(3{\cdot}14)(0{\cdot}375)(1) = 2{\cdot}4 \text{ m}^2$

(iii) $9 \times 2{\cdot}4 = \frac{21{\cdot}6}{576} \times 100 = 3{\cdot}75\%$

(c) (i) $\frac{1}{3}\pi r^2 h = (0{\cdot}333)\pi(2)(2)(6) = 8\pi \text{ cm}^3$

> Volume of a cone: $= \frac{1}{3}\pi r^2 h$
> See page 10 of *Formulae and Tables*.

(ii) Amount of sand $= 4\pi \text{ cm}^3$

Rate of flow $\frac{4}{45}\pi \text{ cm}^3$ per second

Time $= \frac{4\pi}{\frac{4}{45}\pi} = 45$ seconds

> $4\pi \div \frac{4\pi}{45} = 4\pi \times \frac{45}{4\pi} = 45$

2. A = Central symmetry

B = Translation

C = Axial symmetry in the y-axis

3. (a) Line 6 has slope of –11

> $4\pi \div \frac{4\pi}{45} = 4\pi \times \frac{45}{4\pi} = 45$

Reason: 6 is the largest coefficient of x when the equations are written as $y = mx + c$.

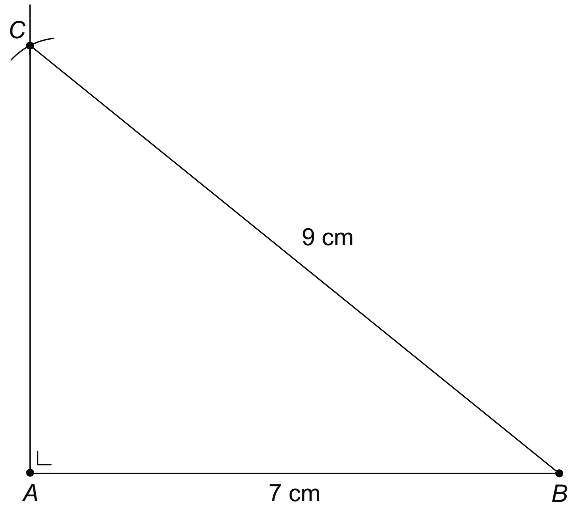
(b) Line 2 and line 5 have slopes of 3.

> Parallel lines have the same slope.

(c) Lines 3 and 4 since $-2 \times \frac{1}{2} = -1$

> For perpendicular lines the product of the slopes should equal −1. ($m_1 m_2 = -1$)

4.



> Steps:
> 1. Draw a line 7 cm in length with a ruler. Label as *AB*.
> 2. With the centre of the protractor on *A*, mark out an angle of 90°. Draw a line which will be 90° to the line *AB*.
> 3. With a compass point on *B*, draw an arc of length 9 cm to cut the 90° line from *A*. Label as C.
> 4. Join *B* to *C*.

5. (a) $(-1, h)$ is on $3(-1) - 4(h) + 7 = 0$

$$-3 + 7 = 4h$$
$$4 = 4h$$
$$1 = h$$

Substitute the point $(-1, h)$ into the line $3x - 4y + 7 = 0$ to find h.

$(k, 0)$ is on $4x + 3y - 24 = 0$

$$4(k) + 3(0) - 24 = 0$$
$$4k = 24$$
$$k = 6$$

Substitute the point $(k, 0)$ into the line $4x + 3y - 24 = 0$ to find k.

(b)

$3x - 4y = -7$ (mult. 3)

$4x + 3y = 24$ (mult. 4)

Solving simultaneously

$$9x - 12y = -21$$
$$16x + 12y = 96$$
$$25x = 75$$
$$x = 3$$

Substitute into top equation

$$3(3) - 4y = -7$$
$$9 - 4y = -7$$
$$9 + 7 = 4y$$
$$16 = 4y$$
$$4 = y$$

Point of intersection $(3, 4) = R$

6. (a)

Die

Spinner	**1**	**2**	**3**	**4**	**5**	**6**
1	(1, 1)	(2, 1)	(3, 1)	(4, 1)	(5, 1)	(6, 1)
2	(1, 2)	(2, 2)	(3, 2)	(4, 2)	(5, 2)	(6, 2)
3	(1, 3)	(2, 3)	(3, 3)	(4, 3)	(5, 3)	(6, 3)

(b) Less than 3 = 1

When the outcomes are added the only outcome giving less than 3 is (1, 1).

(c) $P(8) = \frac{2}{18} = \frac{1}{9}$

There are two outcomes that, when added, give a total of 8 and they are (5, 3) and (6, 2). There are 18 outcomes in total.

$$\text{Probability} = \frac{2}{18} = \frac{1}{9}$$

7. (a) $|\angle AOB| = 100°$ or twice $|\angle ACB|$

(b) $|\angle ADB| = 130°$ or half $|\angle AOB| = 260°$

(c) $|\angle OAB| = 40°$

$|\angle BAD| = 25°$

so $|\angle OAD| = 65°$

8. **(a)** Discrete **(b)** Discrete **(c)** Continuous **(d)** Continuous **(e)** Discrete

9. **(a)** P let $y = 0$ $\quad 3x - 0 + 9 = 0$

$$3x = -9$$

$$x = -3 \qquad (-3, 0) = P$$

Q let $x = 0$ $\quad 3(0) - 2y + 9 = 0$

$$-2y = -9$$

$$y = 4\frac{1}{2} \qquad \left(0, 4\frac{1}{2}\right) = Q$$

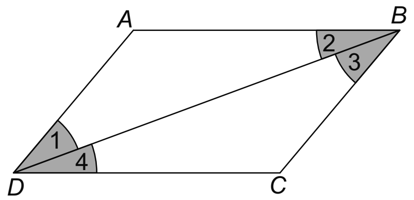
(b) $P \rightarrow Q \rightarrow (?, ?)$

$(-3, 0) \quad \left(0, 4\frac{1}{2}\right) \quad (3, 9)$

$+3 + 4\frac{1}{2}$

$(3, 9)$

10. **Diagram**



Given: A parallelogram $ABCD$

To prove: $|AB| = |DC|$ and $|AD| = |BC|$

Construction: Draw the diagonal $[BD]$ and mark in the angles 1, 2, 3 and 4.

Proof: ← This is a standard proof of this theorem

In the triangles ABD and BCD

$|\angle 1| = |\angle 3|$ Alternate

$|\angle 2| = |\angle 4|$ Alternate

$|DB| = |DB|$ Common

Hence $\Delta ABD \equiv \Delta BCD$

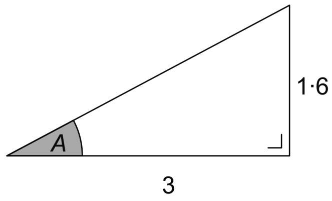
Therefore $|AB| = |DC|$

$|AD| = |BC|$ Corresponding side

Also $|\angle DAB| = |\angle DCB|$ Corresponding angles

and $|\angle ABC| = |\angle ADC|$

11. (a)

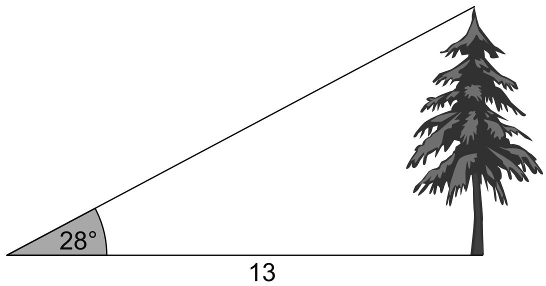


$\text{Tan } A = \frac{1 \cdot 6}{3}$

Tan A = 0·53333

A = 28°

(b)



$\text{Tan } 28° = \frac{\text{Tree}}{13}$

Tree = 13 tan 28 ← Calculator: Key as you see: Answer = 6·91222

= 6·9 m

12. (a) $\frac{3 + 1 + 9 + x + 5}{5} = 6$

$18 + x = 30$

$x = 12$

(b)

Marks	0–40	40–60	60–80	80–100
No. of students	3	9	9	4

(c) Mean $= \frac{(3 \times 20) + (9 \times 50) + (9 \times 70) + (4 \times 90)}{25}$ ← 20, 50, 70 and 90 are the mid-interval values

$= \frac{60 + 450 + 630 + 360}{25}$

$= \frac{1500}{25}$

$= 60$

13. (a) $A = (2, 3)$

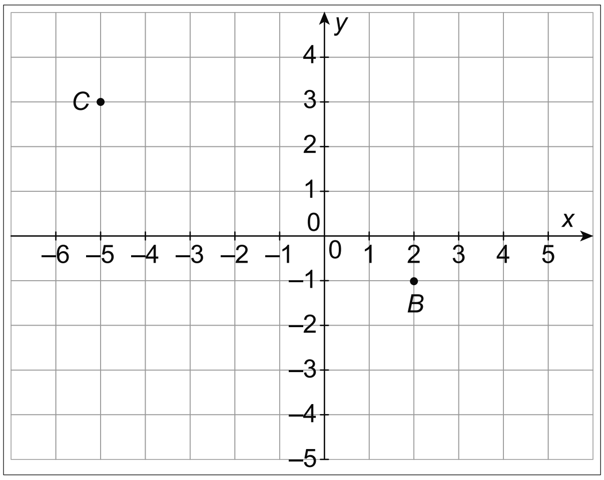
(b) $(–2, –1) \rightarrow (0, –1) \rightarrow (2, –1)$

$B = (2, –1)$

(c) $(-5, -3) \rightarrow (-5, 0) \rightarrow (-5, 3)$

$C = (-5, 3)$

The answer to part **(b)** and **(c)** are plotted on the Cartesian plane.



(d) Slope of $BC = \frac{3+1}{-5-2} = \frac{-4}{7}$

Slope formula: See page 18 of *Formulae and Tables*.

$$y + 1 = -\frac{4}{7}(x - 2)$$

$$7y + 7 = -4x + 8$$

$$4x + 7y = 1$$

$$7y = -4x + 1$$

$$y = -\frac{4}{7}x + \frac{1}{7}$$

Equation formula: $y - y_1 = m(x - x_1)$
See page 18 of *Formulae and Tables*.

14. (a) $\frac{2}{3} = \frac{x}{9}$

$\cos C = \frac{\text{Adjacent}}{\text{Hypotenuse}} = \frac{x}{9}$

$3x = 18$

$x = 6$

(b) $\cos C = \frac{6}{9}$

$C = \cos^{-1}\left(\frac{2}{3}\right)$

$C = 48°$

15. (a) $60° + 90° = 150°$

(b) 15°

Since $180 - 150 = 30° \div 2$

16.

	A	B	C	D
The data is skewed to the right.	✗	✗	✗	✓
The data is skewed to the left.	✗	✗	✓	✗
There is a single mode.	✗	✓	✓	✓

Sample 4 Educate.ie P2

17. **(a)** $(7x - 8) + (5x + 8) + (10x - 10) + (3x - 5) = 360$

$$25x - 15 = 360$$

$$25x = 375$$

$$x = 375 \div 25$$

$$x = 15°$$

(b) $|\angle ABC| = 360° - (7(15) - 8)$

$= 360 - 97$

$= 263°$

Educate.ie Sample 5

Paper 2

1. **(a)** $2(3\cdot14)(8) + 20 + 20 = 90$ cm

> The lengths of two semicircles added give the length of one circle: Length of a circle = $2\pi r$
> See page 8 of *Formulae and Tables.*

(b) **(i)** $\pi r^2 h = \pi(14)(14)(57) = 11\,172\pi$ cm^3

> Volume of a cylinder: $= \pi r^2 h$
> See page 10 of *Formulae and Tables*.

(ii) $\frac{1}{3}\pi r^2 h = \frac{1}{3}\pi(14)(14)(57) = 3724\pi$ cm^3

> Volume of a cone: $= \frac{1}{3}\pi r^2 h$
> See page 10 of *Formulae and Tables*.

(iii) $11{,}172\pi - 3724\pi = \frac{7448\pi}{111{,}172\pi} = \frac{2}{3}$

(c) **(i)** $\frac{\pi r^2}{4} = \frac{\pi(80)(80)}{4} = 1600(3\cdot14)$ m^2

$= 5024$ m^2

(ii) $\pi r^2 = (3\cdot14)(1\cdot2)(1\cdot2) = 4\cdot52$ m^2

(iii) $\frac{3}{4}(4\cdot52) + 5024$ m^2

$= 3\cdot39 + 5024$

$= 5027\cdot39$ m^2

2. **(a)** 110° since the sum of the angles in the polygon *ABED* is 360°

(b) 90° since $|\angle BAC| = |\angle CAD| = 35°$ and $|\angle ADC| = 55°$

(c) Since $|BE|$ is $\perp$ to $|AB|$ and $|DE|$ then

$|\angle ABC| = |\angle DEC|$ … both 90°

$|\angle BAC| = |\angle DCE|$ … both 35°

$|\angle BCA| = |\angle CDE|$ … both 55°

(d) $|AC|^2 = |AB|^2 + |BC|^2$

> Use Pythagoras's Theorem

$|AC|^2 = (5)^2 + (3\cdot5)^2$

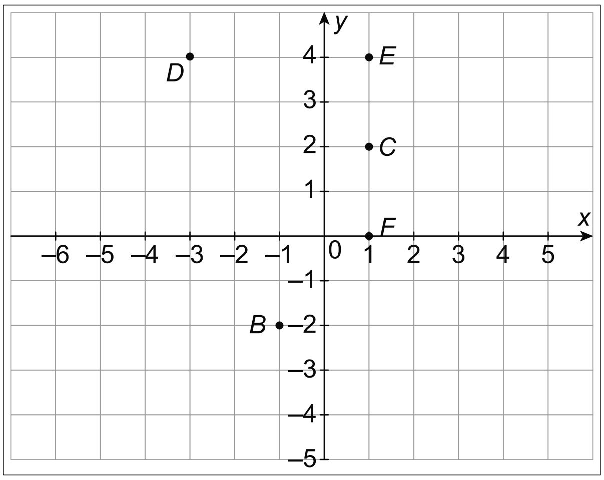
$|AC|^2 = 25 + 12\cdot25$

$|AC|^2 = 37\cdot25$

$|AC| = \sqrt{37\cdot25} = 6\cdot1$

3. **(a)** $A = (-5, 0)$

(b)



(c) $\left(\frac{-1+1}{2}, \frac{-2+2}{2}\right) = \left(\frac{0}{2}, \frac{0}{2}\right) = (0, 0)$ ← Midpoint formula: See page 18 of *Formulae and Tables*.

(d) $\frac{2-0}{1+5} = \frac{2}{6} = \frac{1}{3}$ ← Slope formula: See page 18 of *Formulae and Tables*.

(e) $y - 0 = \frac{1}{3}(x + 5)$ ← Equation formula: $y - y_1 = m(x - x_1)$ See page 18 of *Formulae and Tables*.

$3y = x + 5$

$y = \frac{1}{3}x + \frac{5}{3}$

(f) $\frac{4-2}{-3-1} = \frac{2}{-4} = -\frac{1}{2}$ ← Slope formula: See page 18 of *Formulae and Tables*.

(g) $y - 2 = -\frac{1}{2}(x - 1)$ ← Equation formula: $y - y_1 = m(x - x_1)$ See page 18 of *Formulae and Tables*.

$2y - 4 = -x + 1$

$2y = -x + 1 + 4$

$2y = -x + 5$

$x + 2y - 5 = 0$

(h) The two lines are perpendicular, meeting at 90° angles to each other. This can be seen in part (**b**) when the points are plotted on the Cartesian plane.

4. **(a)** The word 'implies' means that when one statement is made, another statement follows on logically from the first statement.

(b) A corollary is a statement that follows readily from a previous theorem.

(c) Multiple possible answers. For example: A diagonal divides a parallelogram into 2 congruent triangles.

(d) An axiom is a rule or statement that we accept without any proof.

(e) Multiple possible answers. For example: There is exactly one line through any two given points.

5. (a)

No. of shots	66–69	69–72	72–75	75–81
No. of rounds	7	19	15	9

(b) $\frac{(7 \times 67{\cdot}5) + (19 \times 70{\cdot}5) + (15 \times 73{\cdot}5) + (9 \times 78)}{50}$ ← 67·5, 70·5, 73·5 and 78 are the mid-interval values.

$= \frac{472{\cdot}5 + 1339{\cdot}5 + 1102{\cdot}5 + 702}{50}$

$= \frac{3616{\cdot}5}{50}$

$= 72$

6. (a) Line 5 because it has a slope of 8

(b) Lines 2 and 6 since both have slopes of 4

(c) Lines 2 and 3 or lines 6 and 3
Since $4x - \frac{1}{4} = -1$
The product of their slopes equals –1.

(d) Cuts the x axis at $y = 0$
$0 = 7x - 14$
$7x = 14$
$x = 2$
(2, 0)
Cuts the y axis at $x = 0$
$y = 7(0) - 14$
$y = -14$
(0, –14)

7. (a) $|\angle ACB| = 75°$ since $180 - 30 = 150 \div 2 = 75°$

(b) $|\angle ACD| = 105°$ since it makes a straight angle

8. (a) $|\angle BOC| = 80°$ since $|\angle OBC| = 50°$ also since Δ is isosceles

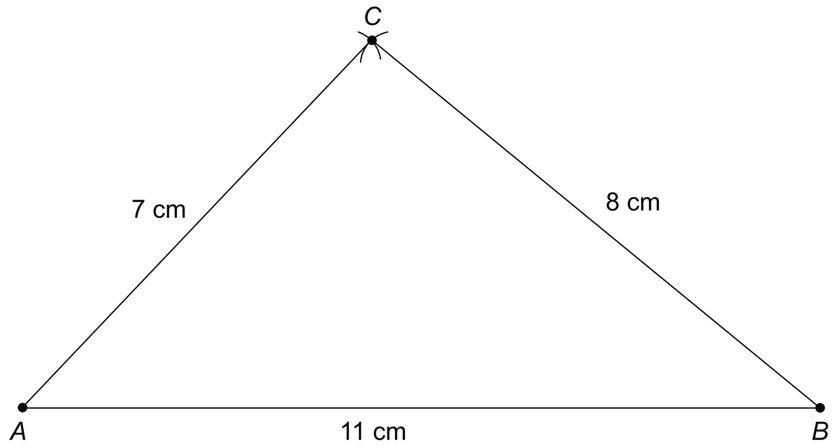
(b) $|\angle BAC| = 40°$ since it is equal to angle $|\angle OCA|$

(c) Since they are both standing on the same arc BD

(d) $|\angle BAE| = |\angle EDC|$ Alternate
$|AB| = |CD|$ Told
$|\angle ABE| = |\angle ECD|$ Alternate
Hence the two triangles are congruent by ASA.

Sample 5
Educate.ie P2

9.



Steps:

1. With a ruler, draw a line segment 11 cm in length. Label as *AB*.
2. Place the compass on the point *A* and mark off an arc that is 7 cm from *A*.
3. Place the compass on the point *B* and mark off an arc that is 8 cm from *B*.
4. Label where the two arcs meet as *C*.

10. (a)

2nd spin \ **1st spin**	**Black**	**White**	**Grey**
Black	BB	WB	GB
White	BW	WW	GW
Grey	BG	WG	GG

(b) 3 ← BB, WW, GG

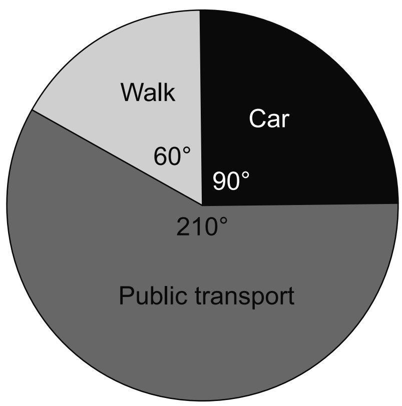
(c) $\frac{6}{9} = \frac{2}{3}$ ← Probability $= \frac{\text{Favourable outcomes}}{\text{Total outcomes}} = \frac{\text{6 with different colours}}{\text{9 total outcomes}}$

11. (a) A pie chart (or a bar chart or a line plot) as one can quickly identify all the different segments.

(b) Public transport $= \frac{35}{60} \times 360 = 210°$

Car $= \frac{15}{60} \times 360 = 90°$

Walk $= \frac{10}{60} \times 360 = 60°$



12. (a) $|\angle ADB| = 45°$ since $|\angle BAD| = 90°$ and $|\angle ABD|$ is also equal to 45°

(b) $|\angle DAC| = 45°$ since ΔACD is an isosceles triangle and we previously know $|\angle ADB|$ is also 45°.

13. **(a)** If one event has m possible outcomes and a second event has n possible outcomes, then the total number of possible outcomes is $m \times n$.

(b) $4 \times 6 \times 2 = 48$

(c) $4 \times 6 = 24$, hence $120 \div 24 = 5$. So there are 5 desserts.

14. **(a)**

1	1	4	5	7	9				
2	1	2	2	2	2	2	3	8	9
3	1	1	3	5	5	7	8		
4	0	1	2						
									Key 4|0 = 40

(b) $23 + 28 = 51 \div 2 = 25{\cdot}5$

(c) 24

(d) $42 - 11 = 31$

(e) 22

(f) He was best in the class.

15. **(a)** $$\frac{(0 \times 7) + (1 \times 9) + (2 \times 11) + (3 \times 12) + (4 \times 7) + (5 \times 4)}{7 + 9 + 11 + 12 + 7 + 4}$$

$$= \frac{0 + 9 + 22 + 36 + 28 + 20}{50} = \frac{115}{50} = 2{\cdot}3$$

(b) $\frac{12 + 7 + 4}{50} \times 100 = \frac{23}{50} \times 100 = 46\%$

(c) 3 days ← The most common number of days missed

16. **(a)** $350 - 62 - 51 - 58 - 54 - 62 = 63$

(b) $\frac{62}{350} = 0{\cdot}18$

$\frac{51}{350} = 0{\cdot}15$

$\frac{58}{350} = 0{\cdot}17$

$\frac{54}{350} = 0{\cdot}15$

$\frac{63}{350} = 0{\cdot}18$

$\frac{62}{350} = 0{\cdot}18$

(c) The die may be slightly biased, but as the die is thrown more often the relative frequency of all the numbers should be $\frac{1}{6} = 0{\cdot}17$. ← Not enough trials were done to decide.

17. €6·60 − €0·90 − €1·14

= €4·56 ÷ 4

= €1·14

Educate.ie Sample 6

Paper 2

1. (a) $2(3{\cdot}14)(4)(8) = 201 \text{ cm}^2$

Curved surface area of cylinder = $2\pi rh$
See page 10 of *Formulae and Tables.*

(b) (i)
$$2L + 2(\pi)\left(\frac{100}{\pi}\right) = 400$$
$$2L + 200 = 400$$
$$L = 100 \text{ m}$$

(ii) Each lap = 400 m so after 1200 m you are at point A, hence the finishing line is at D.

(iii) 1500 m = 3 mins 26 sec
1500 m = 206 sec
$7{\cdot}3$ m = 1 sec
Speed = $7{\cdot}3$ m/s

(c) (i) $\frac{4}{3}(\pi)(2)(2)(2) = 10\frac{2}{3}\pi \text{ cm}^3$

Volume of a sphere: $= \frac{4}{3}\pi r^3$
See page 10 of *Formulae and Tables.*

(ii) $\pi r^2 h = \pi(5)(5)(6) = 150\pi \text{ cm}^3$

Volume of a cylinder = $\pi r^2 h$
See page 10 of *Formulae and Tables.*

(iii)
$$\pi r^2 h = 10\frac{2}{3}\pi$$
$$25\pi h = 10\frac{2}{3}\pi$$
$$h = \frac{10\frac{2}{3}}{25}$$
$$h = 0{\cdot}43 \text{ cm}$$

2. (a) $180 - 84 = 96°$

Opposite angles of a cyclic quadrilateral add to 180˚.

(b) $180 - 73 = 107°$

3. (a)
$$\sin A = 0{\cdot}5$$
$$A = \sin^{-1}(0{\cdot}5)$$
$$A = 30°$$

You could also use trigonometric ratios. See page 13 of *Formulae and Tables.*

(b)
$$\cos 30° = \frac{20}{x}$$
$$x \cos 30 = 20$$
$$x = \frac{20}{\cos 30}$$
$$x = 23{\cdot}09 \text{ m}$$

Sample 6
Educate.ie P2

(c) $\tan 30 = \dfrac{h}{20}$

$h = 20 \tan 30$

$h = 11{\cdot}55$ m

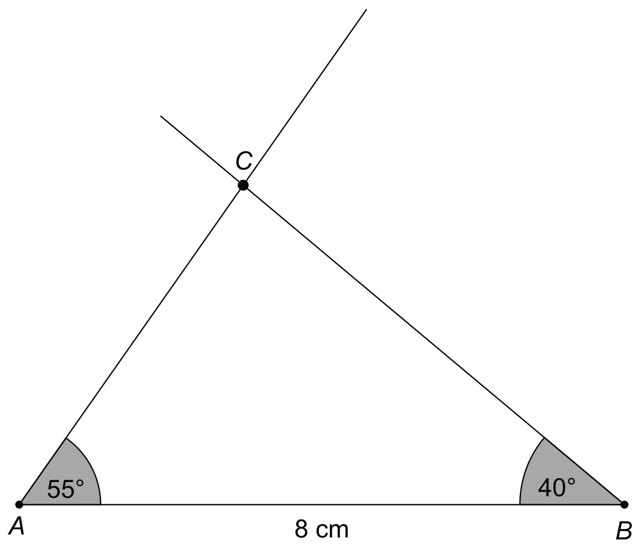
4. (a) A theorem is a rule or statement that you can prove by following a certain number of logical steps or by using a previous theorem or axiom that you already know.

(b) Vertically opposite angles are equal in measure (multiple possible answers).

(c) The converse of a theorem is the reverse of the theorem.

(d) If two angles in a triangle are equal then the triangle is isosceles (multiple possible examples).

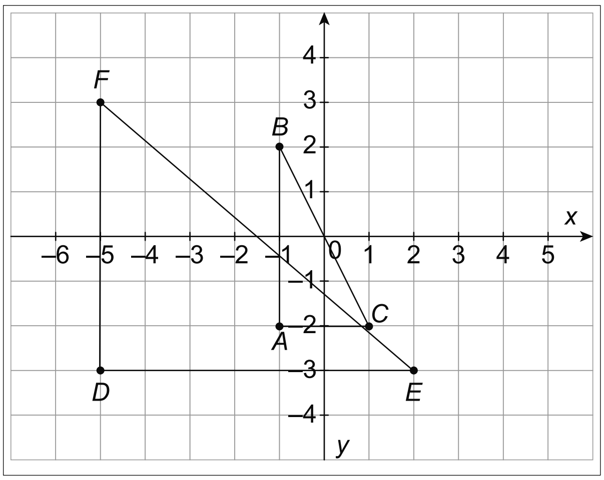
5.



Steps:

1. With a ruler, draw a line segment 8 cm in length. Label as *AB*.
2. Place the centre of a protractor at the point *A* and mark off an angle of 55˚.
3. Draw a line from *A* at 55˚.
4. Place the centre of a protractor at the point *B* and mark off an angle of 40˚.
5. Draw a line from *B* at 40˚ until it crosses the 55˚ line. Label this *C*.

6. (a)



(b) Area $= \dfrac{1}{2}$base $\times \perp$ height

$= \dfrac{1}{2}(2) \times 4$

$= 4$ units^2

Area of a triangle $= \dfrac{1}{2}$(base)(perp. height)

(c) See diagram above.

(d) Area $= \frac{1}{2}$ base $\times \perp$ height

$= \frac{1}{2}(7) \times 6$

$= 3{\cdot}5 \times 6$

$= 21$ units^2

(e) 4:21

(f) **Answer:** No ← The triangles are not equal in all respects so they are not congruent.

Reason: The lengths of the sides of ΔABC are not equal to ΔDEF.

7. (a) Line 5, as the slope is equal to 3.

(b) Lines 1 and 2, as both slopes are 2.

(c) Lines 1 and 3 or lines 2 and 3 since

$2 \times -\frac{1}{2} = -1$

(d) Cuts x axis at $y = 0$

$4x - 3(0) = 12$

$4x = 12$

$x = 3 \quad (3, 0)$

Cuts y axis at $x = 0$

$4(0) - 3y = 12$

$-3y = 12$

$y = -4 \quad (0, -4)$

8. (a) $FBO \rightarrow HDO$

(b) $FBO \rightarrow FAO$

(c) $FBO \rightarrow HCO$

9. (a) A population is when everybody is used to collect data, for example, the entire school. A sample is when only a part of the population is used to collect data, for example, the 3rd year students in a school.

(b) **(i)** Face-to-face interview

(ii) Postal

(iii) Telephone

(iv) Online

(c) Questionnaire should

(i) Be relevant to the survey you're undertaking

(ii) Use clear, simple language

(iii) Be as brief as possible

(iv) Have no leading questions

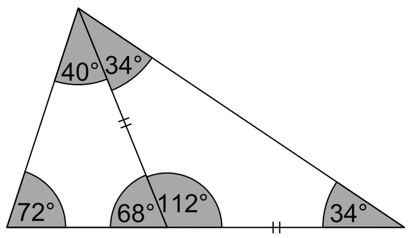
10. (a) $28 \times 30 = 840$ m ← Speed = Distance ÷ Time: Distance = Speed × Time

(b) $\sin 20° = \text{Height} \div 840$

Height = 840 sin 20 ← Calculator: Key as you see.

Height = 287 m

11. (a)



$X = 112°$

$Y = 34°$

← Based on the facts that:
(a) three angles of a triangle add to 180˚.
(b) in a triangle, angles across from equal sides are equal in measure.

(b) $|\angle ABC| = |\angle AMN| = 180° - 115° = 65°$

since both are corresponding angles

(c) $180 - 65 - 65 = 50°$

since $|AB| = |AC|$ makes ΔABC isosceles

12. (a) $\dfrac{2+5+11}{3} = \dfrac{18}{3} = 6$ years

(b) $360 - 150 - 30 - 60 = 120°$

(c) $150° = 10$ students

$30° = \dfrac{10}{150} \times 30$

$= 2$ students

(d) $30° = 2$ students

$360° = \dfrac{360}{30} \times 2 = 24$ students

(e) Spanish = 60° = 4 students. Hence the number who do not study Spanish is 24 – 4 = 20 students.

(f) $\dfrac{20}{24} = \dfrac{5}{6}$

13. $\dfrac{1 + x + 4 + 3}{4} = 2$

$8 + x = 8$

$x = 0$

14. **(a)**

Charolais		Limousin
2	75	
	76	
	77	1 2 3 5
3	78	2 2 5 5
4 3	79	0 3 5
	80	1
9 8 7 7	81	2 6
7 3 0	82	1 7
9 7 2	83	
7 1	84	Key 82|1 = 821 kg

(b) Charolais is better as they are generally the heavier animals. The heaviest Charolais is 847 kg compared to the heaviest Limousin is 827 kg. The heavier the animal, the better the price a farmer gets from the butcher.

15. **(a)** $\sqrt{(6-4)^2+(-2-6)^2}$

$= \sqrt{(2)^2+(4)^2}$

$= \sqrt{20}$

$= 2\sqrt{5}$

> Distance formula: $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$
> See page 18 of *Formulae and Tables.*

(b) $M = \left(\frac{-1+3}{2}, \frac{2+4}{2}\right) = \left(\frac{2}{2}, \frac{6}{2}\right) = (1, 3)$

> Slope formula: See page 18 of *Formulae and Tables.*

(c) $\frac{4-2}{3+1} = \frac{2}{4} = \frac{1}{2}$

(d) Slope (m) = –2   Point (1, 3)

$y - 3 = -2(x - 1)$

$y - 3 = -2x + 2$

$2x + y - 3 - 2 = 0$

$2x + y - 5 = 0$

> Equation of a line formula: $y - y_1 = m(x - x_1)$
> See page 18 of *Formulae and Tables.*

(e)

$x - 2y = 0$

$2x + y = 5$

$x - 2y = 0$

$4x + 2y = 10$

$5x = 10$

$x = 2$

$2 - 2y = 0$

$2 = 2y$

$1 = y$

> Solving simultaneously

Point (2, 1)

16. **(a)** $3 \times 2 \times 4 = 24$

> This is the Fundamental Principle of Counting. Choosing from 3 and then choosing from 2 and then choosing from 4 gives a total of $3 \times 2 \times 4$ choices.

(b) $\frac{2}{24} = \frac{1}{12}$

Educate.ie Sample 7

Paper 2

1. **(a)** $3x + 2x + 3x + 2x = 200$

$10x = 200$

$x = 20$

Length = 3(20) = 60 cm

Breadth = 2(20) = 40 cm

Area = $60 \times 40 = 2400$ cm^2

> Ratio 3:2 means a total of 5 parts, $\frac{3}{5}$ and $\frac{2}{5}$
> Half the perimeter = 100
> $\frac{1}{5}$ of 100 = 20: $\frac{3}{5}$ of 100 = 60: $\frac{2}{5}$ of 100 = 40
> Length = 60 cm
> Breadth = 40 cm

(b) **(i)** $(7{\cdot}5)^2 = (6)^2 + (r)^2$

$56{\cdot}25 - 36 = r^2$

$\sqrt{20{\cdot}25} = r$

$4{\cdot}5 \text{ cm} = r$

(ii) $\pi rl + \pi r^2$

$= (3{\cdot}14)(4{\cdot}5)(7{\cdot}5) + (3{\cdot}14)(4{\cdot}5)(4{\cdot}5)$

$= 170$ cm^2

> Total surface area of a cone = Curved surface area of cone + Area of the circle at bottom

(c) **(i)** Volume $= \pi r^2 h + \frac{2}{3}\pi r^3$

$= \pi(6)(6)(14) + (0{\cdot}66)\pi(6)(6)(6)$

$= 504\pi + 144\pi$

$= 648\pi$ cm^3

(ii) Volume of water $= 648\pi \div 3 = 216\pi$

$144\pi + \pi(6)(6)H = 216\pi$

$36H = 72$

$H = 2$ cm

Depth = 6 cm + 2 cm = 8 cm

2. **(a)** *XYZ*, *XZY*, *ZXY*, *ZYX*, *YZX*, *YXZ*

(b) $\frac{1}{6}$

(c) $\frac{2}{6} = \frac{1}{3}$

> Probability $= \dfrac{\text{Favourable outcomes}}{\text{Total outcomes}}$

(d) 0

> In the question, all horses are said to finish the race.

3. $|\angle BAC| + 47° + 114° = 180°$

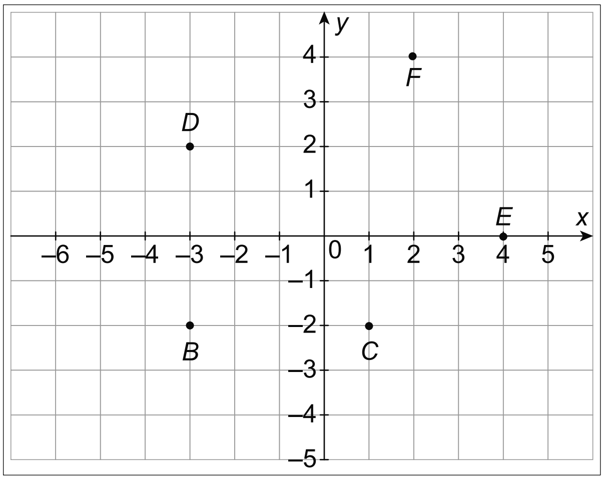
$|\angle BAC| = 180° - 161°$

$|\angle BAC| = 19°$

> Based on the fact that, given parallel lines with a transversal, alternate angles are equal.

4. (a) $(2, -4)$

(b)



(c) Right-angled isosceles triangle since $|BC| = |BD| = 4$ units and $|BD| \perp |BC|$.

5. (a)

Male		Female
	52	2
	53	1 6
5 5	54	5 6
2	55	5
8 3	56	0 4
7	57	3
6 0	58	4
5 4 3	59	0
0	60	
3	61	Key 59|0 = 590

(b) Clearly the male circumferences are much larger than the female circumferences, hence male heads are generally larger.

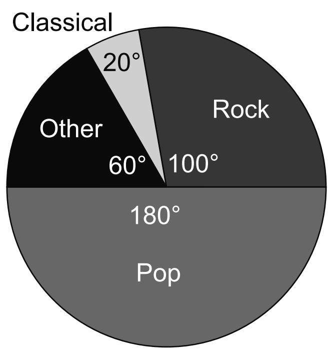
6. A pie chart could be used as there would only be four sectors easily identified.

Pop: $\frac{45}{90} \times 360 = 180°$

Rock: $\frac{25}{90} \times 360 = 100°$

Classical: $\frac{5}{90} \times 360 = 20°$

Other: $\frac{15}{90} \times 360 = 60°$



7. $\dfrac{(4 \times 2{\cdot}5) + (22 \times 7{\cdot}5) + (14 \times 12{\cdot}5) + (x \times 17{\cdot}5) + (6 \times 22{\cdot}5)}{4 + 22 + 14 + x + 6}$

> 2·5, 7·5, 12·5, 17·5 and 22·5 are the mid-interval values.

$= \dfrac{10 + 165 + 175 + 17{\cdot}5x + 135}{46 + x}$

$\Rightarrow \dfrac{485 + 17{\cdot}5x}{46 + x} = \dfrac{11{\cdot}10}{1}$

$485 + 17{\cdot}5x = 510{\cdot}6 + 11{\cdot}1x$

$17{\cdot}5x - 11{\cdot}1x = 510{\cdot}6 - 485$

$6{\cdot}4x = 25{\cdot}6$

$x = \dfrac{25{\cdot}6}{6{\cdot}4}$

$x = 4$

8. **(a)** Line 5 because the slope is 11

(b) Line 2 and line 6, both slopes are –7

(c) Cuts x axis at $y = 0$

$0 = 2x + 6$

$2x = -6$

$x = -3$

$(-3, 0)$

Cuts y axis at $x = 0$

$y = 2(0) + 6$

$y = 6$

$(0, 6)$

9. **(a)** $x - 2y - 3 = 0$

> Rewriting the equation as
> $y = mx + c$
> m will be the slope of the line.

$-2y = -x + 3$

$y = \frac{1}{2}x - \frac{3}{2}$

Slope $= \frac{1}{2}$

(b) Slope –2 Point $(-2, 5)$

> Equation formula:
> $y - y_1 = m(x - x_1)$.
> See page 18 of *Formulae and Tables.*

$y - 5 = -2(x + 2)$

$y - 5 = -2x - 4$

$2x + y = 1$: K

(c)

> Solving simultaneously

$x - 2y = 3$

$\underline{2x + y = 1}$

$x - 2y = 3$

$\underline{4x + 2y = 2}$

$5x = 5$

$x = 1$

$1 - 2y = 3$

$1 - 3 = 2y$

$-2 = 2y$

$-1 = y$

Point of intersection $(1, -1)$

(d) (–2, 5) → (1, –1) → (4, –7)

+3 –6

Answer (4, –7)

10. (a) $\Delta PQT \equiv \Delta SWR$ By ASA

Since $|\angle PQT| = |\angle WSR|$ Told

$|PQ| = |SR|$ By theorem

$|\angle QPT| = |\angle WRS|$ Alternate

Therefore $|PT| = |WR|$

(b) $\Delta PSW \equiv \Delta QTR$

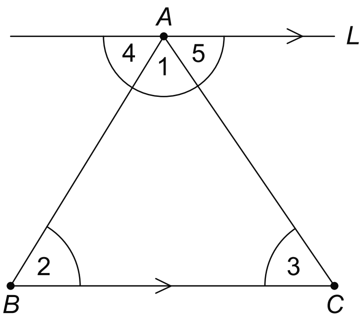
Since $|PS| = |QR|$ By theorem

$|\angle SPW| = |\angle QRT|$ Alternate

$|PW| = |TR|$ Since $|PT| = |WR|$ in part **(a)**

Therefore they are congruent by SAS.

11. Diagram



Given: The triangle *ABC* with angles marked 1, 2 and 3

To prove: $|\angle 1| + |\angle 2| + |\angle 3| = 180°$

Construction: Draw a line *L* through *A* parallel to *BC*.
Mark the angles 4 and 5.

Proof: $|\angle 2| = |\angle 4|$ and $|\angle 3| = |\angle 5|$ Alternate angle ← Standard proof

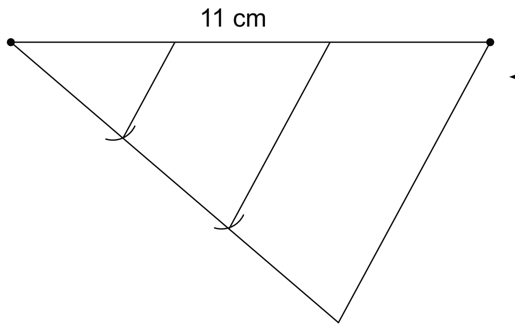
$|\angle 4| + |\angle 1| + |\angle 5| = 180°$ Straight angle

For $|\angle 4|$ and $|\angle 5|$ substitute $|\angle 2|$ and $|\angle 3|$

$|\angle 2| + |\angle 1| + |\angle 3| = 180°$

$|\angle 1| + |\angle 2| + |\angle 3| = 180°$

12.



Steps:

1. Draw a line 11 cm long.
2. Draw another line at an angle to the first line.
3. With a compass, draw three arcs of equal length to cut this line.
4. Join the outside arc to the end point of the 11 cm line.
5. Through the other arcs, draw lines parallel to the line going through the first arc. The 11 cm line will now be divided into three equal segments.

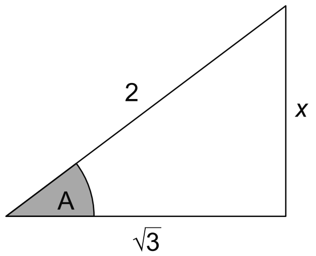
13. $x = 54°$ alternate angle

$y + 54 + 38 = 180°$ (sum of angles in Δ)

$y = 180° - 92°$

$y = 88°$

14. $\cos A = \dfrac{\text{Adjacent}}{\text{Hypotenuse}} = \dfrac{\sqrt{3}}{2}$



$(2)^2 = x^2 + (\sqrt{3})^2$

$4 = x^2 + 3$

$1 = x$

$\tan = \dfrac{\text{Opposite}}{\text{Adjacent}} = \dfrac{1}{\sqrt{3}}$

15. **(a)** $\sin |\angle CBD| = \dfrac{4}{6}$

$|\angle CBD| = \sin^{-1}(0{\cdot}666)$

$|\angle CBD| = 42°$

(b) $\tan |\angle CAD| = \dfrac{4}{9}$

$|\angle CAD| = \tan^{-1}(0{\cdot}444)$

$|\angle CAD| = 24°$

(c) $\sin x = \dfrac{1}{2}$

$x = \sin^{-1}(0{\cdot}5) = 30°$ ← You could also use trigonometric ratios. See page 13 of *Formulae and Tables.*

$\cos 30° = \dfrac{\sqrt{3}}{2}$

16. **(a)** $8x - 8 = 5x + 10$ ← The two angles at Q marked by the dots are equal.

$8x - 5x = 10 + 8$

$3x = 18$

$x = 6°$

(b) Angle $|\angle PQR|$

$(8(6) - 8) + (5(6) + 10)$

$(48 - 8) + (30 + 10)$

$40 + 40$

$= 80°$

2014 SEC Paper 2 (Phase 3)

1. **(i)**

HHH	THH
HHT	THT
HTH	TTH
HTT	TTT

← Try to list the outcomes systematically.

(ii) $\Pr(2\,H, 1\,T) = \frac{3}{8}$

← Probability = $\frac{\text{Favourable outcomes}}{\text{Total outcomes}}$
Count your answers from the table above.

(iii) Answer: No

Reason: $\Pr(3\,H) = \frac{1}{8}$ but $\Pr(2\,H, 1\,T) = \frac{3}{8}$

← Again, use the table above to count the events.

Or:

Reason: There is only 1 way to get three heads. There are 3 ways to get two heads and one tail.

(iv) Answer: Yes

Reason: $\Pr(H\,H\,H) = \frac{1}{8}$ and $\Pr(H\,T\,H) = \frac{1}{8}$

Or:

Reason: There is only one way to get each event.

2. **(i)** Volume = $13 \times 8 \times 6 = 624$ cm^3

← The volume of a cuboid is given by Length × Width × Height

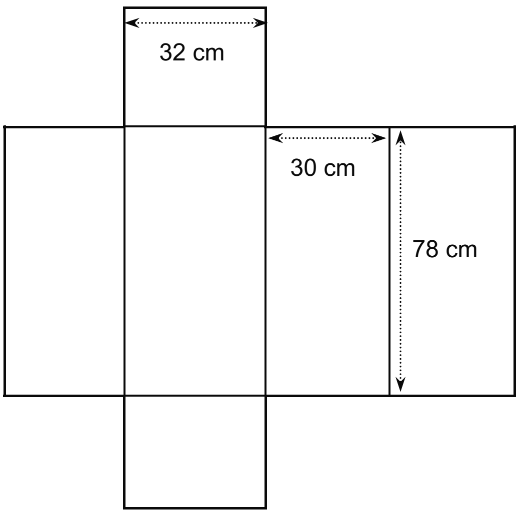
(ii) Box **A**:

Volume = $32 \times 30 \times 78 = 74\,880$ cm^3

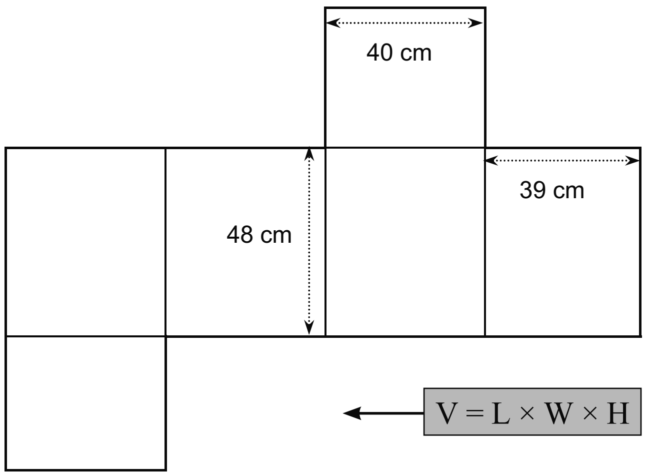
Box **B**:

Volume = $48 \times 40 \times 39 = 74\,880$ cm^3

Box **A**



Box **B**



← V = L × W × H

(iii) Box **A**: $32 \div 8 = 4$; $30 \div 6 = 5$; $78 \div 13 = 6$; so Box **A** can be filled completely.

Box **B**: $48 \div 6 = 8$; $40 \div 8 = 5$; $39 \div 13 = 3$; so Box **B** can be filled completely.

Total: $4 \times 5 \times 6 = 120$ individual phone boxes.

← Also $8 \times 5 \times 3 = 120$ (Box **B**)

(iv)

Box **A**:	Box **B**:
Surface Area = 2 (32 × 30 + 32 × 78 + 30 × 78) = 11 592 cm^2	Surface Area = 2 (48 × 40 + 48 × 39 + 40 × 39) = 10 704 cm^2

S.A. = 2 (Front + Top + Side)

(v) Use Box **B**. The cost is given per m^2, so convert surface area to m^2 (or cost to per cm^2).

1 cm = 0·01 m, so 1 $cm^2 = 0{\cdot}01^2$ m^2 = 0·0001 m^2.

Surface area = 10 704 cm^2 = 10 704 × 0·0001 m^2 = 1·0704 m^2

Cost of box = €1·0704 × 0·67

= €0·717168

= €0·72, to the nearest cent

(vi) Cost of Box A = € (11 592 × 0·0001 × 0·67)

= €0·776664

= €0·78, to the nearest cent

Saving per annum = €(0·78 − 0·72) × 140 × 12

= €(0·06) × 1680

= €100·80

Or:

Difference in area = (11 592 − 10 704) cm^2 = 888 cm^2 = 0·0888 m^2

Saving per annum = €0·67 × 0·0888 × 140 × 12 = €99·95

3.

IQ Test 1				
86	104	89	105	96
96	103	94	104	119
115	79	97	111	108

IQ Test 2				
83	120	105	111	114
99	111	108	106	97
97	102	94	108	117

(i)

IQ Test 1						IQ Test 2				
				9	7					
			9	6	8	3				
	7	6	6	4	9	4	7	7	9	
8	5	4	4	3	10	2	5	6	8	8
		9	5	1	11	1	1	4	7	
					12	0				
Key: 9|7 = a score of 97										

Make sure to order the scores in the plot.

(ii) Range of IQ Test 1 = 119 − 79 = 40 Range of IQ Test 2 = 120 − 83 = 37

Range = Highest Score – Lowest Score

(iii) 15 data points in each set, so the median is the $\frac{15+1}{2}$ = 8th data point.

Median of IQ Test 1 = 103

Median of IQ Test 2 = 106

Median: Middle value when the scores are arranged in ascending/descending order

(iv) Mean of IQ Test 1 = $\frac{1506}{15}$ = 100·4

Mean of IQ Test 2 = $\frac{1572}{15}$ = 104·8

Mean: Sum of the scores divided by the number of scores

(v) In general, the scores in IQ Test 2 are slightly higher than in IQ Test 1, as both the mean and median are higher for IQ Test 2.

Measures of Central Tendency: Mean or Median
Measure of Variability: Range

The scores are slightly more spread out in IQ Test 1 than in IQ Test 2, as the range is bigger for IQ Test 1;
or
The spread of scores is very similar, as the two ranges are almost the same.

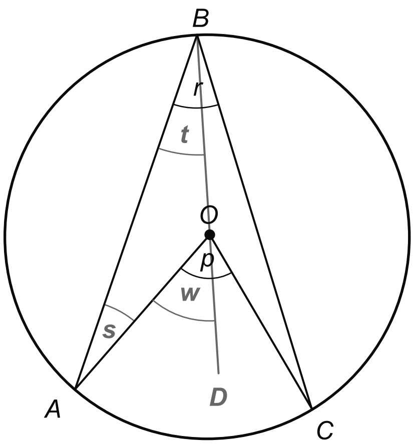
(vi) Answer: No.

Explanation: The person who got 119 on IQ Test 1 could have got less, e.g. 94, on IQ Test 2.

Or:

Explanation: The maximum score on IQ Test 1 is greater than the minimum score on IQ Test 2.

4. (a) Diagram:



Given: A circle with centre *O*. Points *A*, *B*, and *C* on the circle. Angles *p* and *r*, as shown.

To Prove: $p = 2r$.

Construction: Join *B* to *O*, and extend to *D*. Mark the angles *s*, *t*, and *w*.

Proof: $\lvert OA\rvert = \lvert OB\rvert$	Radii of circle	Step 1
$\therefore s = t$	Isosceles triangle	Step 2
$w = s + t$	Exterior angle	Step 3
$\therefore w = 2t$		Step 4

Standard Proof

2014
SEC P2

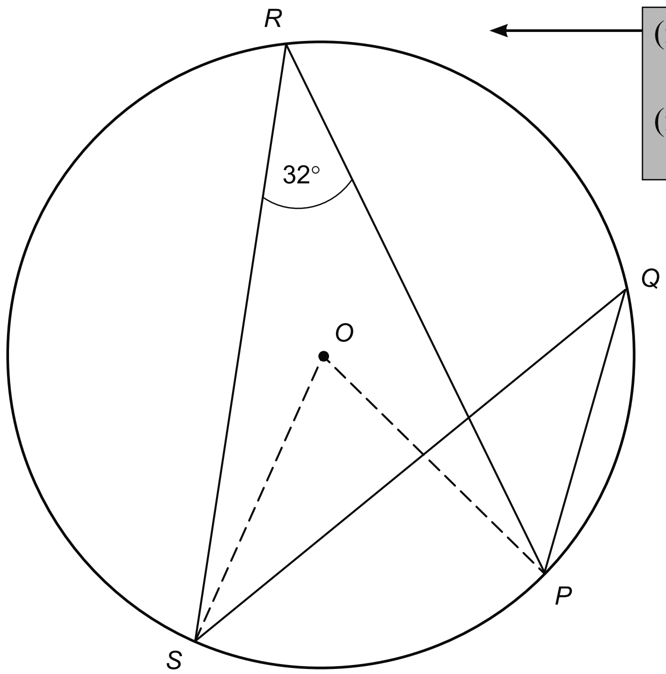
Similarly, $(p - w) = 2(r - t)$.

So $p = (p - w) + w$

$= 2(r - t) + 2t$

$= 2r$ Step 5

(b) **(i)**



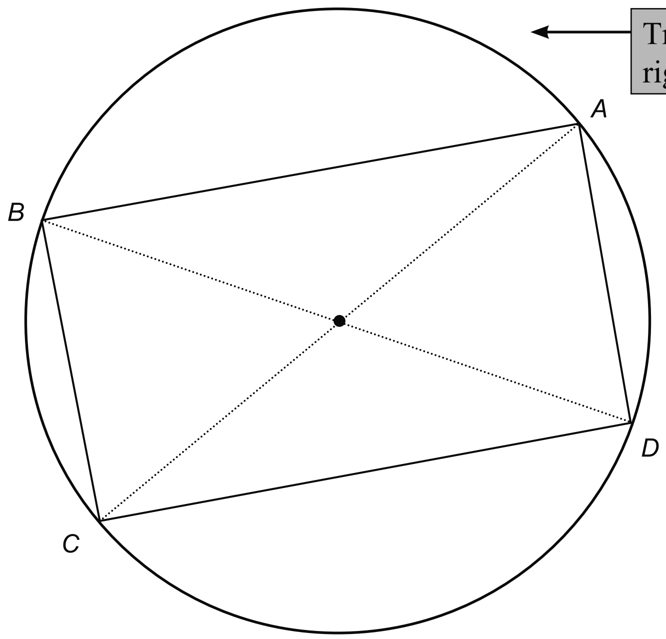
(i) Angle at the centre theorem (above in part (a))

(ii) Angles standing on the same arc are equal.

$|\angle SOP| = 2 \times 32° = 64°$

(ii) $|\angle SQP| = |\angle SRP| = 32°$

(c)



Try to show that the opposite angles are right angles.

We just need to prove that the four angles are 90°.

$|\angle BAD| = |\angle BCD| = 90°$, as $[BD]$ is a diameter.

Similarly, $|\angle CBA| = |\angle CDA| = 90°$

So $ABCD$ is a rectangle.

5. (a)

Credit Card	Debit Card	Debit Card	Cash	Debit Card
Credit Card	Cash	Cash	Credit Card	Debit Card
Debit Card	Debit Card	Cheque	Cash	Cash
Cash	Cash	Debit Card	Cash	Credit Card

(i) Categorical Nominal

(ii)

Method of Payment	Credit Card	Debit Card	Cash	Cheque
Frequency	4	7	8	1

(iii) Mode = Cash ← Mode: Which appears most often?

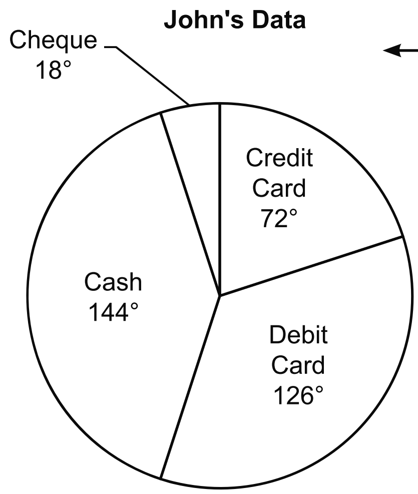
(iv) He cannot add up his values and divide by 20. ← The Mode is the only measure of centre for this type of data.

(v) Credit Card: $\frac{4}{20} \times 360° = 72°$

Debit Card: $\frac{7}{20} \times 360° = 126°$

Cash: $\frac{8}{20} \times 360° = 144°$

Cheque: $\frac{1}{20} \times 360° = 18°$.

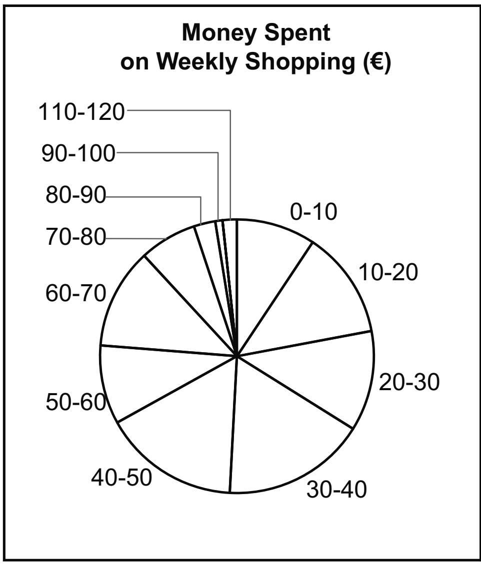


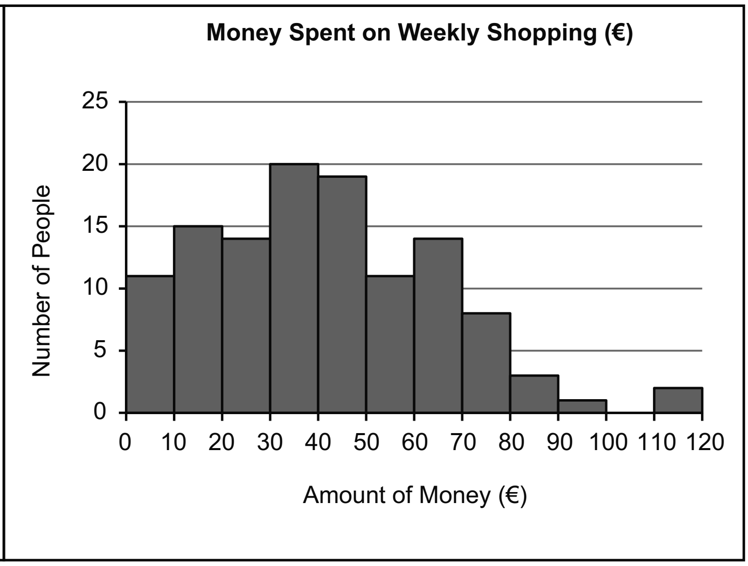
← Use proportions of 360° to decide on the angle measures.

(b) Margaret's data may be biased because her sample is probably not representative. She will probably have a lot more people answering "Lidl" than she should.

(c) **(i)** Answer: Pie chart

Reason: It's easy to see where half the pie chart is (180°).

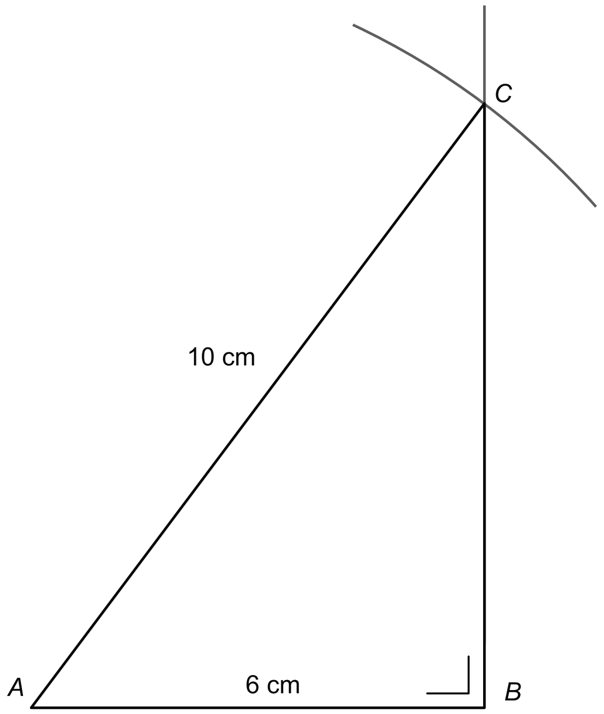




(ii) Answer: Bar chart/Histogram

Reason: It's easy to see which bar is highest.

6. (i)



Standard construction

Note: It is also possible to work out the length of the third side, [*BC*], using the Theorem of Pythagoras, and then construct [*BC*] and [*AC*].

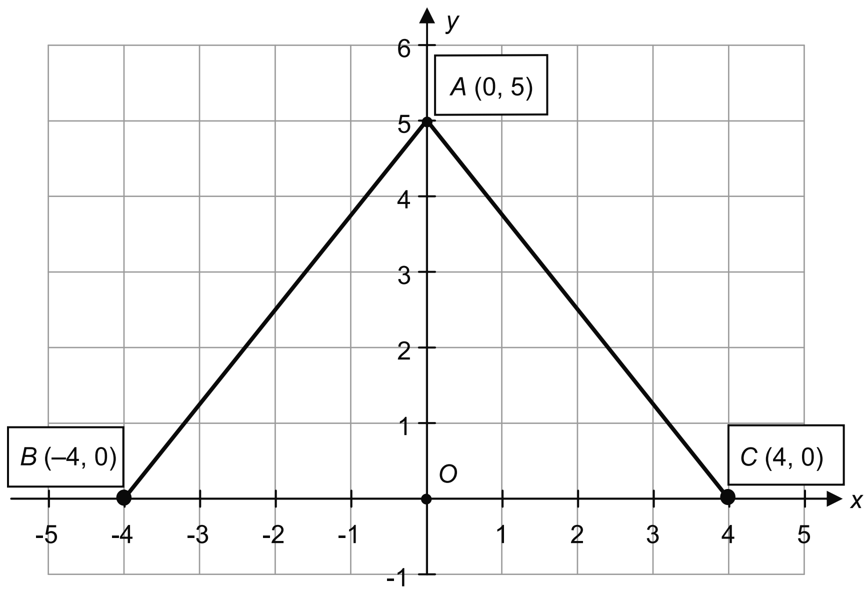
(ii) $|\angle CAB| = 53°$ ← Use your protractor.

(iii) $\cos(53°) = 0{\cdot}6018\ldots = 0{\cdot}602$, correct to three decimal places

(iv) They are not the same because $|\angle CAB| = \cos^{-1}\left(\frac{6}{10}\right) = 53{\cdot}1301°$.

So if X is a whole number then $\cos X$ can never be exactly $0{\cdot}6$.

7. (i)



(ii) $|AB| = \sqrt{4^2 + 5^2} = \sqrt{41}$

Similarly, $|AC| = \sqrt{41}$ ← Or use the Length formula. See page 18 of *Formulae and Tables*.

Total length of metal bar $= 2\sqrt{41} = 12{\cdot}80\ldots = 12{\cdot}8$ m, correct to one decimal place.

(iii) *AB*: *AC*:

Slope $= \frac{\text{Rise}}{\text{Run}} = \frac{5}{4}$ or 1·25 Slope $= \frac{5-0}{0-4} = -\frac{5}{4}$ or $-$ 1·25.

Or use the Slope formula. See page 18 of *Formulae and Tables.*

(iv) Answer: No

Reason: Product of slopes $= \frac{5}{4} \times -\frac{5}{4} = -\frac{25}{16} \neq -1$

Perpendicular: Product of the slopes is –1.

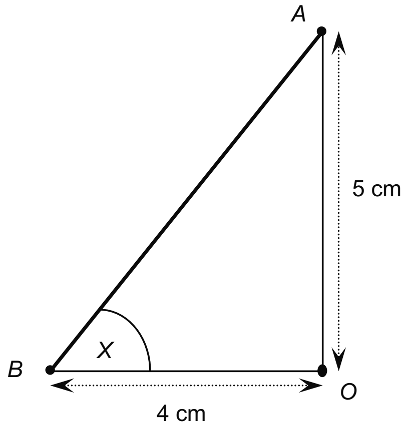
Or:

Reason: When you invert one slope and change the sign, you don't get the other slope.

(v) $\tan X = \frac{5}{4}$

$|\angle X| = \tan^{-1}\left(\frac{5}{4}\right) = 51{\cdot}340... = 51{\cdot}34°$, correct to two decimal places.

Use the inverse (shift) key on your calculator to get $\tan^{-1}$.



(vi) Recall from **(ii)** that $|AB| = \sqrt{41}$ m.

Increase $|\angle X|$ by 20% to get $|\angle X'|$:

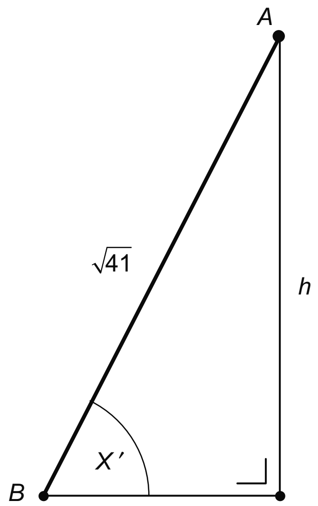
$|\angle X'| = 51{\cdot}34 \times 1{\cdot}2 = 61{\cdot}608°$

From the diagram, $\sin X' = \sin 61{\cdot}608 = \frac{h}{\sqrt{41}}$.

$\Rightarrow h = \sqrt{41} \sin 61{\cdot}608$

$= 5{\cdot}632$

$= 5{\cdot}6$ m, correct to one decimal place.



2014 SEC P2

8. (i)

Method 1:	Method 2:
$-3y = -x + 6$ Step 1	Slope $= \frac{-a}{b}$
$3y = x - 6$	$= \frac{-1}{-3}$
$y = \frac{1}{3}x - 2$ Step 2	
$\Rightarrow$ Slope $= \frac{1}{3}$ Step 3	$= \frac{1}{3}$

Use $y = mx + c$

(ii) Substitute in $(1, -2)$ to l: LHS $= 1 - 3(-2) - 6 = 1 \neq 0 =$ RHS.

Point not on l.

If a point is on a line, when you substitute it into the equation of the line it will satisfy the equation (LHS = RHS)

(iii) Slope of $k = \frac{1}{3}$ Point on $k = (1, -2)$

Equation of k *Or*: Equation of k:

Equation of a Line formula. See page 18 of *Formulae and Tables*

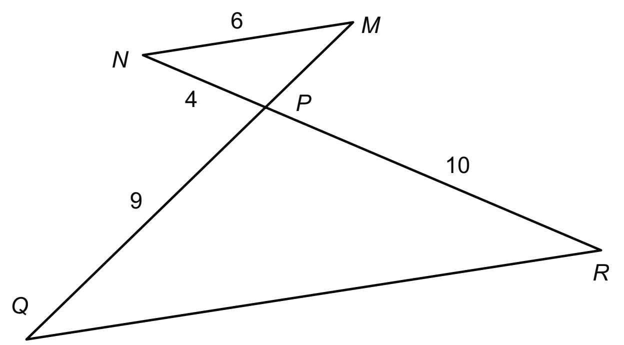
$y - (-2) = \frac{1}{3}(x - 1)$ $x - 3y + c = 0$

$\Rightarrow \quad y = \frac{x}{3} - \frac{7}{3}$ $\Rightarrow 1 - 3(-2) + c = 0$

or $x - 3y - 7 = 0$ $\Rightarrow \quad c = -7$

$\Rightarrow \quad x - 3y - 7 = 0$

9.



(i) Proof: $|\angle MNP| = |\angle PRQ|$ (given)

$|\angle NPM| = |\angle QPR|$ (vertically opposite)

$|\angle NMP| = |\angle PQR|$ (third angle)

$\Rightarrow$ The triangles are similar.

Similar Triangles: They have the same angles.

(ii) Answer: Yes

Reason: $|\angle MNP| = |\angle PQR|$ *or* $|\angle NMP| = |\angle PQR|$ *or* alternate angles are equal.

Given $|MN| = 6$, $|NP| = 4$, $|QP| = 9$, and $|PR| = 10$, find:

(iii) $|QR|$

By similar triangles ΔMNP and ΔQRP.

In Similar Triangles, the corresponding sides are in proportion.

$$\frac{|QR|}{6} = \frac{10}{4}$$

$$\Rightarrow |QR| = 6 \times \frac{10}{4} = 15$$

(iv) $|QM|$

By similar triangles ΔMNP and ΔQRP:

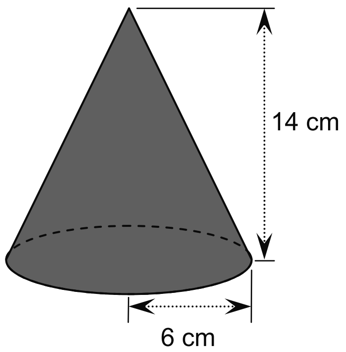
$\frac{|PM|}{9} = \frac{6}{15}$ or $\frac{4}{10}$ Or: $\frac{|PM|}{4} = \frac{9}{10}$

$\Rightarrow |PM| = \frac{18}{5}$ or 3·6 $\Rightarrow |PM| = 4 \times \frac{9}{10} = \frac{18}{5}$ or 3·6

$\Rightarrow |QM| = 9 + 3{\cdot}6 = \frac{63}{5}$ or 12·6 $\Rightarrow |QM| = 9 + 3{\cdot}6 = \frac{63}{5}$ or 12·6.

10. (i) Volume $= \frac{1}{3}\pi(6)^2 \times 14 = 168\pi$ cm^3

Volume of a Cone formula: $V = \frac{1}{3}\pi r^2$ h
See page 10 of *Formulae and Tables*.



(ii) Volume of upper (removed) portion:

$\frac{1}{3}\pi(3)^2 \times 7 = 21\pi$ cm^3

Subtract the volume of the smaller (missing) cone from your answer in part **(i)** to get the volume of the frustum.
Ratio = Fraction or proportion

Volume of frustum:

$168\pi - 21\pi = 147\pi$ cm^3

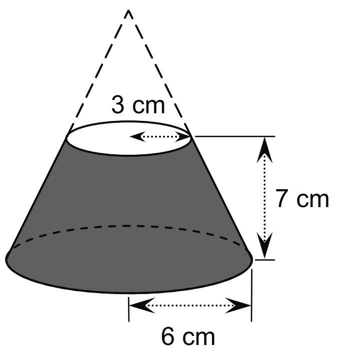
Or:

Volume of frustum $= \frac{1}{3}\pi h[R^2 + Rr + r^2]$

$= \frac{1}{3}\pi \times 7(6^2 + 6 \times 3 + 3^2) = 147\pi$ cm^3

Required ratio:

$\frac{147\pi}{168\pi} = \frac{7}{8}$ *or* 7:8 *or* 0·875



2014 SEC P2

2014 SEC Sample Paper 2 (Phase 2)

1. (i) Area = Length × Width ← Area of rectangle = Length × Width

= 8 m × 4 m

= 32 m^2

(ii) Total length needed

= 5(semicircles) + length

= 5(3·14 × 2) + 8

= 31·4 + 8

= 39·4 metres

(iii) Length: = 12·5 m

since

8 + 2 + 2 + 2(0·25)

Length + Diameter of circle + 2 buried pieces

Area = Length × Width

= 12·5 × 6·78

= 84·75 m^2

Width 6·78 m

since

$\frac{\cancel{2}\pi r}{\cancel{2}} + 2(0{\cdot}25)$

(3·14)(2) + 0·5

(iv) Volume = Cylinder ÷ 2

$= \frac{\pi r^2 h}{2}$ ← Volume of Cylinder = $\pi r^2 h$. See page 10 of *Formulae and Tables.*

$= \frac{(3{\cdot}14)(2)(2)(8)}{2}$

= 50·24 m^3

(v) Area of bed = (8 – 0·4) × (4 – 0·4)

= 7·6 × 3·6

= 27·36 m^2

Volume of topsoil = 27·36 × 0·25 = 6·84 m^3

Cost of topsoil = 6·84 × 0·75 × €80

= €410·40

2. (i) 600 – (92 + 101 + 115 + 98 + 105) = 89 ← Total frequency must add to 600.

(ii) Based on the above results, I disagree. Each number should have a $\frac{1}{6}$ or 16·7% chance of occurring. Currently, 5 has a 14·8% chance of occurring, whereas 3 has a 19·2% chance of occurring.

(iii) Answer: 152

Reason: the probability of getting an even number from six hundred throws is

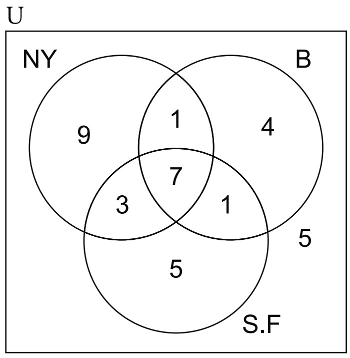
$\frac{101 + 98 + 105}{600} = \frac{304}{600} = \frac{152}{300}$ ← Probability = $\frac{\text{Favourable outcomes}}{\text{Total outcomes}}$

3. **(i)** Example 1: Black jeans, White shirt, Black jumper and Boots

Example 2: Black jeans, Red shirt, Black jumper and Flip-flops

(ii) $3 \times 4 \times 2 \times 3 = 72$ outfits ← This is the Fundamental Principle of Counting.

4. **(i)**



(ii) $\frac{5}{33} = \frac{1}{7}$ ← Probability $= \frac{\text{Favourable outcomes}}{\text{Total outcomes}}$

(iii) $\frac{9}{35}$

(iv) $\frac{9+3+4+1+1+7}{35} = \frac{25}{35} = \frac{5}{7}$ ← Always simplify.

5. **(i)**

No. of hours	0–2	2–4	4–6	6–8	8–10	10–12	12–14	14–16	16–18	18–20	20–22
No. of students	1	31	18	13	11	3	1	1	6	1	4

(ii) 2 – 4 hours ← The mode is the most common interval: 2 – 4 occurs 31 times.

(iii) $(1 \times 11) + (3 \times 31) + (5 \times 18) + (7 \times 13) + (9 \times 11) + (11 \times 3) + (13 \times 1) +$
$(15 \times 1) + (17 \times 6) + (19 \times 1) + (21 \times 4) = \frac{650}{100} = 6{\cdot}5$ hours

(iv) (1) He is using a much smaller sample.

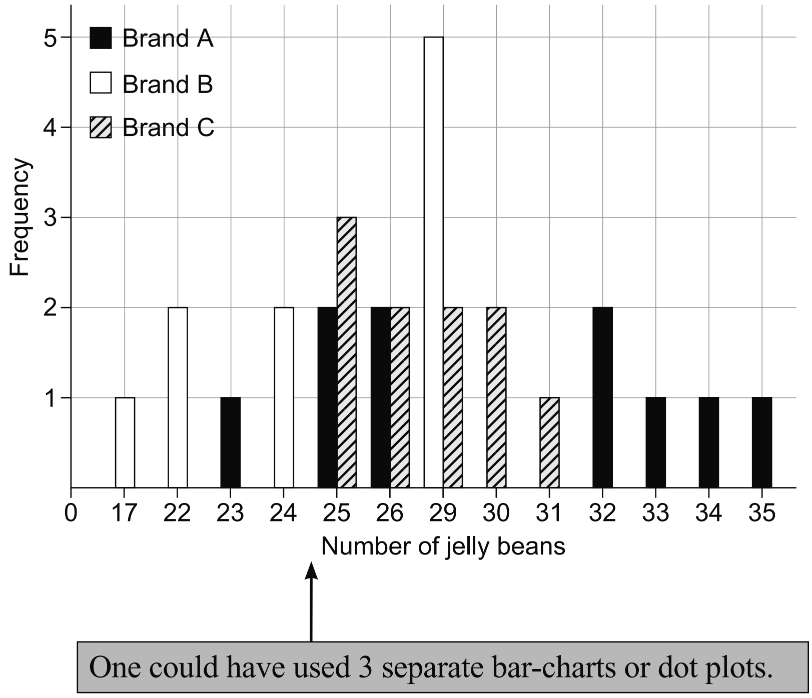
(2) His sample consists of First Year boys only and ignores other years and the opinions of girls.

(3) His survey is being conducted after the mid-term break, when students would have had more free time and probably spent many more hours on social networking sites.

2014 Sample SEC P2

6. **(i)**

No. of jelly beans	17	22	23	24	25	26	29	30	31	32	33	34	35
Brand A	0	0	1	0	2	2	0	0	0	2	1	1	1
Brand B	1	2	0	2	0	0	5	0	0	0	0	0	0
Brand C	0	0	0	0	3	2	2	2	1	0	0	0	0



(ii) Example answer: If I had to choose, I would buy Brand C. In Brand C, the mean number of sweets is $\frac{276}{10}$ = 27·6 sweets, compared with a mean of 24·5 sweets for Brand B and 29·1 for Brand A. The reason I didn't pick Brand A, even though it has a greater mean, is because Brand C has a range of 6 (31–25) unlike Brand A (35–23) and Brand B (29–17), which both have a range of 12, double that of Brand C.

This means there is a greater difference in the number of sweets between the biggest and smallest packages.

When buying sweets, I'd expect a consistent number of sweets in any brand package I buy.

$\Rightarrow$ I would choose Brand C.

7. (i) $\frac{93 \cdot 725}{360} \times 3\,165 = 824$ schools

$\frac{\text{Angle}}{360°} \times$ Total no. of schools

(ii) Example answer: I disagree, as the first pie chart represents 3 165 primary schools, but the second chart only represents 729 post-primary schools.

Both angles are approximately 45°.

Primary: $\frac{45}{360} \times 3\,165 = 396$ schools

Post-primary: $\frac{45}{360} \times 729 = 91$ schools

8. (i) Answer: No

Reason: No two lengths are equal in measure, hence, an isosceles triangle with two sides of equal measure cannot be constructed.

(ii) In a parallelogram, opposite sides are of equal length, but no two strips are of equal measure. Hence, they cannot be used to form a parallelogram.

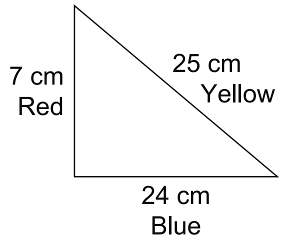
(iii) Use Pythagoras's theorem:

Is $(25)^2 = (24)^2 + (7)^2$?

$625 = 576 + 49$

$625 = 625$

$\Rightarrow$ It is a right-angled triangle.



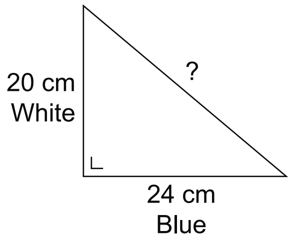
(iv) $(\text{Missing side})^2 = (20)^2 + (24)^2$

$(\text{Missing side})^2 = 400 + 576$

$\text{Missing side} = \sqrt{976}$

$= 31{\cdot}2409987$

$= 31{\cdot}24 \text{ cm}$



9. (i) The sin of $|\angle EAB|$ = opposite ÷ hypotenuse which can be written as

$$\frac{|BE|}{|AE|} = \frac{80}{120} \textbf{ or } \frac{|CD|}{|AD|} = \frac{200}{120 + |ED|}$$

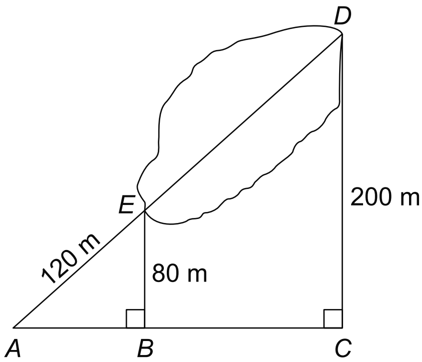
(ii) $\sin|\angle EAB| = \dfrac{80}{120} = \dfrac{200}{120 + |DE|}$

$80(120 + |DE|) = 200(120)$

$9600 + 80|DE| = 24\,000$

$80\,|DE| = 24\,000 - 9600 = 14\,400$

$|DE| = 180$ m



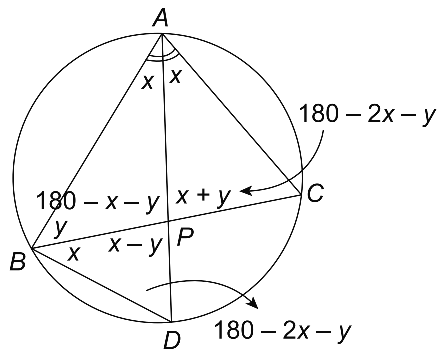
10. (i) ADB is similar to ΔAPC:

$|\angle BAP| = |\angle PAC|$ told.

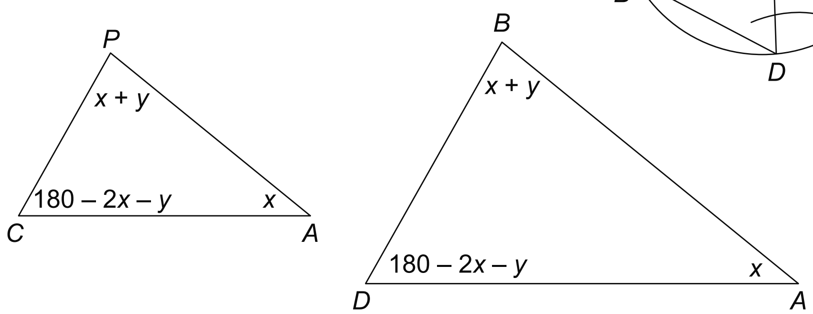
$|\angle ABD| = |\angle APC|$ … $(x + y)$

$|\angle ADB| = |\angle ACP|$ … $(180 - 2x - y)$

Similar by A, A, A (3 angles the same)



(ii)



$$\frac{\text{Small } \Delta}{\text{Big } \Delta} \cdot \frac{|AC|}{|AD|} = \frac{|EC|}{|BD|} = \frac{|AP|}{|AB|}$$

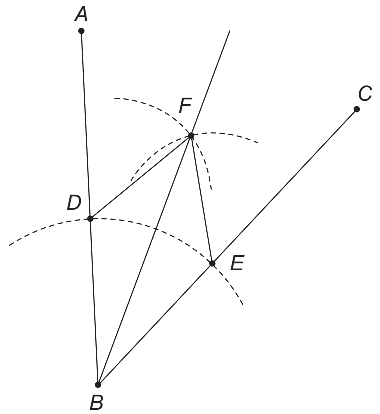
Cross multiply

$|AC| \cdot |BD| = |AD| \cdot |PC|$

Proportional sides in similar triangles

2014 Sample SEC P2

11.



(i) $|DE| = |EF|$ because the arc BE has the same radius as the arc EF. F is a point on two circles of equal radii.

(ii) In the triangle BDF and the triangle BEF

$|DE| = |EF|$ …………..Part **(i)**

$|BF| = |BF|$ …………..Same line

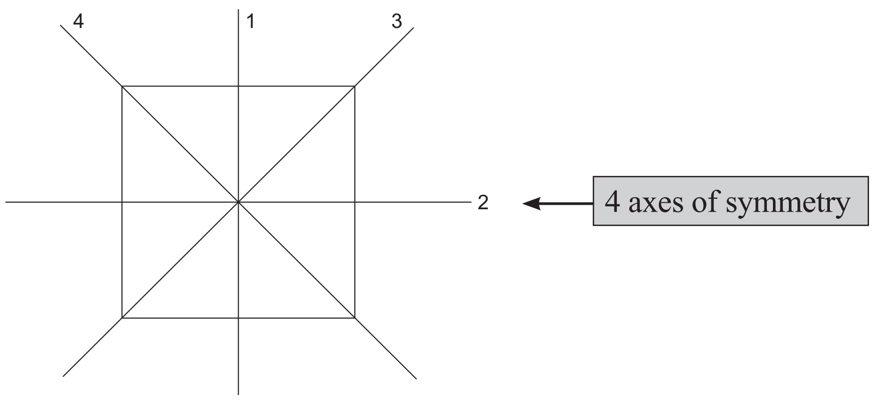
$|BD| = |BE|$ …………..On same arc

Therefore the triangle BDF and the triangle BEF…………….. $SSS = SSS$

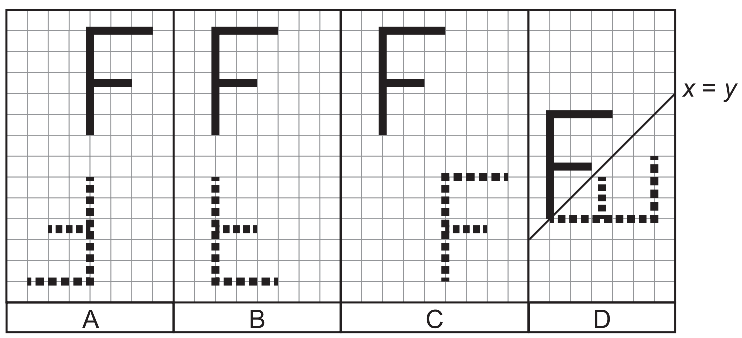
Therefore $|\angle DBF| = |\angle EBF|$

Therefore BF bisects the angle ABC.

12. (a)



(b)



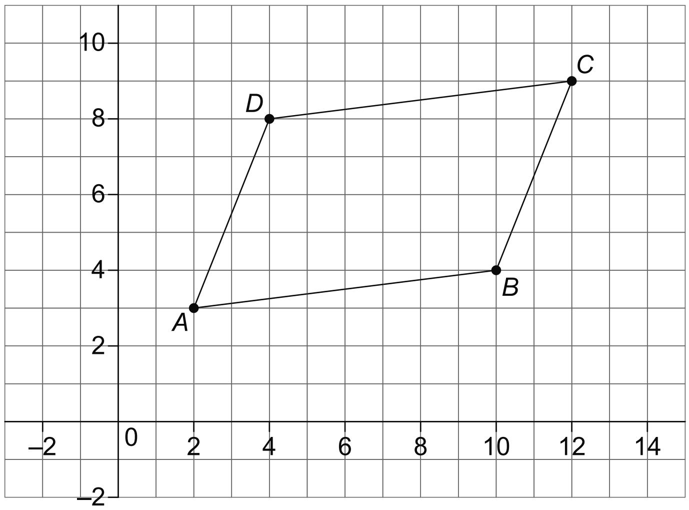
A Central symmetry

B Axial symmetry (through the x-axis)

C Translation

D Axial symmetry (through the line $x = y$)

13. (i)



(ii)

$$|AD| = |BC|$$

$$(2, 3)(4, 8) = (10, 4)(12, 9)$$

$$\sqrt{(4-2)^2 + (8-3)^2} = \sqrt{(12-10)^2 + (9-4)^2}$$

$$\sqrt{(2)^2 + (5)^2} = \sqrt{(2)^2 + (5)^2}$$

$$\sqrt{4 + 25} = \sqrt{4 + 25}$$

$$\sqrt{29} = \sqrt{29}$$

> Distance Formula:
> $\sqrt{(x_2 - x_1)^2 + (y_2 - y)^2}$
> See page 18 of *Formulae and Tables*.

(iii) E = midpoint of $|(2, 3)(12, 9)| = \left(\frac{12+2}{2}, \frac{3+9}{2}\right) = (7, 6)$

F = midpoint of $|(10, 4)\ (4, 8)| = \left(\frac{10+4}{2}, \frac{4+8}{2}\right) = (7, 6)$

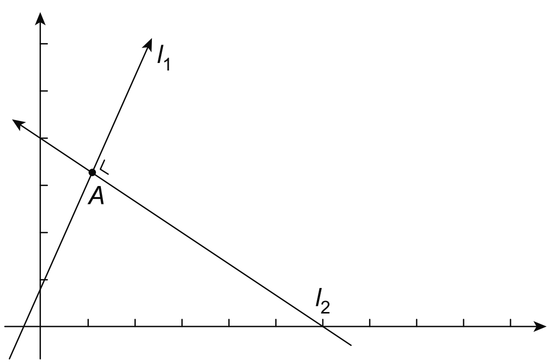
> Mid-point Formula:
> See page 18 of
> *Formulae and Tables*.

Since the midpoint of both diagonals $|AC|$ and $|BD|$ is (7, 6)

$\Rightarrow$ the diagonals bisect each other.

(iv) No, we cannot. We would have to prove that opposite sides and opposite angles are equal.

14. (i) and **(ii)**



15. (a) Answer: False/incorrect

Reason: tan 60° = 1·732, which is greater than 1.

(b) Answer: True/correct

Reason: sin 30° = 0·5, but sin 60° = 0·8.

(c) False/incorrect

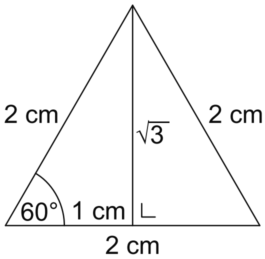
Reason: cos 30° = 0·866, but cos 60° = 0·5.

(d) $(2)^2 = l^2 + h^2$ ← Theorem of Pythagoras

$3 = h^2$

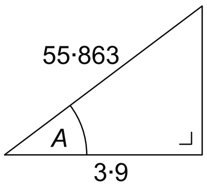
$\sqrt{3} = h$

$$\sin 60° = \frac{\text{Opposite}}{\text{Hypotenuse}} = \frac{\sqrt{3}}{2}$$



$$\boxed{\sin 60° = \frac{\sqrt{3}}{2}}$$

16. The Leaning Tower of Pisa



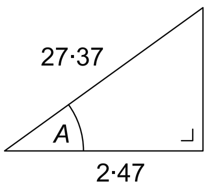
$\cos A = \dfrac{3{\cdot}9}{55{\cdot}863}$

$\cos A = 0{\cdot}069813651$

$A = \cos^{-1}(0{\cdot}069813651)$

$A = 86°$

Tower of Suurhusen Church



$\cos A = \dfrac{2{\cdot}47}{27{\cdot}37}$

$\cos A = 0{\cdot}090244793$

$A = \cos^{-1}(0{\cdot}090244793)$

$A = 84{\cdot}822°$

The most tilted tower is the tower of Suurhusen Church, which makes an angle of 84·822°.

17. (i) $x + 4x + 90 = 180$

$5x = 90$

$x = 90 + 5 = 18°$

Hence, the other angle = 4(18) = 72°

Answer: 18°, 72°

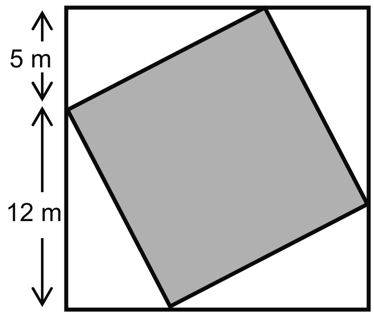
(ii) Slope = tan x

= tan 18°

= 0·324919696

= 0·325

2013 SEC Paper 2 (Phase 3)



1. (a) $h^2 = a^2 + b^2$ ← Apply Pythagoras's Theorem.

Length = $\sqrt{12^2 + 5^2} = 13$ m

(b) Area = $\frac{1}{2}(12)(5) = 30$ m^2 ← Area of a Triangle Formula $A = \frac{1}{2} b \times h$ See page 9 of *Formulae and Tables.*

(c) Area of patio = $13^2 = 169$ m^2

or

Area of patio = $17^2 - (4 \times 30) = 169$ m^2 ← Area of a Square formula $A = l \times l = l^2$

(d) Area of flagstone = $0{\cdot}8 \times 0{\cdot}5 = 0{\cdot}4$ m^2

Number of flagstones = $\frac{169}{0{\cdot}4} = 422{\cdot}5$ (or 423)

Extra 20% = $422{\cdot}5 \times 0{\cdot}2 = 84{\cdot}5$ or 120% = $422{\cdot}5 \times 1{\cdot}2$

Total number of flagstones = 507

or

Extra 20% = $423 \times 0{\cdot}2 = 84{\cdot}6$ or 120% = $423 \times 1{\cdot}2$

Total number of flagstones = 507·6

or

Area to cover = $169 \times 1{\cdot}2 = 202{\cdot}8$

Total number of flagstones = $\frac{202{\cdot}8}{0{\cdot}4} = 507$

2. (a)

Aerobics class		Swimming class
9	0	7 8
8 7 5 3	1	2 3 4 5 6 8
7 4 4 2 0	2	2 2 3 4 4 4 9
9 7 7 7 3 2	3	1 3 5 6 6 8
9 8 8 5 2 2 1	4	1 2 5 7 8
8 6 4 3 1	5	1 2 3
3 1	6	2

Key: 1|5 means 15 ← Don't forget to fill in the key.

(b) Aerobic median: $\frac{37 + 39}{2} = 38$ ← Median is the middle value if the results are arranged in ascending/descending order.

Swimming median: $\frac{29 + 31}{2} = 30$

(c) Mean or Mode ← Mean and mode are also considered measures of "centre".

(d) An older age group take Aerobics class.

2013 SEC P2

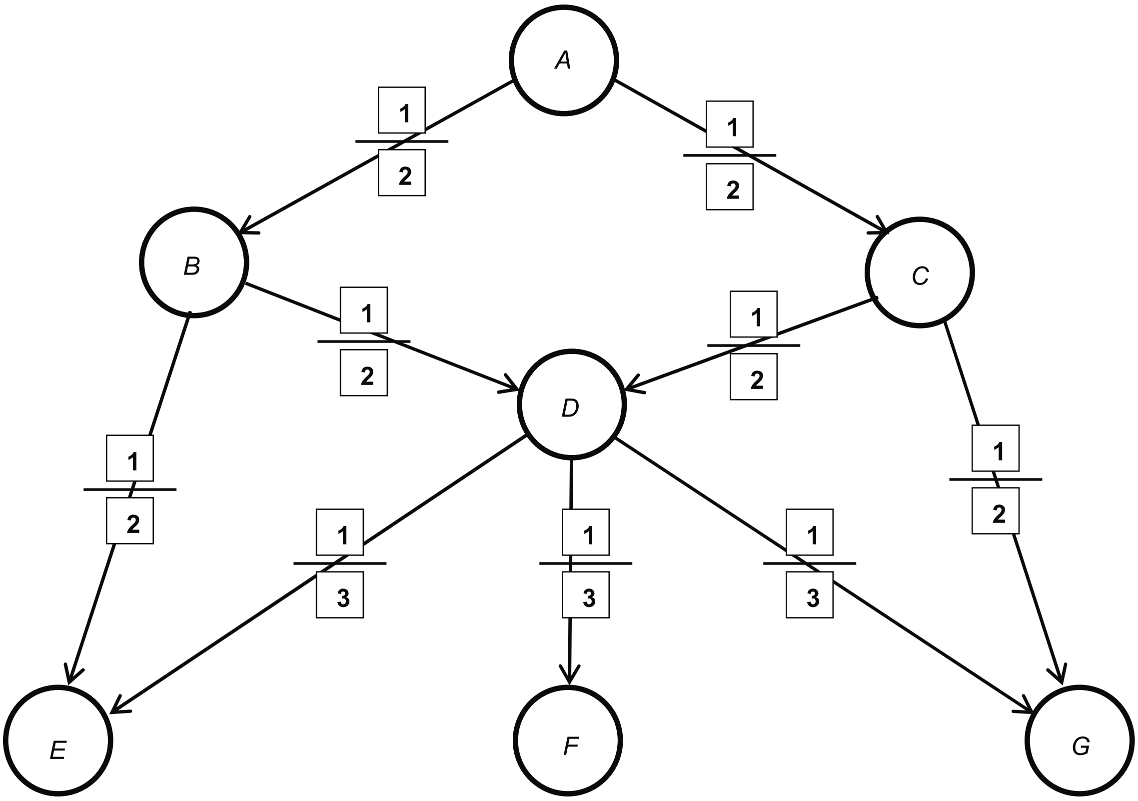
or

A younger age group take Swimming class.

or

Similar

3. (a)



(b) (i)

1. $A \to B \to E$
2. $A \to B \to D \to E$
3. $A \to C \to D \to E$

} 3 ways ← Count all the individual paths from A to E.

(ii)

1. $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$
2. $\frac{1}{2} \times \frac{1}{2} \times \frac{1}{3} = \frac{1}{12}$
3. $\frac{1}{2} \times \frac{1}{2} \times \frac{1}{3} = \frac{1}{12}$

Probability $= \frac{1}{4} + \frac{1}{12} + \frac{1}{12}$

$= \frac{5}{12}$

Calculate the probabilities of the different paths separately e.g. A-B-E means A to B and B to E $= \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$ ("and" means multiply), etc.

4. (a) Different strips: $2 \times 3 \times 5 = 30$

(b) Different strips: $2 \times 2 \times 4 = 16$

This is the Fundamental Principle of Counting. Choosing from 2 and then choosing from 3 and then choosing from 5 gives a total of $2 \times 3 \times 5$ choices.

5. (a) Girls – phone under pillow = 35% of 4171

$= 4171 \times 0{\cdot}35$

$= 1459{\cdot}85$

$= 1460$ (or $1459{\cdot}85$ or 1459)

Use the females' pie chart.

(b) Total number of students = 7150

Boys – phone under pillow = 23% of 2979

"Overall": get the number from both pie charts to calculate the percentage.

$= 685{\cdot}17$

$= 685$ (or $685{\cdot}17$ or 686)

Total $= 1460 + 685 = 2145$ (or $2145{\cdot}02$)

Percentage $= \dfrac{2145}{7150} \times 100 \left(\text{or } \dfrac{2145{\cdot}02}{7150} \times 100\right)$

$= 30\%$ (or $30{\cdot}0002\%$).

(c) Angle = 30% of 360° ← Use the solution from part (b). Remember there are 360° in a pie-chart.

$= 360 \times 0{\cdot}3$

$= 108°$ (or $108{\cdot}00072)°$

6. (a)

Salary (€1000)	0–10	10–20	20–30	30–40	40–50	50–60	60–70
No. of Employees	1	6	12	9	2	1	1

[Note: 10–20 means €10 000 or more but less than €20 000, etc.]

(b) The mid-interval values are 5000, 15 000, 25 000, 35 000, 45 000, 55 000, 65 000

Mean ← 5, 15, 25, 35, 45, 55 and 65 are the mid-interval values.

$$= \frac{(5000 \times 1) + (15\,000 \times 6) + (25\,000 \times 12) + (35\,000 \times 9) + (45\,000 \times 2) + (55\,000 \times 1) + (65\,000 \times 1)}{32}$$

$$= \frac{5000 + 90\,000 + 300\,000 + 315\,000 + 90\,000 + 55\,000 + 65\,000}{32}$$

$$= \frac{920\,000}{32}$$

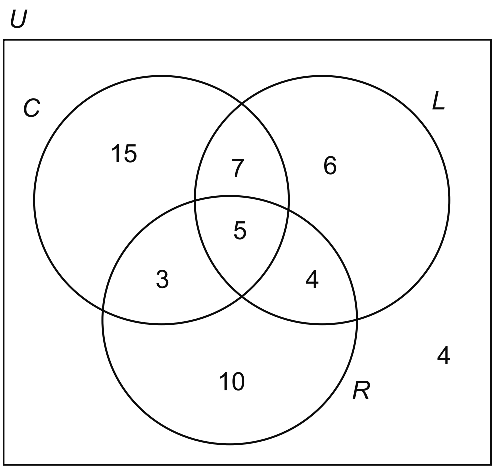
= €28,750

(c) **(i)** Add up all the individual salaries and divide by 32.

(ii) Answer: Adding up individual salaries and dividing by 32

Reason: This gives the actual mean as estimates (mid-intervals) are not used.

7. (a)



← Fill in the intersection of all 3 sets first.

(b) Probability person did not vote $= \dfrac{4}{54}$ or $\dfrac{2}{27}$ ← Probability $= \dfrac{4 \text{ (did not vote)}}{54 \text{ (total)}} = \dfrac{2}{27}$

2013 SEC P2

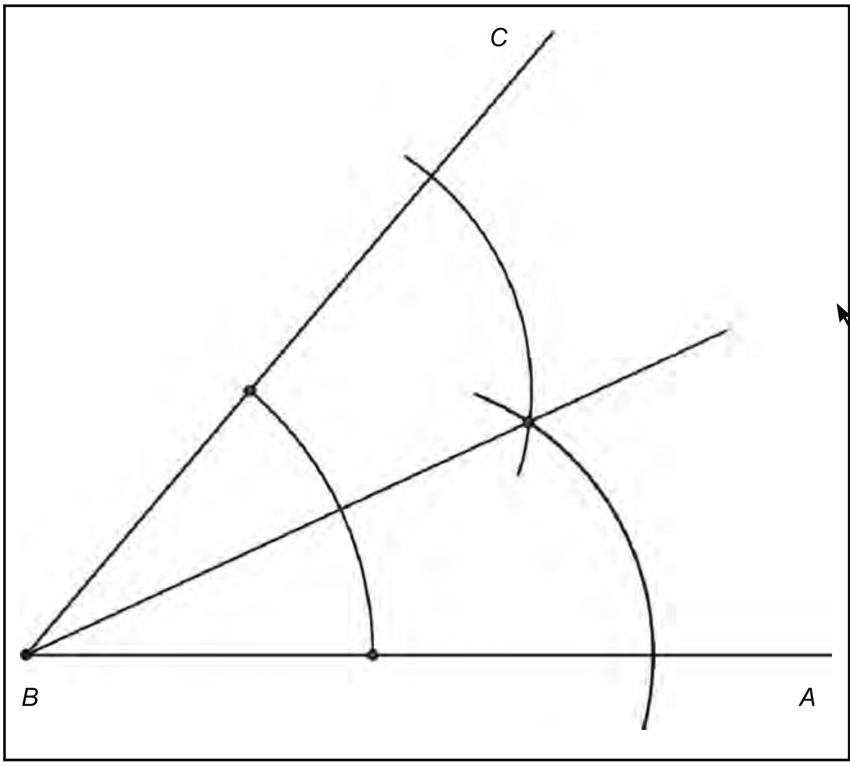
(c) Probability person voted for at least two parties $= \dfrac{3+5+7+4}{54} = \dfrac{19}{54}$

Probability $= \dfrac{\text{19 (voted for two or three parties)}}{\text{54 (total)}}$

(d) Probability person voted for the same party $= \dfrac{15+6+10}{54} = \dfrac{31}{54}$

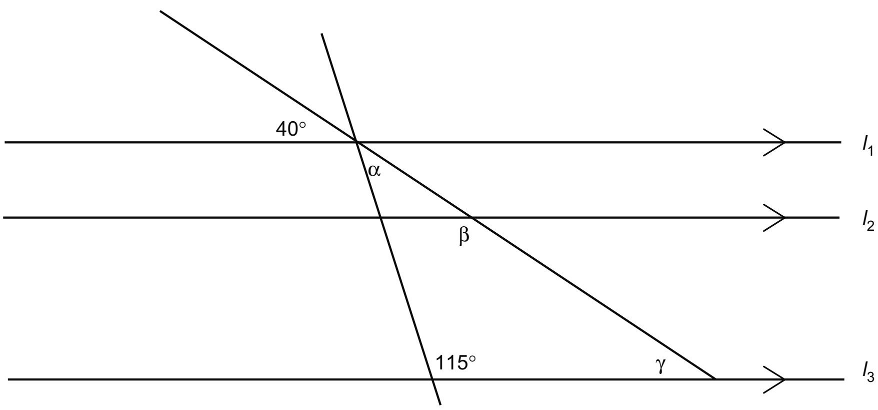
Probability $= \dfrac{\text{31 (voted for C only or L only or R only)}}{\text{54 (total)}}$

8.



1. Place the tip of the compass on the vertex of the angle and draw an arc which cuts each arm of the angle at two different points.
2. Place the tip of the compass on one of the points where the *first arc* cuts an arm of the angle.
3. Place the compass' tip on the point of intersection of the *first arc* and the other arm, without changing the width of the compass.
4. Draw a line connecting the vertex of the angle and the point of intersection of the *second* and the *third* arcs.
5. This line segment bisects the angle.

9.



$\alpha = 180 - (115 + 40) = 25°$ ← A straight angle measures 180°.

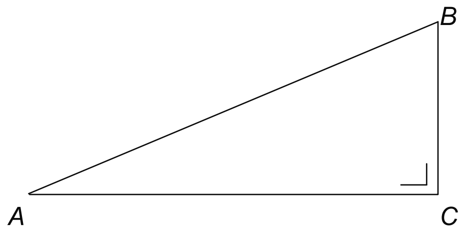
$\beta = 180 - 40 = 140°$ ← Alternate angles are equal.

$\gamma = 40°$ ← The three angles in a triangle sum to 180°.

10. (a) $h^2 = a^2 + b^2$ ← Apply Pythagoras's Theorem.

$2^2 = |AC|^2 + 1^2$

$\Rightarrow |AC| = \sqrt{2^2 - 1^2} = \sqrt{3}$

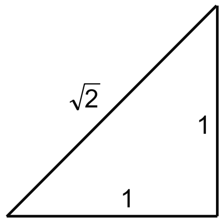


(b) $\cos \angle BAC = \dfrac{\sqrt{3}}{2}$ ← $\cos A = \dfrac{\text{Adjacent}}{\text{Hypotenuse}}$

$|\angle BAC| = 30°$

(c) Hypotenuse $= \sqrt{1^2 + 1^2} = \sqrt{2}$

$\cos 45° = \dfrac{1}{\sqrt{2}}$



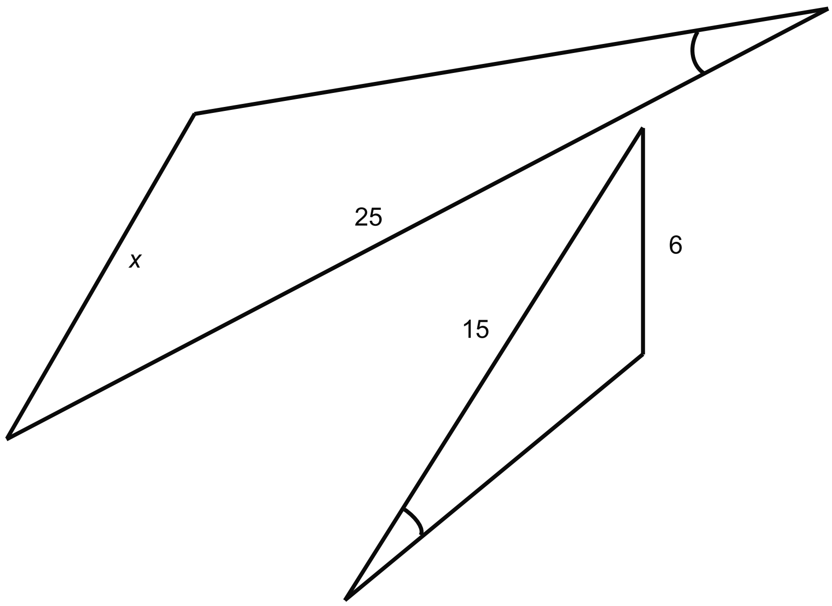
(d) $\cos 75°' = \dfrac{\sqrt{6} - \sqrt{2}}{4} = 0{\cdot}2588$

$\cos 45° + \cos 30° = \dfrac{1}{\sqrt{2}} + \dfrac{\sqrt{3}}{2} = 0{\cdot}7071 + 0{\cdot}8660 = 1{\cdot}5731$

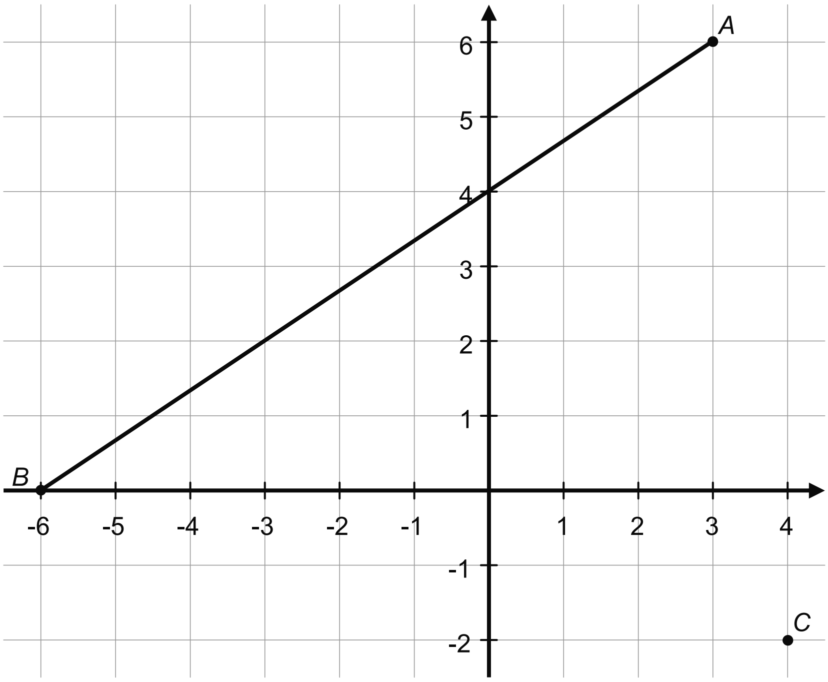
$(0{\cdot}2588 \neq 1{\cdot}5731)$

11. $\dfrac{x}{6} = \dfrac{25}{15}$ ← Similar triangles have corresponding sides in proportion.

$\Rightarrow x = 10$



12.



(a) $A(3, 6)$ $B(-6, 0)$ $C(4, -2)$

(b) $D = \left(\frac{3-6}{2}, \frac{6+0}{2}\right) = \left(-\frac{3}{2}, 3\right)$ ← Midpoint formula: See page 18 of *Formulae and Tables.*

(c) Slope $AB = \frac{0-6}{-6-3} = \frac{2}{3}$ ← Equation formula: $y - y_1 = m(x - x_1)$ See page 18 of *Formulae and Tables.*

Equation AB: $y - 0 = \frac{2}{3}(x + 6)$ or $y - 6 = \frac{2}{3}(x - 3)$

or

$y = \frac{2}{3}x + 4$

$2x - 3y + 12 = 0$

(d) Perpendicular slope $= -\frac{3}{2}$ ← Remember: to find a perpendicular slope, invert and change the sign.

Line through C: $y + 2 = -\frac{3}{2}(x - 4)$

$3x + 2y - 8 = 0$

or

The line is of the form $3x + 2y + c = 0$

$(4, -2)$: $3(4) + 2(-2) + c = 0 \Rightarrow c = -8$

$3x + 2y - 8 = 0$

(e) E the point of intersection of two lines $2x - 3y + 12 = 0$ (i) ← Use simultaneous equations to find E.

$3x + 2y - 8 = 0$ (ii)

$2 \times$ (i) $4x - 6y = -24$ or $y = \frac{2x + 12}{3}$

$+ 3 \times$ (ii) $9x + 6y = 24$ $\Rightarrow 3x + 2\left(\frac{2x + 12}{3}\right) - 8 = 0$

$\Rightarrow 9x + 4x + 24 - 24 = 0$

$\Rightarrow x = 0$ and $y = 4$

(f) $|CD| = \sqrt{\left(4 + \frac{3}{2}\right)^2 + (-2-3)^2} = \sqrt{55{\cdot}25}$ or $7{\cdot}433$

$|CE| = \sqrt{(4-0)^2 + (-2-4)^2} = \sqrt{52}$ or $7{\cdot}211$

$|CE|$ is the shorter distance.

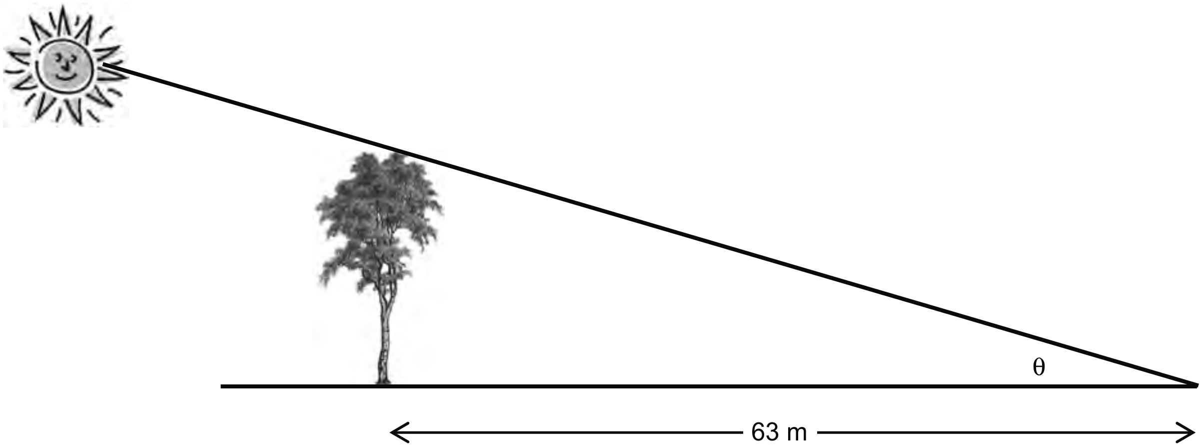
or

$|CE|$ (is the perpendicular distance and therefore is the shorter distance.)

$|CE| = \sqrt{(4-0)^2 + (-2-4)^2} = \sqrt{52}$ or $7{\cdot}211$

Length formula: See page 18 of *Formulae and Tables.*

13.



$\tan\theta = \frac{32}{63}$

$\Rightarrow \theta = 26{\cdot}9277$

$\Rightarrow \theta = 26°55{\cdot}39{\cdot}64''$

$= 26°56'$

or

$\tan\alpha = \frac{63}{32}$

$\Rightarrow \alpha = 63{\cdot}0723$

$\Rightarrow \theta = 90 - 63{\cdot}0723$

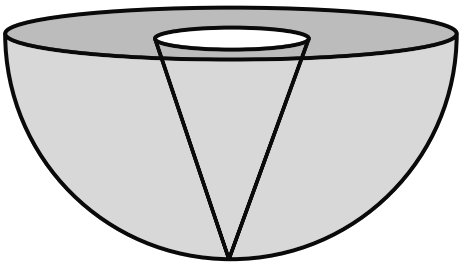
$= 26{\cdot}9277$

$= 26°55'39{\cdot}64''$

$= 26°56'$

$\tan A = \frac{\text{Opposite}}{\text{Adjacent}}$

14.



(a) Volume of hemisphere $= \frac{2}{3}\pi r^3$

$= \frac{2}{3} \times \pi \times 12^3$

$= 1152\pi \text{ cm}^3$

Note: This formula is found by halving the volume of a sphere. See page 10 of *Formulae and Tables.*

(b) Volume of cone $= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \pi \times 4^2 \times 12$

$= 64\pi \text{ cm}^3$

Volume of a cone: $= \frac{1}{3}\pi^2 h$

See page 10 of *Formulae and Tables.*

(c) Remaining metal $= 1152\pi - 64\pi$

$= 1088\pi$

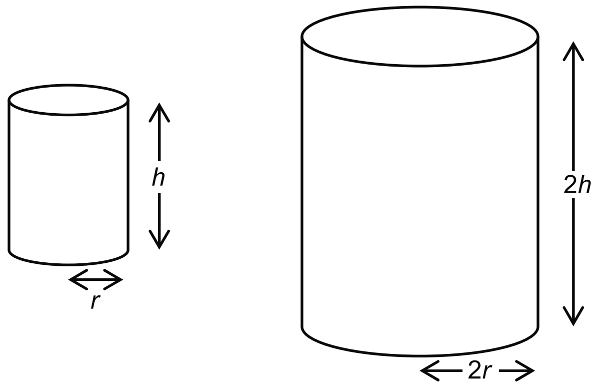
Volume of cone = 64π

$$\text{Number of cones} = \frac{1088\pi}{64\pi}$$
$$= 17$$

or

$$\text{Number of cones} = \frac{1152\pi}{64\pi} - 1$$
$$= 18 - 1$$
$$= 17$$

15.



(a) Curved surface area of a cylinder $= 2\pi rh$

Curved surface area of cylinder $= 2\pi rh$
See page 10 of *Formulae and Tables.*

Curved surface area of small cylinder $= 2 \times \pi \times r \times h$
$= 2\pi rh$

Curved surface area of large cylinder $= 2 \times \pi \times (2r) \times (2h)$
$= 8\pi rh$

Ratio $= 2\pi rh : 8\pi rh$
$= 1 : 4$

(b) Volume of a cylinder $= \pi r^2 h$

Volume of a cylinder $V = \pi r^2 h$
See page 10 of *Formulae and Tables.*

Volume of small cylinder $= \pi \times r^2 \times h$
$= \pi r^2 h$

Volume of large cylinder $= \pi \times (2r)^2 \times (2h)$
$= 8\pi r^2 h$

Ratio $= \pi r^2 h : 8\pi r^2 h$
$= 1 : 8$

2012 SEC Paper 2 (Phase 2)

1. $0{\cdot}3 \times 0{\cdot}26 = 0{\cdot}078\ \text{m}^2$

$0{\cdot}078 \times 100 = 7{\cdot}8\ \text{m}^2$

$300 \times 260 = 78\,000$

$78\,000 \times 100 = 7\,800\,000$

$= 7{\cdot}8\ \text{m}^2$

10 mm = 1 cm
1000 mm = 1 m
$1\,000\,000\ \text{mm}^2 = 1\ \text{m}^2$

2. **(a)** $\pi r^2 h = V$ ← 1 litre = $1000\ \text{cm}^3$

$\pi \times r^2 \times 70 = 50\,000$ ← 0.7 m = 70 cm

$r^2 = \dfrac{50\,000}{\pi \times 70}$

$r^2 = 227{\cdot}36$

$r = 15$

Volume of a Cylinder formula:
$V = \pi r^2 h$
See page 10 of *Formulae and Tables*.

Diameter = 30 cm

(b) $60 \times 35 \times 15 = 31\,500\ \text{cm}^3$ or $31{\cdot}5$ L

Change all to centimetres.
Remember 1 m = 100 cm.

(c) $\pi \times 15 \times 15 \times h = 31\,500$

$h = \dfrac{31\,500}{\pi \times 15 \times 15}$

$h = 44{\cdot}6$ cm

$\dfrac{31{\cdot}5}{50} \times 100 = 63\%$

63% of the height of 70 cm is 44·1 cm

Method 1: Compare volumes
Method 2: Compare percentages

3. **(a)** $\dfrac{24}{4} = 6$ cm ← The height consists of four side lengths of the cube.

(b) $2(6 \times 6 \times 6) + 2\left(\dfrac{4}{3}\pi \times 3^3\right)$ ← The volume is made up of 2 spheres and 2 cubes.

$= 432 + 226$

$= 658\ \text{cm}^3$

Volume of a Sphere formula:
$V = \dfrac{4}{3}\pi r^3$
See page 10 of *Formulae and Tables*.

(c) $24 \times 6 \times 6 - 658{\cdot}2 = 864 - 658{\cdot}2 = 205{\cdot}8$

$\dfrac{205{\cdot}8}{864} \times \dfrac{100}{1} = 23{\cdot}82\%$

$6 \times 6 \times 6 = 216$

$\dfrac{4}{3} \times \pi \times 3^3 = 113{\cdot}097$

$216 - 113{\cdot}097 = 102{\cdot}9$

$102{\cdot}9 \times 2 = 205{\cdot}8$

$\dfrac{205{\cdot}8}{864} \times 100 = 23{\cdot}82\%$

$\dfrac{\text{Remaining}}{\text{Total}} \times 100\%$

4. (a) Answer = Diagram D

$2\pi r = 2 \times \pi \times 3{\cdot}5 = 22$ cm

Need a piece 10 cm × 22 to make this cylinder.
Only D has this.

Surface Area of Cylinder = Rectangle + Two Circular ends

(b) Area of sheet: $18 \times 23 = 414$

Surface area of cylinder: $10 \times 22 + 2(\pi \times 3{\cdot}5^2) = 220 + 77 = 297$

Metal remaining: $414 - 297 = 117$ cm^2

Volume of a Cylinder formula:
$V = \pi r^2 h$
See page 10 of *Formulae and Tables*.

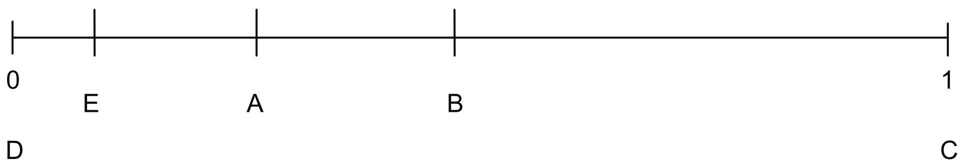
(c) $V = \pi \times 3{\cdot}5^2 \times 10 = 384{\cdot}85$ cm^3

5. (a)

Event		Probability
A club is selected in a random draw from a pack of playing cards.	A	$\frac{1}{4}$
A tossed fair coin shows a tail on landing.	B	$\frac{1}{2}$ OR evens OR $\frac{50}{50}$
The sun will rise in the east tomorrow.	C	1 OR certain
May will follow directly after June.	D	0 OR impossible
A randomly selected person was born on a Thursday.	E	$\frac{1}{7}$

$$\text{Probability} = \frac{\text{Favourable outcomes}}{\text{Total outcomes}}$$

(b)



6. (a)

Male actors		Female actors
9	2	5 6 8 9 9 9
8 8 7 7 6 2	3	0 2 3 3 3 5 5 6 9
8 7 6 5 5 3 0	4	2 5 5 9
4 2 0 0	5	
0 0	6	1

Key: 2|5 is 25 years old

Don't forget to fill in the key

(b) Same shape of distribution

Discuss shape/range/unusual numbers etc.

No one is over 61.

No one is under 24.

The range is similar in both.

There is an outlier in the female winners.

No female is in her 50s.

The females are younger.

(c) **(i)**

Male	Female
Sum = 887	Sum = 714
Mean = 44·35	Mean = 35·7

Use numbers to back up your arguments.

The mean age of women is lower so the statement is true for mean age.

(ii)

Male	Female
$\frac{(45 + 45)}{2}$	$\frac{(33 + 33)}{2}$
Median = 45	Median = 33

The median age of women is lower so the statement is true for median age.

(d)

Male	Female
50 − 37·5 = 12·5	40·5 − 29 = 11·5

I.Q.R. = Upper Quartile–Lower Quartile

7. **(a)** $\frac{2}{3}$ or $\frac{240}{360}$

Probability = $\frac{2 \text{ (blue)}}{3 \text{ (total)}}$

(b) $\frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$

"and": Multiply the individual probabilities.

(c) $\frac{1}{2} \times \frac{2}{3} = \frac{1}{3}$

"and": Multiply the individual probabilities.

8. **(a)** $\frac{1}{6}$

Probability = $\frac{1 \text{ (one shows on the die)}}{6 \text{ (total)}}$

(b) 500 − (70 + 82 + 90 + 91 + 81) = 86

(c) $\frac{70}{500} = 0{\cdot}14$ $\frac{90}{500} = 0{\cdot}18$

$\frac{82}{500} = 0{\cdot}16$ $\frac{91}{500} = 0{\cdot}18$

$\frac{86}{500} = 0{\cdot}17$ $\frac{81}{500} = 0{\cdot}16$

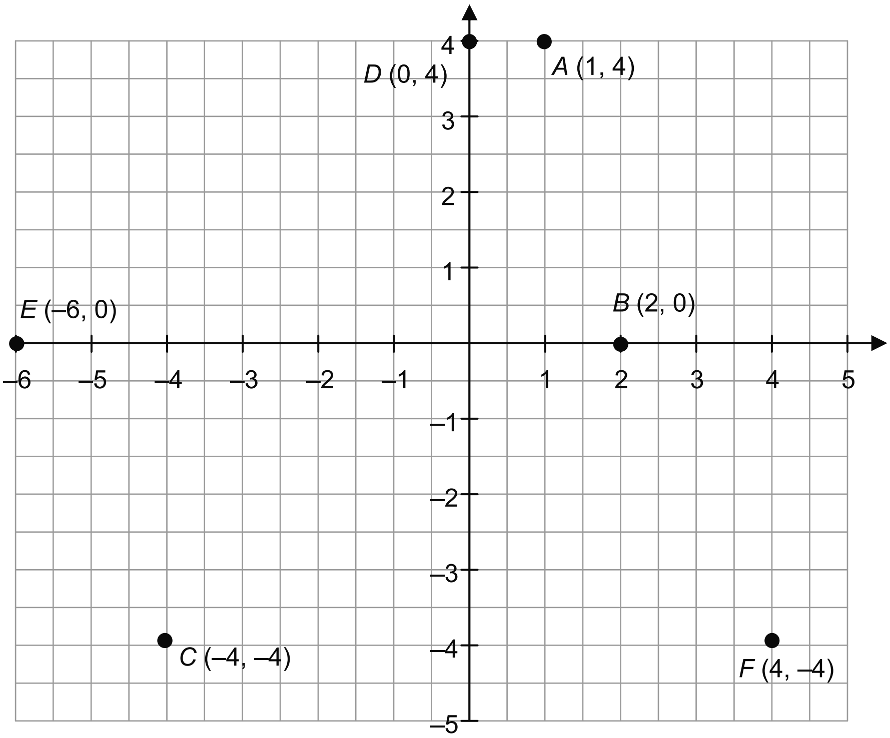
Relative Frequency: How often something happens divided by all the outcomes

(d) 0·166 V 0·14

Experimental error

2012 SEC P2

9. (a) and (b)



(c) $\left(\frac{0+4}{2}, \frac{4-4}{2}\right) = (2, 0)$ ← Midpoint formula: See page 18 of *Formulae and Tables*.

(d) $\frac{-4-0}{4-2} = \frac{-4}{2} - 2$ ← Slope formula: See page 18 of *Formulae and Tables.*

(e) $y = -2x + 4$ $\quad$ $y - 0 = -2(x - 2)$

$y = -2x + 4$ ← Equation formula: $y - y_1 = m(x - x_1)$ See page 18 of *Formulae and Tables.*

(f) $\frac{0-(-4)}{-6-(-4)} = \frac{4}{-2} = -2$ ← Slope formula: See page 18 of *Formulae and Tables*.

(g) $y - 0 = -2(x + 6)$ ← Equation formula: $y - y_1 = m(x - x_1)$ See page 18 of *Formulae and Tables.*

$y = -2x - 12$

$2x + y + 12 = 0$

(h)

Area of ΔBCE	Area of ΔBCF
$\frac{1}{2}(8)(4)$	$\frac{1}{2}(8)(4)$
16	16

← Area of a Triangle Formula $A = \frac{1}{2} b \times h$ See page 9 of *Formulae and Tables*.

Ratio: 1:1 or 1/1

(i) Answer: Yes ← Congruent: identical in all respects

Reason: *CFBE* is a parallelogram and *CB* is a diagonal which divides the parallelogram into two congruent triangles.

or

SSS or SAS or ASA argument

10. (a) Line 3 OR $y = 5x + 20$ ← Use $y = mx + c$

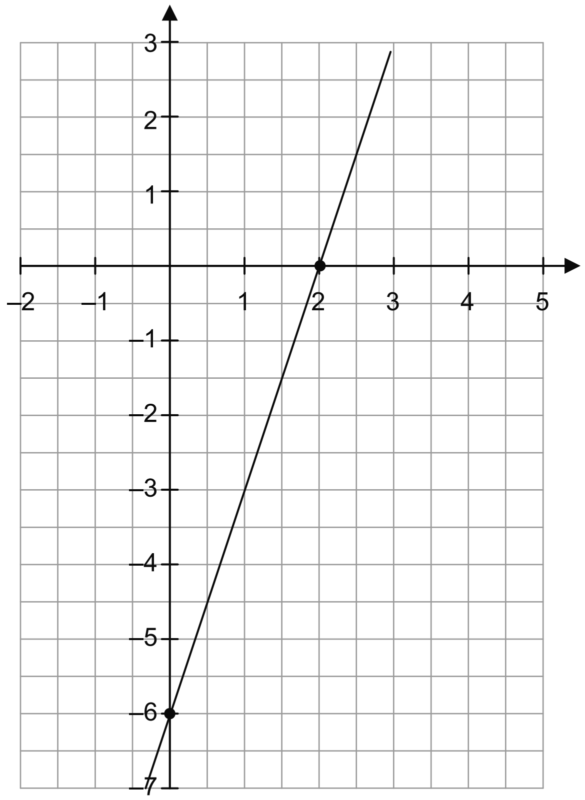
5 is the biggest number in front of x for any of the lines.

(b) Line 1 and Line 2 ← Parallel lines have equal slopes.

$y = 3x - 6$ and $y = 3x + 12$

They have the same slope (3).

(c)



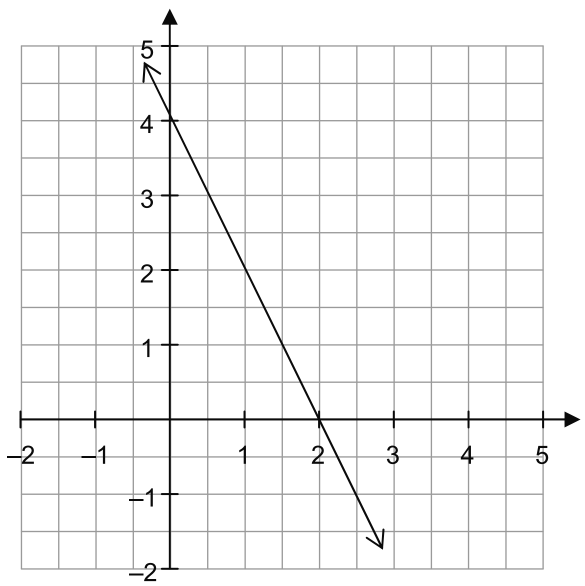
$y = 3x - 6$

$x = 0, y = -6$

$y = 0, x = 2$

$(2, 0), (0, -6)$

(d)



slope $= \dfrac{0-4}{2-0} = -2$

y-intercept = 4

equation $y = -2x + 4$

Answer = Line 5 ($y = -2x + 4$)

← Equation formula: $y - y_1 = m(x - x_1)$ See page 18 of *Formulae and Tables*.

2012 SEC P2

(e) $y = 4x - 16$ $\quad m = \dfrac{20 - 12}{9 - 7} = 4$

$y = 4(7) - 16 = 12$ $\quad y - 12 = 4(x - 7)$

$y = 4(9) - 16 = 20$ $\quad y = 4x - 16$

$y = 4(10) - 16 = 24$

Answer = Line 6

Find the equation which is satisfied by all the points.

(f) $y = x - 7$ ← Simultaneous Equations

$y = 4x - 16$

$0 = 3x - 9$

$x = 3$

$y = 3 - 7 = -4$

$(3, -4)$

Answer: $x = 3$

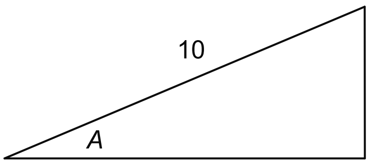
(g) Line 4

$y = (3) - 7 = -4$

Line 6

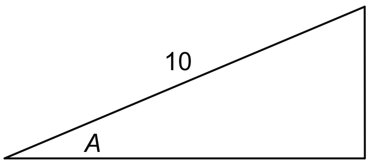
$y = 4(3) - 16 = -4$

11. (a)



1. Draw a rough diagram.
2. Remember that $\sin A = \dfrac{\text{Opposite}}{\text{Hypotenuse}} = \dfrac{4}{10}$
3. Draw a base of any length.
4. Use your protractor/set square to draw a perpendicular line to the base.
5. Mark off 4 cm on this line using your compass.
6. Use the compass to draw a 10 cm line segment from the top of the 4 cm line segment to the base.
7. Label angle A and the side lengths 4 cm and 10 cm.

(b)

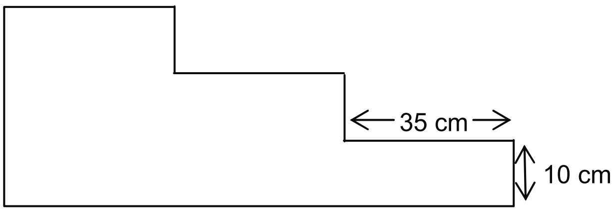


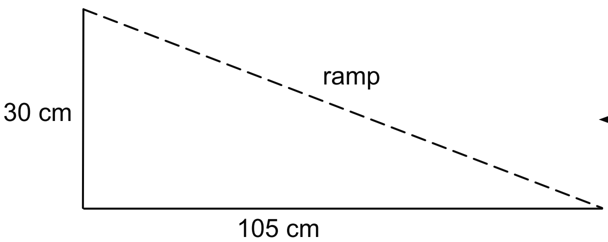
$\cos A = \dfrac{\text{Adjacent}}{\text{Hypotenuse}}$

$\sqrt{100 - 16} = \sqrt{84}$

$\cos A = \dfrac{\sqrt{84}}{10}$

12.





Apply Pythagoras's Theorem

$\text{Ramp} = \sqrt{30^2 + 105^2}$

$= 109{\cdot}2 \text{ cm}$

OR

$H^2 = 35^2 + 10^2$

$H = \sqrt{1225 + 100}$

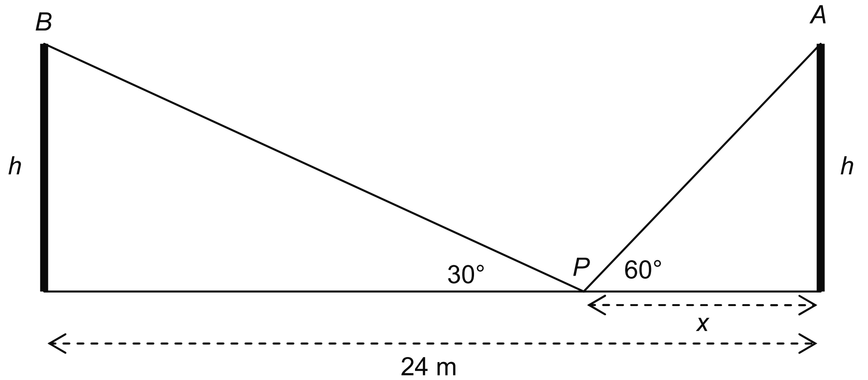
$H = \sqrt{1325}$

$\text{Ramp} = 3 \times H$

$\text{Ramp} = 3 \times \sqrt{1325}$

$\text{Ramp} = 109{\cdot}2 \text{ cm}$

13.



(a)

$\tan 60° = \frac{h}{x}$ | $\tan 60° = \frac{h}{x}$ | $\tan 30° = \frac{h}{24 - x}$ ← $\tan A = \frac{\text{Opposite}}{\text{Adjacent}}$

$\sqrt{3} = \frac{h}{x}$ | $1{\cdot}732 = \frac{h}{x}$ | $\frac{1}{\sqrt{3}} = \frac{h}{24 - x}$

$h = \sqrt{3}\,x$ | $h = 1{\cdot}732\,x$ | $h = \frac{24 - x}{\sqrt{3}}$

(b)

$\tan 30° = \frac{h}{24 - x}$ | $\tan 30° = \frac{h}{24 - x}$

$\frac{1}{\sqrt{3}} = \frac{\sqrt{3}x}{24 - x}$ | $\frac{1}{1{\cdot}732} = \frac{1{\cdot}732x}{24 - x}$

$3x = 24 - x$ | $3x = 24 - x$

$4x = 24$ | $4x = 24$

$x = 6$ | $x = 6$

$h = 6\sqrt{3} \text{ m}$ | $h = 6\sqrt{3} \text{ m or } 6(1{\cdot}732) \text{ m} = 10{\cdot}39 \text{ m}$

2012 SEC P2

14. Given: A circle with centre O, with points A, B and C on the circle

To Prove: $|\angle BOC| = 2|\angle BAC|$

Construction: Join A to O and extend to R

Proof: In the triangle AOB

$|\angle AO| = |\angle OB|$ Radii

$\Rightarrow |\angle OBA| = |\angle OAB|$ Theorem 2 (isosceles Δ)

$|\angle BOR| = |\angle OBA| + |\angle OAB|$ Theorem 6 (exterior angle)

$\therefore |\angle BOR| = |\angle OAB| + |\angle OAB|$

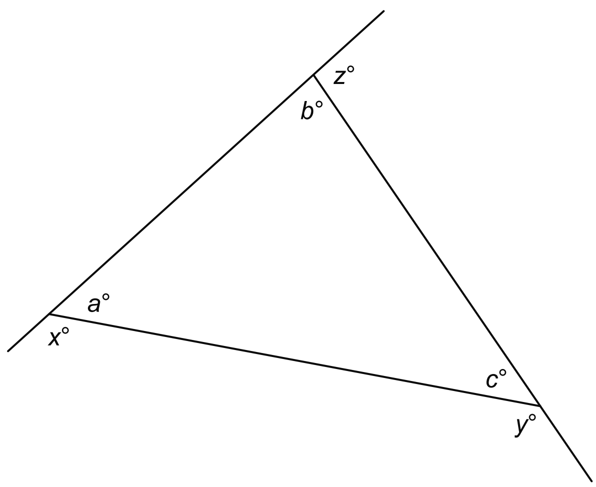
$\therefore |\angle BOR| = 2|\angle OAB|$

Similarly $|\angle ROC| = 2|\angle OAC|$

$\therefore |\angle BOC| = 2|\angle BAC|$

Standard Proof

15. (a)



Label clearly any angles you add in.

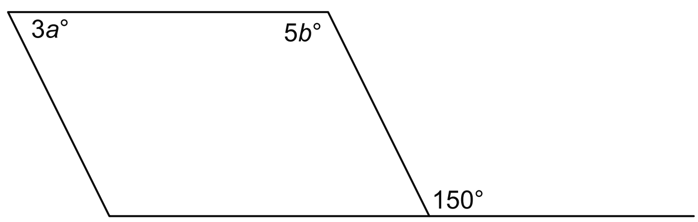
$x = b + c$ (external angle) $a + b + c = 180$ (Triangle)

$y = a + b$ (external angle) $x + y + z = 360$

$z = a + c$ (external angle)

$x + y + z = 2(a + b + c)$

(b)



Alternate angles are equal.

$5b = 150$(alt)

$b = 30$

$3a + 5b = 180$

$3a = 30$

$a = 10$

2011 SEC Paper 2 (Phase 1)

1. (a) Semicircular lengths = $2\pi r = 14\pi$ Step 1

Straight lengths = 2(28) or 56 Step 2

Total length = $14\pi + 56$

= 99·98

= 100 cm Step 3

Both semicircular ends together form a full circle.

(b) (i) Volume of cylinder = $\pi r^2 h$

$= \pi(27)^2(70)$

$= 51030\pi$ cm^3

Don't forget to give your answer correct to the nearest whole number.

(ii) Volume of cone = $\frac{1}{3}\pi r^2 h$

$= \frac{1}{3}\pi(27)^2(70)$

$= 17010\pi$ cm^3

or

Volume of cone = $\frac{1}{3}(51030\pi)$

$= 17010\pi$ cm^3

Volume of a Cylinder formula: See page 10 of *Formulae and Tables*.

'Leave your answer in terms of π'. This means leave π in your answer.

Volume of a Cone formula: See page 10 of *Formulae and Tables*.

(iii) Remainder = $51030\pi - 17010\pi$

$= 34020\pi$

Volume of Cylinder – Volume of Cone

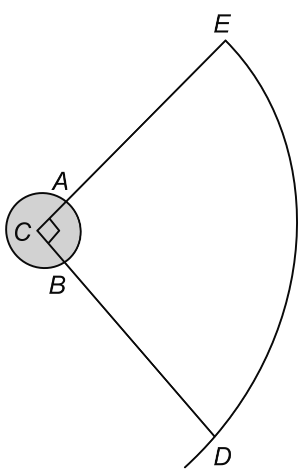
Fraction = $\frac{34020\pi}{51030\pi}$

$= \frac{34020}{51030}$ or $\frac{2}{3}$

or

$1 - \frac{1}{3} = \frac{2}{3}$

(c)



(i) Area of $CDE = \frac{1}{4}\pi r^2 = \frac{1}{4}\pi(100)^2$ ← CDE is one quarter of the disc.

$= 2500\pi$

$= 7853{\cdot}9816$

$= 7853{\cdot}98 \text{ m}^2$

(ii) Area of throwing zone $= \pi r^2 = \pi(1)^2$ ← Circle of radius 1 m

$= 1\pi$

$= 3{\cdot}1416$

$= 3{\cdot}14 \text{ m}^2$

(iii) Area of the shot-put zone $= \frac{3}{4}$ (Area of throwing zone) + Area CDE

$\frac{3}{4}$ (Area of throwing zone) $= \frac{3}{4}(3{\cdot}14) = 2{\cdot}355$

Area of shot-put zone $= 2{\cdot}355 + 7853{\cdot}98$

$= 7856{\cdot}335$

$= 7856{\cdot}34$

or

$\frac{3}{4}$ (Area of throwing zone) $= 0{\cdot}75\pi$

Area of shot put zone $= 0{\cdot}75\pi + 2500\pi = 2500{\cdot}75\pi$

$= 7856{\cdot}337828$

$= 7856{\cdot}34 \text{ m}^2$

or

$\frac{1}{4}$ (Area of throwing zone) $= \frac{1}{4}(3{\cdot}14) = 0{\cdot}785$

Area of shot put zone $= 7853{\cdot}98 + 3{\cdot}14 - 0{\cdot}785$

$= 7856{\cdot}335$

$= 7856{\cdot}34$

or

$\frac{1}{4}$ (Area of throwing zone) $= 0{\cdot}25\pi$

Area of shot put zone $= 2500\pi + 1\pi - 0{\cdot}25\pi = 2500{\cdot}75\pi$

$= 7856{\cdot}337828$

$= 7856{\cdot}34 \text{ m}^2$

2.

Blood Group	Percentage in Irish population	Blood groups to which transfusions can be safely given	Blood groups from which transfusions can be safely received
O−	8	All	O−
O+	47	O+, AB+, A+, B+	O+ and O−
A−	5	A−, A+, AB+, AB−	A− and O−
A+	26	A+ and AB+	A+, O−, O+, A−
B−	2	B−, B+, AB−, AB+	B− and O−
B+	9	B+ and AB+	B+, B−, O−, O+
AB−	1	AB− and AB+	AB−, O−, A−, B−
AB+	2	AB+	All

Source: Irish Blood Transfusion Service

(a) $\frac{1}{100}$ ← $1\% = \frac{1}{100}$

(b) B− (2%) + O− (8%) = 10%

OR $B-\left(\frac{2}{100}\right) + O-\left(\frac{8}{100}\right) = \frac{10}{100}$ ← $= \frac{1}{10}$ (reduce to simplest form where possible in probability answers)

(c) O+(47%) + AB+(2%) + A+(26%) + B+(9%) = 84% = $\frac{84}{100}$ ← $= \frac{21}{25}$

(d) O− can only receive blood from other O− people. This is only 8% of the population, therefore this category needs to be encouraged to donate blood.

or

O− can safely give blood to all other groups and so is the best to have if there is any shortage of blood.

3.

Colour	Frequency	Relative frequency	Daily frequency (Part (e) below)
Red	70	$\frac{70}{500}$ or 0·14	336
Blue	100	$\frac{100}{500}$ or 0·2	480
Yellow	45	$\frac{45}{500}$ or 0·09	216
White	55	$\frac{55}{100}$ or 0·11	264
Black	90	$\frac{90}{500}$ or 0·18	432
Silver	140	$\frac{140}{500}$ = 0·28	672
Total	500	$\frac{500}{500}$ or 1	2400

(a) 500 − (70 + 100 + 45 + 55 + 140)

= 500 − 410

= 90 black cars

(b) Done in table ← Relative Frequency: How often something happens divided by all the outcomes

(c) Method: The sum of the relative frequencies should total to 1.

OR The percentages should sum to 100%.

Check: Candidate to show his/her check

(d) $\frac{70}{500} = 0{\cdot}14 = 14\%$ ← $\frac{\text{No. of red cars}}{\text{Total No. of cars}}$

2011 SEC P2

(e) $\frac{70}{500} \times 2400 = 336$ $\frac{55}{500} \times 2400 = 264$

$\frac{100}{500} \times 2400 = 480$ $\frac{90}{500} \times 2400 = 432$

$\frac{45}{500} \times 2400 = 216$ $\frac{140}{500} \times 2400 = 672$

OR

$2400 \div 500 = 4{\cdot}8$

$70 \times 4{\cdot}8 = 336$

$100 \times 4{\cdot}8 = 480$

$45 \times 4{\cdot}8 = 216$

$55 \times 4{\cdot}8 = 264$

$90 \times 4{\cdot}8 = 432$

$140 \times 4{\cdot}8 = 672$

(f) No. A test is reliable if repeated runs of the test would give the same results. There is no reason to say that if this test was run again it would be different because of the sample not being random. The colour of a vehicle is random and running the test at different times of the day or on different days would not necessarily make the test any more reliable.

4. (a) $5 \times 9 = 45$ OR $180 \div 5 = 36$ OR $180 \div 5 = 36$

$180 \div 45 = 4$ dessert choices $36 \div 9 = 4$ dessert choices $36 \div 9 = 4$ dessert choices

(b) $S \times M \times D$ ← Fundamental Principle of Counting

$4 \times 8 \times 4 = 128$ different 3-course lunches

5.

Airplane	A	B	C	D	E	F	G
Distance (cm)	188	200	250	30	380	330	302

(a) Median = 250 OR C ← Median: Middle value when the values are arranged in order of size

(b) $\frac{180 + 200 + 250 + 30 + 380 + 330 + 302}{7} = \frac{1680}{7} = 240$ cm ← Mean: Sum of all values divided by total number of values

(c) $280 \times 7 = 1750$ OR $1750 - 1680 = 70$

$1680 - 30 = 1650$ $70 + 30 = 100$ cm

$1750 - 1650 = 100$ cm

OR

$$\frac{188 + 200 + 250 + 30 + 380 + 330 + 302 + x}{7} = 250$$

$$\frac{1650 + x}{7} = 250$$

$$1650 + x = 1750$$

$$x = 100 \text{ cm}$$

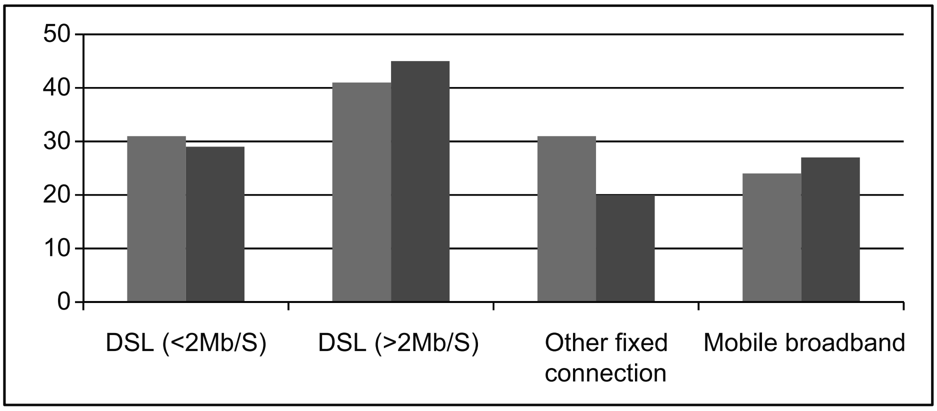
(d) The minimum distance is anything greater than 250 cm.

OR $x > 250$ cm, $x \in \mathbb{R}$

6.

	2008	2009
	%	%
Broadband connection	84	84
By type of connection		
DSL (<2Mb/S)	31	29
DSL (>2Mb/S)	41	45
Other fixed connection	31	20
Mobile broadband	24	27

(a) Bar Charts



(b) The ‘fixed connection’ went down a lot.
The DSL > 2Mb (faster connection) went up.
The DSL < 2Mb (slower connection) went down.
There was no increase in broadband connection.
Mobile broadband went up slightly.

> A comparison bar chart is very useful with this type of data.

7. **(a)**

Test 2		Test 1
	2	7 9
9 9 8 8 7 3	**3**	3 6 7 7 8
9 9 9 8 6 4 4 4 0	**4**	0 1 4 5 5 5 6 7 8 9
9 8 2 2 1 1 0	**5**	1 1 2 2 3 9
2 1	**6**	0

The stem-and-leaf diagram need not be sorted.

> Test 1: Before practice
> Test 2: After practice

> A key should be included with all stem and leaf plots e.g. 5|2 means 52 sit-ups.

(b) 24

(c) Test 1 60 − 27 = 33 Test 2 62 − 33 = 29

> The range is the largest value minus the smallest.

(d) Yes. Only 3 people did worse after practising. 2 did the same and 19 did better.
Yes, there is a higher average.
Yes, the median is higher.
Yes, there is a general shift of data upwards.
Most students did better after the exercise.

(e) **Compared Favourably**: The class average improvement is 2·67. John's improvement is 3. Therefore he improved by more than the average improvement of his classmates.

Compared Unfavourably: There were 8 people below him before the practice. There were only 7 people below him after the practice. Therefore he moved down relative to his classmates.

8.

Number of days absent	None	One	Two	Three	Four	Five
Number of students	9	2	3	4	1	0

(a) 5 ← 24 – 19

(b) The 9 students (or 'none') who missed no days would not change. The 5 who were absent on the Friday would fall under one of the other five categories, since they had missed at least one day (the Friday).

(c)

Smallest possible number of days missed						
Number of days absent	None	One	Two	Three	Four	Five
Number of students	9	7	3	4	1	0

Largest possible number of days missed						
Number of days absent	None	One	Two	Three	Four	Five
Number of students	9	2	3	4	1	5

(d)

Number of days absent	None	One	Two	Three	Four	Five
Number of students	9	2	5	4	3	1
Number of degrees	135°	30°	75°	60°	45°	15°

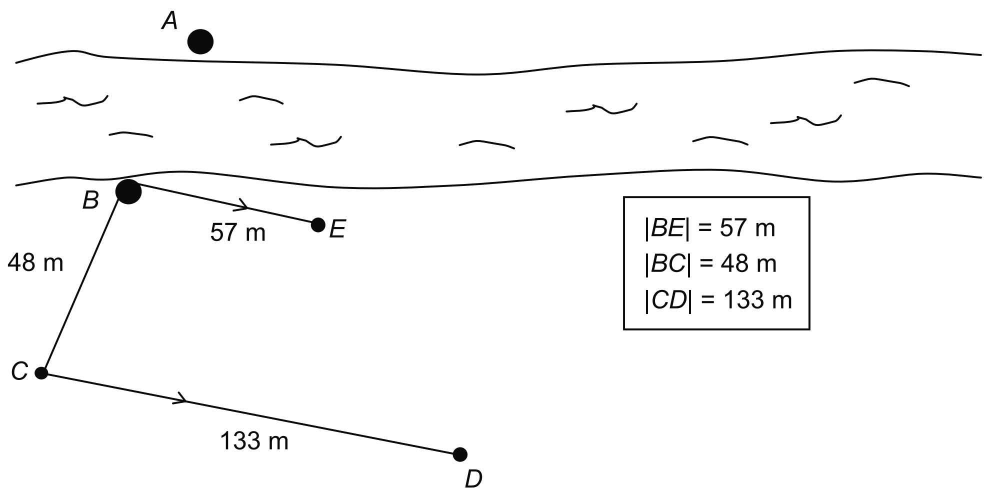
$\frac{135}{360} \times 24 = 9$

$$\frac{9 \times 0 + 2 \times 1 + 5 \times 2 + 4 \times 3 + 3 \times 4 + 1 \times 5}{24}$$

$$= \frac{0 + 2 + 10 + 12 + 12 + 5}{24}$$ ← Mean: sum of all values divided by total number of values

$$= \frac{41}{24} = 1{\cdot}7 \text{ days}$$

9.



(a) Peg C must be collinear with the two trees, A and B. Pegs E and D must be collinear with each other and the tree A. Also $[BE]$ must be parallel to $[CD]$.

Allows us to use similarity to find $|AB|$

(b) $\dfrac{|BA|}{|AC|} = \dfrac{|BE|}{|CD|}$

$\dfrac{|AB|}{48 + |AB|} = \dfrac{57}{133}$

$\Rightarrow 133|AB| = 2736 + 57|AB|$

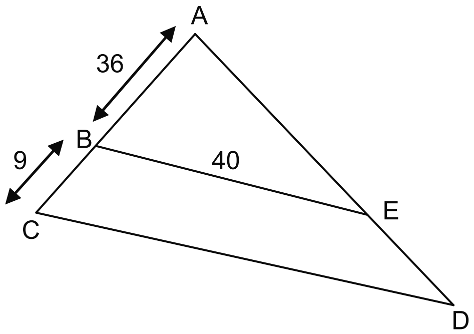
$\Rightarrow 76|AB| = 2736$

$\Rightarrow |AB| = 36$ m

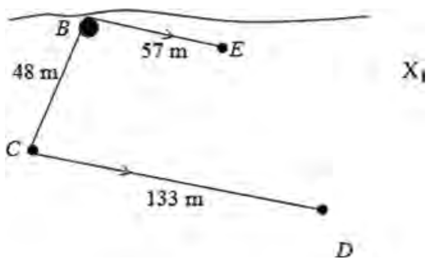
A line drawn parallel to one side of a triangle divides the other two sides in the same proportion.

(c) $\dfrac{|CD|}{40} = \dfrac{45}{36}$

$|CD| = 50$ m



(d)



Create a parallelogram $CBEX$ using strings,

where $|CB| = |XE| = 48$ m

And $|BE| = |CX| = 57$ m

Then extend $[CX]$ until D is collinear with E and A.

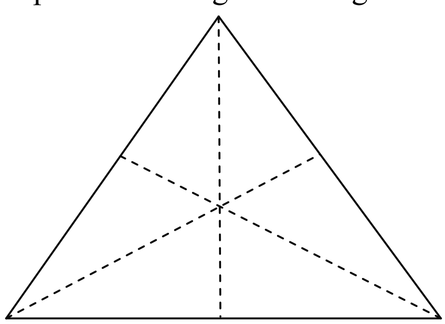
OR

Create a parallelogram BX_1DC, where

$|BX_1| = |CD| = 133$ m and

$|BC = |X_1D| = 48$ m

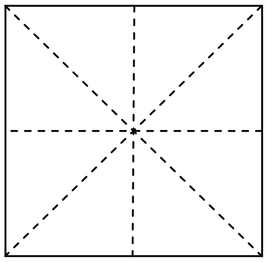
10. (a) Equilateral triangle showing the three axes of symmetry



Axis of Symmetry: line along which the shape will 'fold' onto itself

2011 SEC P2

(b) A square showing the four axes of symmetry



11. (a) To Prove: [AE bisects $\angle DAC$.

Proof:

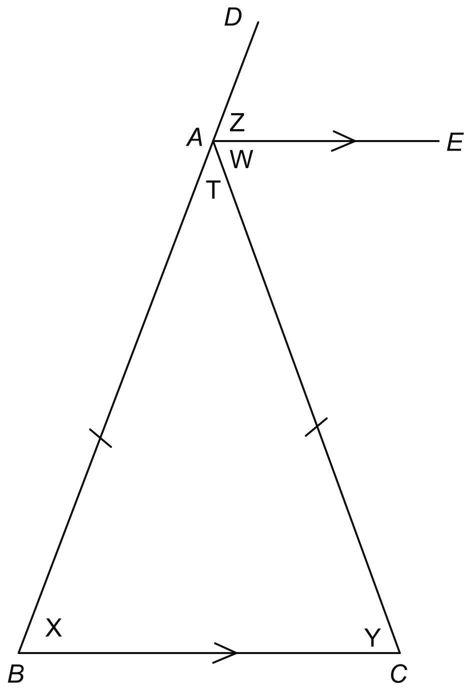
$|\angle Y| = |\angle W|$ Alternate

$|\angle Y| = |\angle X|$ Isosceles

$|\angle X| = |\angle Z|$ Corresponding

$\Rightarrow |\angle W| = |\angle Z|$

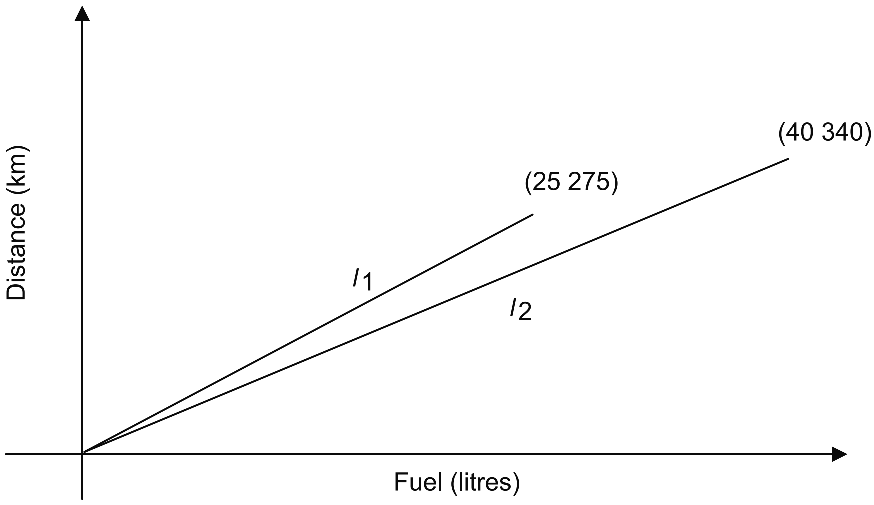
Therefore [AE bisects $\angle DAC$



Make sure to label all angles clearly. You should also justify with a reason, any geometric statement that you make.

(b) No, the result in **(a)** would not still apply. Angle Y would not be equal to angle X.

12.



(a) $\frac{275-0}{25-0}$

Slope of $l_1 = 11$

$\frac{340-0}{40-0}$

Slope of $l_2 = 8.5$

OR

Slope $= \frac{\text{Rise}}{\text{Run}}$

$= \frac{275}{25}$

$= 11$

Slope $l_1 = 11$

Slope $= \frac{\text{Rise}}{\text{Run}}$

$= \frac{340}{40}$

$= 8.5$

Slope $l_2 = 8.5$

Slope formula: See page 18 of *Formulae and Tables*.

(b) The higher slope for l_1 indicates that you get more km per litre at the lower speed.
OR More fuel is used at the higher speed.

Slope is the rate of change of distance with respect to fuel consumption.

(c) l_1:

$y = mx$

$200 = 11x$

$\Rightarrow x = 18{\cdot}18$ litres

$18{\cdot}18 \times 149{\cdot}9 = €27{\cdot}25$

l_2:

$y = mx$

$200 = 8{\cdot}5x$

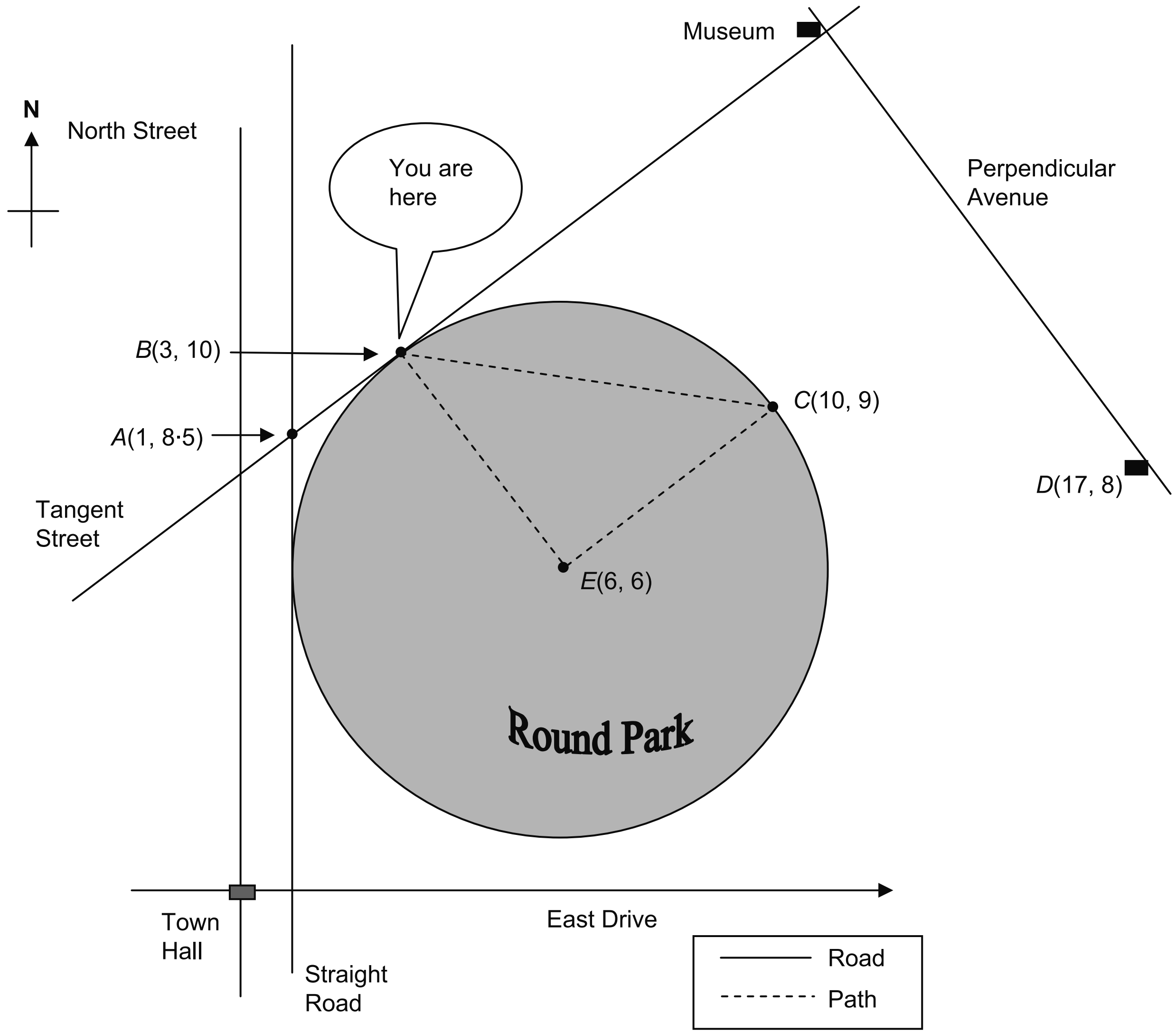
$\Rightarrow x = 23{\cdot}53$ litres

$23{\cdot}53 \times 149{\cdot}9 = €35{\cdot}27$

$35.27 - 27.25 = €8.02$ OR 802c

Calculate the costs separately and subtract.

13.



(a) $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$\sqrt{(10 - 3)^2 + (9 - 10)^2}$

$\sqrt{(7)^2 + (-1)^2}$

$\sqrt{49 + 1}$

$\sqrt{50}$

$7{\cdot}07$ km

Distance formula: See page 18 of *Formulae and Tables*. You could also use Pythagoras's theorem.

2011 SEC P2

(b) $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $\quad 5 + 5 = 10$

$\sqrt{(10 - 6)^2 + (9 - 6)^2}$ $\quad 10 - 7{\cdot}07 = 3$ km

$\sqrt{(4)^2 + (3)^2}$

$\sqrt{9 + 16}$

$\sqrt{25}$

5

> Distance formula: See page 18 of *Formulae and Tables*.

(c) $m = \dfrac{y_2 - y_1}{x_2 - x_1}$

$= \dfrac{10 - 8{\cdot}5}{3 - 1} = \dfrac{1{\cdot}5}{2} \left(\text{or } \dfrac{3}{4}\right)$

$y - y_1 = m(x - x_1)$

$y - 10 = \dfrac{1{\cdot}5}{2}(x - 3)$ or $y - 8{\cdot}5 = \dfrac{1{\cdot}5}{2}(x - 1) \left(\text{or } \dfrac{3}{4} \text{ used as slope}\right)$

$3x - 4y + 31 = 0$ Equation of Tangent Street

> Slope formula: See page 18 of *Formulae and Tables*.

(d) Perpendicular slope $= \dfrac{-2}{1{\cdot}5} \left(\text{or } -\dfrac{4}{3}\right)$

$y - y_1 = m(x - x_1)$

$y - 8 = \dfrac{-2}{1{\cdot}5}(x - 17) \left(\text{or } \dfrac{-4}{3} \text{ used as slope}\right)$

$4x + 3y - 92 = 0$

> Perpendicular slopes: invert and change sign

> Equation of a Line formula: See page 18 of *Formulae and Tables*.

(e) $3x - 4y + 31 = 0$ Museum at (11, 16)

$4x + 3y - 92 = 0$

> Solve simultaneous equations

(f) North to Tangent Street (7·75 km) and then on to the Museum (13·75 km) i.e. distance from (0, 7·75) to (11, 16)

7·75 + 13·75 = 21·5 km

East for 1 km to Straight Road. Then North to *A* (8·5 km). Then from *A* to the Museum i.e. distance from (1, 8·5) to (11, 16) = 12·5 km

1 + 8·5 + 12·5 = 22 km

14. (a) 40 × 2·54 = 101·6 cm

(b) $(9x)^2 + (16x)^2 = 101{\cdot}6^2$

$81x^2 + 256x^2 = 10322{\cdot}56$

$337x^2 = 10322{\cdot}56$

$x^2 = 30{\cdot}63$

$x = 5{\cdot}534$

$\Rightarrow$ length = 16 × 5·534 = 88·55 = 89 to nearest cm

$\Rightarrow$ height = 9 × 5·534 = 49·81 = 50 to nearest cm

> The ratio is 16:9. Apply Pythagoras's theorem. Make sure to give your answer correct to the nearest cm.

(c) $(4x)^2 + (3x)^2 = 101{\cdot}6^2$

$16x^2 + 9x^2 = 10322{\cdot}56$

$25x^2 = 10322{\cdot}56$

$x^2 = 412{\cdot}9024$

$x = 20{\cdot}32$

> Apply Pythagoras's theorem and subtract the answers.

$\Rightarrow$ length = 4 × 20·32 = 81·28 = 81 to nearest cm

$\Rightarrow$ height = 3 × 20·32 = 60·96 = 61 to nearest cm

(81 × 61) − (89 × 50) = 491 cm^2

15. (a) $2\pi r = 7{\cdot}07$

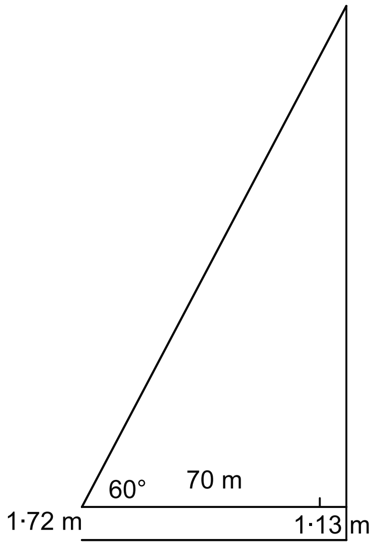
$6{\cdot}28r = 7{\cdot}07$

$\Rightarrow r = 1{\cdot}12579$ m

OR: The circumference can be used to calculate the radius, which will give the full distance that Maria is from the centre of the base of the spire.

> Height is measured from the centre of the base to the top of the spire.

(b)



> Maria's distance to the centre of the base is (70 + 1·13) m.
> Apply the Tan ratio $\frac{\text{Opposite}}{\text{Adjacent}}$ to solve.

$\tan 60° = \dfrac{x}{71{\cdot}13}$

$x = 123{\cdot}2$

Spire = 123·2 + 1·72 = 124·92 = 125 m

2015 SEC Supplementary Questions

1. Approximating the island with a conical shape, 916 m is represented with approx. 4 cm in the photograph.

$\therefore$ 229 m = 1 cm

The width of the island is approx. 11·3 cm.

$\therefore$ The radius is 5·65 cm = 1293·85 m

Volume of a Cone $= \frac{1}{3}\pi r^2 h$ ← See page 10 of *Formulae and Tables*.

$V = \frac{1}{3}\pi\,(1293{\cdot}85)^2\,(916)$

$V = 1{\cdot}6 \times 10^9$ m^3

2. **(a)** $n + 1, n + 2, n + 3$ ← Natural numbers differ by 1.

(b) $(n + 1) + (n + 2) + (n + 3)$

$= 3n + 6$

$\frac{3n + 6}{3} = n + 2$

i.e. divides evenly by 3

(c) $(n) + (n + 1) + (n + 2) + (n + 3)$

$= 4n + 6$

Since $4n + 6$ is not evenly divisible by 4, the sum of four natural numbers will never be evenly divisible by 4.

3. **(a)**

3	2	3
2	4	2
3	2	3

← Drawing lines can help to see where these numbers come from.

(b) Two lines

Each square can be reached by at least one vertical and at least one horizontal line.

(c) Maximum is 4 for ODD values of *n*. ← Draw grids for *n* an odd number and *n* an even number to understand this solution.

Maximum is 3 for EVEN values of *n*.

The only squares which belong to 4 lines are the “centre” squares in any grid. Centre squares will only occur for odd values of *n*.

4. Let the selling price be €220 and the VAT be 10%.

€220 = 110%

Divide both sides by 110.

€2·00 = 1%

Multiply both sides by 100.

€200 = Price before VAT

$\therefore$ VAT = €220 − €200 = €20

5. **(a)** **If** a triangle is right angled, **then** it has sides 3 cm, 4 cm and 5 cm.

> "Reverse" the order of Maisy's statement to build its converse.

(b) False

There are right-angled triangles with different side lengths, e.g. 5 cm, 12 cm and 13 cm.

> Not all converses are true statements.

6. **(a)** $5(x-3)$ ← H.C.F.

$2(3-x)$

(b) $\dfrac{5(x-3)}{2(3-x)} = \dfrac{5(x-3)}{-2(x-3)} = \dfrac{-5}{2}$ which is constant $\therefore$ proportional

> $\dfrac{(x-3)}{(x-3)} = 1$

(c) $\dfrac{x^2+3x+2}{2x+2} = \dfrac{(x+1)(x+2)}{2(x+1)} = \dfrac{x+2}{2}$ which is a variable

$\therefore$ Not proportional

> $\dfrac{(x+1)}{(x+1)} = 1$

> Factorise above and below

7. Let b = No. of hours in Bob's Bakery.

Let c = No. of hours in Ciara's Café.

> Solve simultaneous equations.

$11{\cdot}50b + 9{\cdot}30c = 362{\cdot}40$

$b + c = 34$… …(Multiply by $11{\cdot}5$)

$11{\cdot}50b + 9{\cdot}30c = 362{\cdot}40$

$11{\cdot}5b + 11{\cdot}5c = 391$… … (Subtract)

$-2{\cdot}2c = -28{\cdot}6$

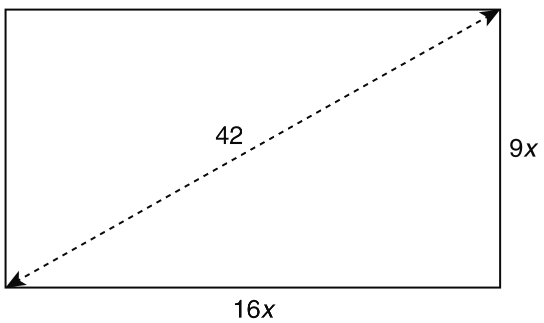
$c = 13$

$\therefore b = 21$ hours

8. **(a)** $\{6, 6, 6, 6, 6\}$

(b) Data sets in which all the elements are identical

9.



> Sides are in proportion 16:9.

$42^2 = (9x)^2 + (16x)^2$

$1764 = 81x^2 + 256x^2$

$1764 = 337x^2$

$\dfrac{1764}{337} = x^2$

$\dfrac{144 \, x 1764}{337} = 144x^2 =$ Area of television

$= 754$ inches^2